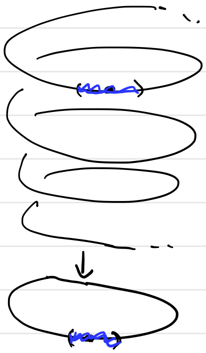
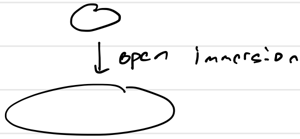
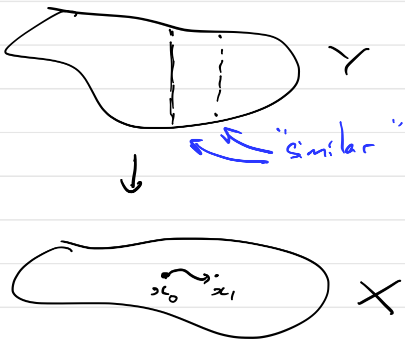
étale $\longleftrightarrow$ local homeomorphism

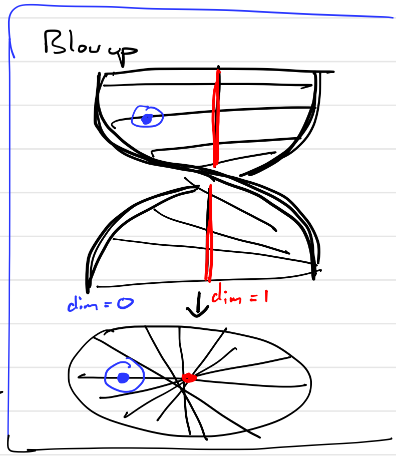


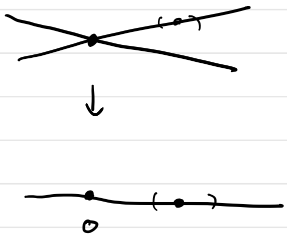


flatness.

local uniformity







$$\frac{k[x,y]}{(x+y)(x-y)}$$

$$k[x]$$

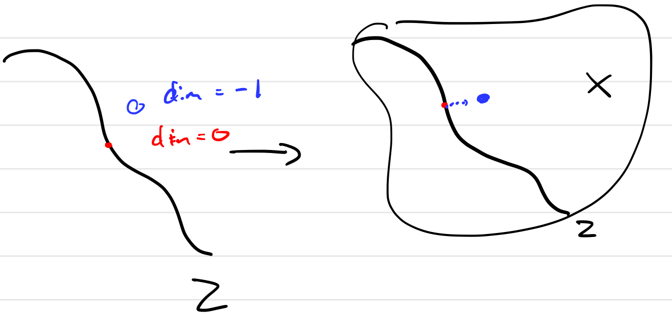
$$\left(\frac{k[x,y]}{(x+y)(x-y)}\right)_{(x,y)} = B_q$$

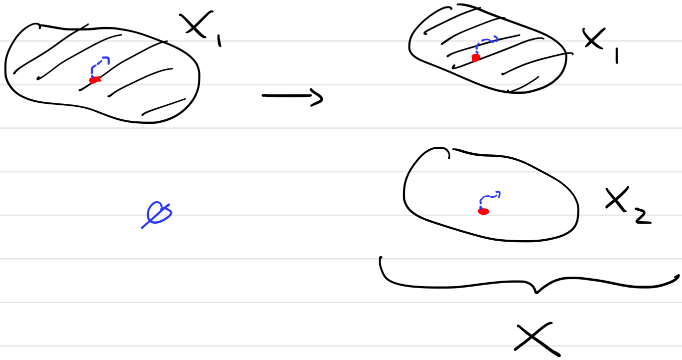
$$\uparrow$$

$$k[x]_{(x)} = A_p$$

$$(x)B_q \neq (x,y)$$

$A \to A/I \quad \longleftrightarrow \quad$ closed immersion $\mathrm{Spec}(A/I) \to \mathrm{Spec}(A)$





Definition

$$\cdots \xrightarrow{d_{i-1}} N_{i-1} \xrightarrow{d_i} N_i \xrightarrow{d_{i+1}} N_{i+1} \xrightarrow{d_{i+2}} \cdots$$

is exact at N_i if

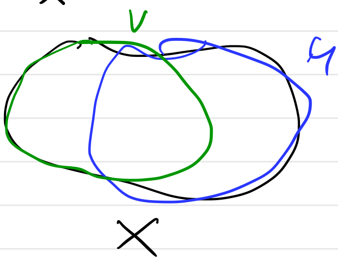
$$\mathrm{image}(d_i) = \mathrm{kernel}(d_{i+1})$$

(Abelian groups or A-modules

Example $\quad A \quad f,g \in A \quad$ s.t. $\quad \langle f,g \rangle = A$

$$\rightsquigarrow \quad \underset{\substack{\| \\ U}}{\operatorname{Spec} A_f} \amalg \underset{\substack{\| \\ V}}{\operatorname{Spec} A_g} \longrightarrow \underset{\substack{\| \\ X}}{\operatorname{Spec} A} \quad \text{open covering}$$

$$\{U \to X,\ V \to X\}$$



$B := A_f \times A_g$ faithfully flat A-algebra.

$$\left[\ 0 \to A \to B \to B \otimes_A B \to B \otimes_A B \otimes_A B \to \cdots\ \right]$$

$$\operatorname{Spec}\left(B \otimes_A \cdots \otimes_A B \right) = (U \amalg V) \times_X (U \amalg V) \times_X \cdots \times_X (U \amalg V)$$

$$= U \amalg U\cap V \amalg \cdots \amalg U \cap V \amalg V$$

$\exists \iff U = X$ or $V = X$

$\Phi : A \to B$ homo. of rings

$\Rightarrow B \in A\text{-mod}$

$$A\text{-Alg.} \xrightarrow{\text{forget}} A\text{-Mod}$$

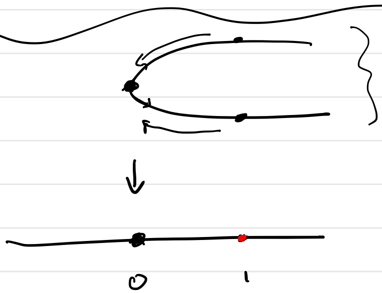
Def. Φ is flat if $\text{forget}(\Phi)$ is flat

Def.' $\Phi : A \to B$ is flat if $B \otimes_A - : A\text{-mod} \to A\text{-mod}$ preserves monomorphisms.

$z \in \mathbb{C}$ $\quad \mathbb{F}_2$

$\downarrow$

$z^2 \in \mathbb{C}$ $\quad \mathbb{F}_2$



x^2

$\uparrow$

x

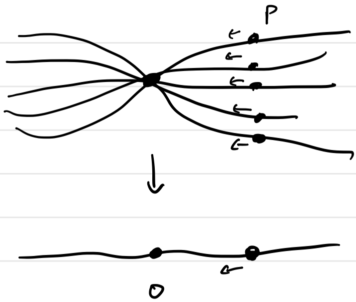
nilpotent

$k[x]/x^2$

$\uparrow$

k

2



$\mathbb{F}_p(t^{1/p})$

$\uparrow$ purely inseparable

$\mathbb{F}_p(t)$

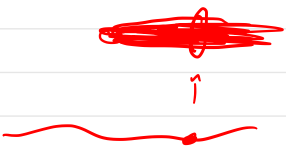
1

$\mathbb{A}^1_{\mathbb{F}_2}$ $\quad x^2$

$\downarrow$ $\quad \uparrow$

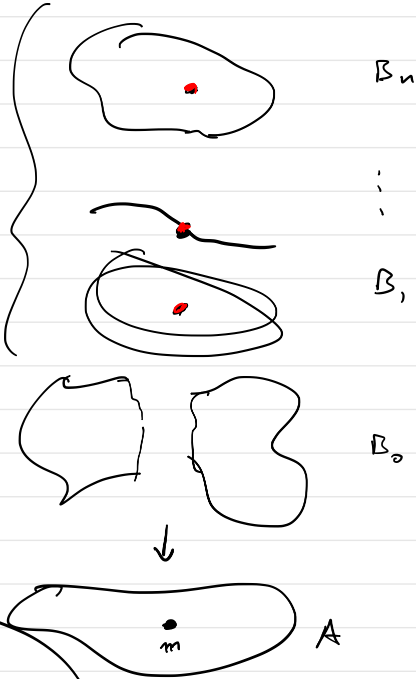
$\quad x$

$\mathbb{A}^1_{\mathbb{F}_2}$



$$\begin{array}{ccc} B & & \operatorname{Spec} B \\ \text{quasi finite} \uparrow & & \downarrow \\ A \ \text{henselian} & & \operatorname{Spec} A \end{array}$$

local



$X = \operatorname{Spec}(A)$ ← local

A is <u>hensel</u> iff

$\forall$ étale $f: Y \to X$ and $y \in f^{-1}(x)$ s.t. $k(y) = k(x)$

(x → closed point of X)

$\exists$ s.t. $y \leftarrow\!\shortmid\, x$

$$\begin{tikzcd} f^{-1}(x) = Y_x \arrow[r] \arrow[d] & Y \arrow[d, "\text{étale}"] \\ x \arrow[u, bend left, "\forall"] \arrow[r] & X \arrow[u, bend left, "\exists"] \end{tikzcd}$$