

Exercise 1. Let C be any curve. Show that the degree map $\mathcal{Z}^1(C) \rightarrow \mathbb{Z}$ which is defined by $\sum n_i Z_i \mapsto \sum n_i$ induces an isomorphism $\mathcal{Z}^1(C)/\mathcal{Z}_{alg}^1(C) \cong \mathbb{Z}$.

Recall that we defined classical motives using integral coefficients (Scholl uses rational coefficients). So for us

$$\text{hom}_{\mathcal{M}}((X, p, m), (Y, q, n)) = q \circ C_{rat}^{n-m+\dim X}(X \times Y) \circ p$$

(whereas Scholl uses $q \circ (C_{rat}^{n-m+\dim X}(X \times Y) \otimes \mathbb{Q}) \circ p$).

Exercise 2. Show that $CH^m(Y) = \text{hom}_{\mathcal{M}}((\text{Spec}(k), \text{id}, -m), (Y, \text{id}, 0))$.

Exercise 3. Show that an adequate equivalence relation \sim for smooth projective varieties which is coarser than rational equivalence is the same thing as a collection of morphisms in \mathcal{M} closed under composition and addition.

Exercise 4. Recall that a pairing $\phi : M \times N \rightarrow L$ of abelian groups is *non-degenerate* if for all $m \in M$ and $n \in N$ the induced maps $\phi(m, -) : N \rightarrow L$ and $\phi(-, n) : M \rightarrow L$ are injective.

Show that numerical equivalence is the finest equivalence relation which makes the pairing $CH^i(X) \times CH^{\dim X - i}(X) \rightarrow CH^{\dim X}(X)$ non-degenerate.