Exercise 1. Using Poincaré Duality, induction, and the fact that

$$H^r(\mathbb{P}^n, \mathbb{Z}) = \begin{cases} 0 & r \text{ odd} \\ \mathbb{Z} & r \text{ even}, r \leq 2n \end{cases}$$

Show that if $H_1, \dots, H_c$ are hypersurfaces such that each $H_1 \cap \cdots \cap H_i$ is smooth, then the cohomology of $X = H_1 \cap \cdots \cap H_c$ is

$$H^r(X, \mathbb{Z}) = \begin{cases} H^{\dim X}(X, \mathbb{Z}) & r = \dim X \\ 0 & r \text{ odd}, r \neq \dim X \\ \mathbb{Z} & r \text{ even}, r \neq \dim X, r \leq d \dim X \end{cases}$$

Exercise 2. Show that the projective variety defined by $st+xy$ in $\mathbb{P}^3$ is smooth. Show that the intersection with the hyperplane defined by $as - bt$ is smooth unless $a = 0$ or $b = 0$, in which case, it is isomorphic to a union of two $\mathbb{P}^1$'s intersecting in a point.

Exercise 3. Consider the quadric $uz - vy + wx$ in $\mathbb{P}^5$.

1. Show that the intersection of this quadric with the two hyperplanes $u = 0$ and $v = 0$ is isomorphic to a union of two $\mathbb{P}^2$'s, say P, Q, intersecting in a $\mathbb{P}^1$.

2. Let P', Q' be the two $\mathbb{P}^2$'s obtained analogously from $y = 0$ and $z = 0$. Using the fact that all hyperplanes in $\mathbb{P}^5$ are rationally equivalent, show that $P + Q \sim_{rat} P' + Q'$, and in particular, $P - P' \sim_{rat} Q - Q'$.

3. Show that $(P - P') \cdot (Q - Q') = -x - y$ for some points x, y.

4. Deduce that $\deg(P - P') \cdot (P - P') = -2 \neq 0$, and in particular, that $P - P'$ is not homologically equivalent to zero.