An A-algebra B is <u>finite type</u> if there is an isomorphism of A-algebras

$$B \cong A[x_1, \dots, x_n]/I$$

for some ideal $I \subseteq A[x_1, \dots, x_n]$, and some $n \in \mathbb{N}$.

An A-algebra B is <u>finite presentation</u> if there is an isomorphism of A-algebras

$$B \cong A[x_1, \dots, x_n]/I$$

for some finitely generated ideal $I \subseteq A[x_1, \dots, x_n]$, and some $n \in \mathbb{N}$.

<u>Remark:</u> If A is noetherian, then $A[x_1, \dots, x_n]$ is noetherian so every ideal $I \subset A[x_1, \dots, x_n]$ is finitely generated so in this case finite type = finite presentation.

<u>Theorem</u> if (A_λ) is a filtered system of rings, and a finite presentation A-algebra $A \to B$ (where $A = \varinjlim A_\lambda$) then $\exists\ \lambda$, and finite presentation A_λ-algebra $A_\lambda \to B_\lambda$ such that $B = A \otimes_{A_\lambda} B_\lambda$.

<u>Remark</u> Every ring is a filtered colimit of finite type $\mathbb{Z}$-algebras.

$$\operatorname{Spec}(A \otimes_{A_\lambda} B_\lambda) = \operatorname{Spec}(A) \times_{\operatorname{Spec}(A_\lambda)} \operatorname{Spec}(B_\lambda)$$

Theorem If $A \xrightarrow{f} B$ is an A-algebra, f is finite presentation if and only if for every filtered system (C_λ) of A-algebras

$$\hom_{A\text{-alg}}(B, \varinjlim C_\lambda) = \varinjlim \hom_{A\text{-alg}}(B, C_\lambda)$$

(i.e., finite presentation A-alg. are the compact objects in the category A-alg.)

Finite type: $A[x_1, \dots, x_n] \xrightarrow{\text{surjection}} B$

finite presentation:

$$\underbrace{\bigoplus_{i=1}^{m} A[x_1, \dots, x_n] \to A[x_1, \dots, x_n]}_{A[x_1,\dots,x_n]\text{ modules}} \underbrace{\twoheadrightarrow B}_{\text{rings}}$$

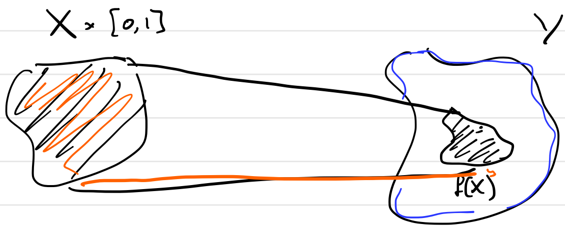
$f : A^\bullet \to B^\bullet$

$$\mathrm{Cone}(f)^n = B^n + A^{n-1}$$

$$d = \begin{bmatrix} d_B & (-1)^n f \\ & d_A \end{bmatrix}$$

① Topoligical version. $f : X \to Y \in \mathrm{Top}$.

$$\mathrm{Cyl}(f) = X \times [0,1] \coprod_{X \times \{1\}} Y$$



$\to \dfrac{X \times [0,1]}{X \times \{0\} \quad X \times \{1\}}$

$Y \longrightarrow$

$$\mathrm{Cone}\big((X,x) \to (Y,y)\big) = \frac{X \times [0,1] \coprod Y}{(x,t) \sim y \quad \forall\, t \in [0,1]}$$

$\downarrow$

$$SX = \frac{X \times [0,1]}{\begin{array}{ll} (x,t) \sim (x,0) & \forall\, t \\ (x',0) \sim (x,0) & \forall\, x' \\ (x',1) \sim (x,1) & \forall\, x' \end{array}}$$

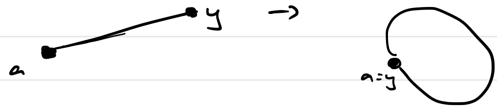
$$Y \to \mathrm{Cone}(f) \to SX$$

$\mathrm{Top.} \xrightarrow{\ \mathrm{Sing}_*\ } \mathrm{Cplx}$

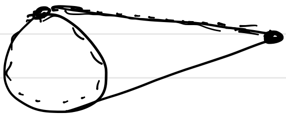
$$\boxed{② \quad B^\bullet \to \mathrm{Cone}(f) \to A^\bullet[1]}$$

$X = \{x, a\} \xrightarrow{f} Y = \{y\}$

$\text{Cone}(f) =$ SX



$X = S^1 \longrightarrow Y = \{y\}$



Cone SX