Exercise 1. Let $\mathcal{O} = k[x,y]_{(x,y)}$ where k is a field. Using the exact sequence $0 \to \mathcal{O} \xrightarrow{x} \mathcal{O} \to \mathcal{O}/(x) \to 0$ show that $\mathrm{Tor}_r^{\mathcal{O}}(\mathcal{O}/(x), \mathcal{O}/(y)) = k$ if $r = 0$, and is zero otherwise. Deduce that $i(V \cdot W; \mathcal{O}) = 1$ where $V = \mathrm{Spec}(k[x,y]/(x))$ and $W = \mathrm{Spec}(k[x,y]/(y))$.

Exercise 2. Let $\mathcal{O} = k[x,y]_{(x,y)}$ where k is a field. Using the exact sequence $0 \to \mathcal{O} \xrightarrow{x} \mathcal{O} \to \mathcal{O}/(x) \to 0$ show that $\mathrm{Tor}_r^{\mathcal{O}}(\mathcal{O}/(x), \mathcal{O}/(x-y^2)) = k[y]/y^2$ if $r = 0$, and is zero otherwise. Deduce that $i(V \cdot W; \mathcal{O}) = 2$ where $V = \mathrm{Spec}(k[x,y]/(x))$ and $W = \mathrm{Spec}(k[x,y]/(x-y^2))$.

Exercise 3. Let $f : X \to Y$ be a morphism of varieties over an algebraically closed field, and let $\Gamma_f \subseteq X \times Y$ be the graph. If $Z \subseteq X$ is a closed subvariety, show that $\Gamma_f \cap (Z \times Y) = (\mathrm{id} \times f)(Z) \subseteq X \times Y$, so in particular, $T \cdot (Z \times Y)$ is defined. Show that $(\pi_Y)_*\{T \cdot Z \times Y)\} = f_* Z$.

Exercise 4. Let X be a variety, $Z, Z' \in \mathcal{Z}^i(X)$, and $W = \sum n_k W_k \in \mathcal{Z}^j(X)$. Using $\sum n_k(\mathbb{P}^1 \times W_k) \in \mathcal{Z}^j(\mathbb{P}^1 \times X)$, show that if $Z \sim Z'$, and $Z \cdot W, Z' \cdot W$ are defined, then $Z \cdot W \sim Z' \cap W$. That is, show that rational equivalence satisfies (R2).

Exercise 5. Using the diagonal $\mathbb{P}^1 \subset \mathbb{P}^1 \times \mathbb{P}^1$, show that for any two points $a, b \in \mathbb{P}^1$ we have $a \sim b$. Deduce that the degree map $\mathcal{Z}^1(\mathbb{P}^1) \to \mathbb{Z}$ defined by $\sum n_i a_i \mapsto \sum n_i$ induces an isomorphism $CH^1(\mathbb{P}^1) \cong \mathbb{Z}$.

Exercise 6. Using the decomposition $\mathbb{P}^n = \mathbb{A}^n \cup \mathbb{P}^{n-1}$, the homotopy property, and the localisation sequence, show by induction on n that $CH_q(\mathbb{P}^n) \cong \mathbb{Z}$ for all $0 \leq q \leq n$.