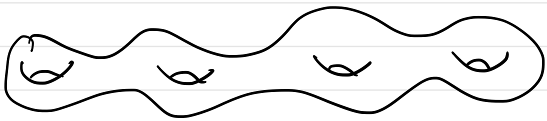
(complex dim.)

$U/\mathbb{C}$ smooth dimension one variety.

$U \subset \mathbb{P}^n_{\mathbb{C}}$ subvariety. Induced topology from the "usual" topology on $\mathbb{C} \cong \mathbb{R}^2$.

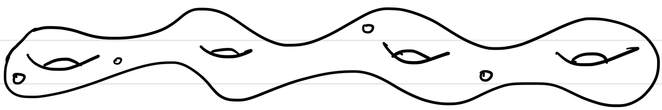
$\rightsquigarrow$ $U(\mathbb{C})$ is smooth two dimensional real manifold.

If $U(\mathbb{C})$ is compact (i.e. U is projective), then as a topological space it looks like



a doughnut with many handles.

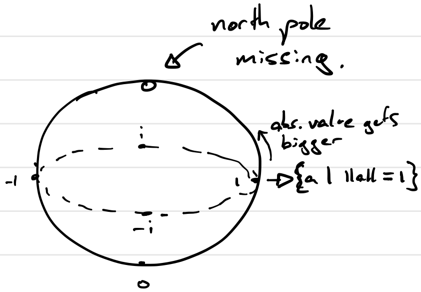
Not compact $\rightsquigarrow$ same picture but with some points removed



Examples

① $\mathcal{U} = \mathbb{A}^1 = \operatorname{Spec} \mathbb{C}[t]$

$U(\mathbb{C}) = \{a \in \mathbb{C}\} \cong \mathbb{R}^2 \cong$

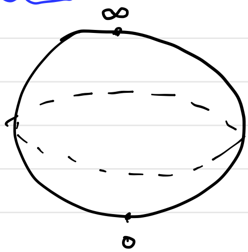


$\mathbb{C}[t] = \{\sum_{i=0}^{n} a_i t^i \mid a_i \in \mathbb{C}\}$

Prime ideals: (0), $(t-a)$, for $a \in \mathbb{C}$

② $\mathcal{U} = \mathbb{P}^1$

$\mathcal{U}(\mathbb{C}) \cong S^2$



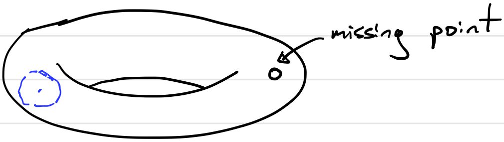
③ $\mathcal{U} = \operatorname{Spec} \dfrac{\mathbb{C}[x,y]}{y^2 = x(x-1)(x+1)}$

four dimensional real manifold

$\mathcal{U}(\mathbb{C}) = \{(a,b) \mid b^2 = a(a-1)(a+1)\} \subseteq \mathbb{C}^2 \cong \mathbb{R}^4$

two dimensional real manifold

inclusion



$a = a_0 + a_1 i$

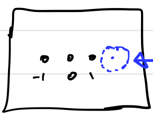
$b = b_0 + b_1 i$

$$\mathcal{U}(\mathbb{C}) \cong \left\{ (a_0, a_1, b_0, b_1) \in \mathbb{R}^4 \;\middle|\; \begin{array}{l} b_0^2 - b_1^2 = a_0(a_0-1)(a_0+1) + \dots \\ 2 b_0 b_1 = a_1^3 + \dots \end{array} \right\}$$

$$\begin{array}{ccc} \mathbb{C} & \to & \mathcal{U}(\mathbb{C}) \\ t & \longmapsto & (t, \sqrt{t^3 - t}) \end{array}$$

only defined up to ± 1

$\mathbb{C}$



can choose a value for $\sqrt{t^3 - t}$ in this ball.