$Y \xrightarrow{f} X$ continuous morphism of topological spaces.

can $\longmapsto$ define a sheaf on X

$$\underset{\text{open}}{U \subset X} \longmapsto \left\{ \begin{array}{l}\text{continuous}\\ \text{sections}\end{array} \quad \begin{tikzcd} & Y \arrow[d, "f"] \\ U \arrow[ur, dashed, "s"] \arrow[r, hook, "i"'] & X \end{tikzcd} \;\middle|\; fs = i \right\}$$

In fact, every sheaf on $X \in \mathrm{Top}$ is of this form. Google "espace étalé" or "étalé space".

So can think of sheaves on X as "spaces" over X.

Today: functoriality.

If $T \xrightarrow{g} S$ is a morphism of topological spaces.

$$T \times_S S' \longleftarrow S'$$

$$\begin{tikzcd} \mathrm{Top}/T \arrow[r, bend right] & \mathrm{Top}/S \arrow[l, bend right] \end{tikzcd} = \left\{ \begin{array}{c} S' \\ \downarrow \\ S \end{array} \text{ continuous} \right\}$$

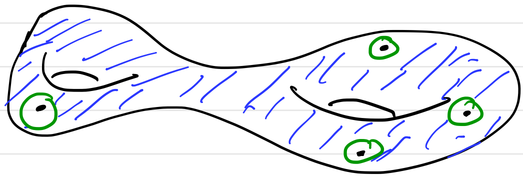
$$\begin{array}{c} T' \\ \downarrow \\ T \end{array} \longmapsto \begin{array}{c} T' \\ \downarrow \\ T \\ \downarrow \\ S \end{array}$$

Sheaf version:

$$\begin{tikzcd} \mathrm{Shv}(T) \arrow[r, bend right, "g_*"'] & \mathrm{Shv}(S) \arrow[l, bend right, "g^*"'] \end{tikzcd}$$

Algebraic curve, 1-dim. scheme, Riemann surface, ...

(proper nontrivial) closed subsets = finite sets of closed points (+ nilpotents)



Precise control over sheaves on curves.

Sheaf on generic point ⟵ Galois theory.
Sheaves on closed points ⟵
+ gluing data

$\left(\mathrm{Shv}(\underset{\text{field}}{K}) = \mathrm{Gal}(K^{sep}/K)\text{-rep.} \right)$

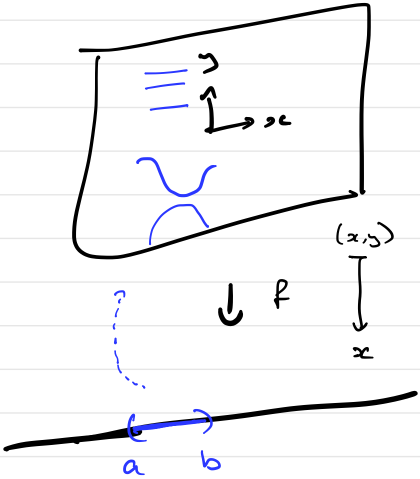
$\mathrm{Op}(Y) \leftarrow \mathrm{Op}(X)$

Top. $\left\{ Y \xrightarrow{f} X \right.$ $\quad F \in \mathrm{Shv}(X) \rightsquigarrow$ sheaf on Y ?

$f^*F : \left(\underset{\text{open}}{V} \subset Y \right) \longmapsto F(\cancel{f(V)})$ maybe not open

$\underset{f(V) \subseteq U}{\underrightarrow{\mathrm{colim}}} F(U)$

$$\begin{tikzcd} & & & \mathbb{R}^2 \arrow[d] \\ (a,b) = U \arrow[urrr, dashed, "\text{continuous}"] & \subset_i & & \mathbb{R}^1 \end{tikzcd}$$



For any continuous $\sigma : (a,b) \to \mathbb{R}$ we can define

$s : c \mapsto (c, \sigma(c)) \in \mathbb{R}^2$

then $f \circ s = i$

$$\begin{tikzcd} & & X \arrow[d, "f"] & \text{top.} \\ \emptyset, & \{*\} \arrow[ur, "x"] & \subset_i \{*\} & \text{one point.} \end{tikzcd}$$

pullback = fibre product

$$\begin{tikzcd} T \times_S X \arrow[r] \arrow[d, "g"'] & X \arrow[d, "f"] \\ T \arrow[r, "i"'] & S \end{tikzcd}$$

$T \times_S X$ is the fibre product of T and X over S.

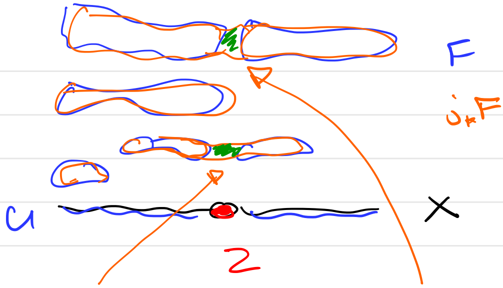
g is the pullback of f along i.

$T \times_S X$ is the pullback of X along i.

$\pi : C' \to C$ functor

$\pi^* F$ is the pullback.

$\pi_* F$ is the push forward.



$i^* j_* F$

fill in the gaps where possible

Constant: $\rightsquigarrow$ all gaps filled.

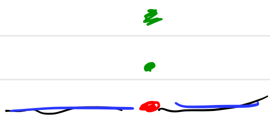
$s(F_1, F_2, \eta) \longrightarrow$

$j_* F_2$

$\downarrow$ $\qquad\qquad\qquad\qquad$ $\downarrow$

$i_* F_1$

$\xrightarrow{\eta}$



$i_* i^* j_* F_2$

$(F_1, F_2, \eta : F_1 \to i_* i^* j_* F_2)$