Tokyo-Seoul Conference in Mathematics, 2023
Topology and Geometric Group Theory
October 6–7, 2023

Lecture Hall in the Mathematical Science Building,
the University of Tokyo
3-8-1 Komaba Meguro-ku, Tokyo 153-8914, Japan

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Organizers: Nariya Kawazumi (The University of Tokyo), Inkang Kim (KIAS),
Sang-hyun Kim (KIAS), Takuya Sakasai (The University of Tokyo).

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Schedule with Titles

October 6 (Friday)

10:00–11:00,
Se-Goo Kim (Kyunghee University)
Topological and smooth concordances of knots

11:20–12:20,
Kazuki Kannaka (RIKEN)
Zariski dense discontinuous surface groups for reductive symmetric spaces

14:00–15:00,
Minkyu Kim (KIAS)
Finite path integral model and toric code based on homological algebra

15:20–16:20,
Mai Katada (The University of Tokyo)
Stable rational cohomology of the IA-automorphism groups of free groups

16:40–17:40,
Hyungryul Baik (KAIST)
Constructing 3-manifolds from 1-dimensional actions

18:00–, **Reception** at the Room 222 ("Common Room")

October 7 (Saturday)

10:00–11:00,
Kazuo Habiro (The University of Tokyo)
Johnson homomorphisms for handlebody groups

11:20–12:20,
Anderson Vera Arboleda (IBS Center for Geometry and Physics)
Around the Le-Murakami-Ohtsuki invariant

14:00–15:00,
Yoshikata Kida (The University of Tokyo)
Orbit equivalence, treeings, and Baumslag-Solitar groups

15:20–16:20,
Inhyeok Choi (KIAS)
Genericity of contracting elements in groups

16:40–17:40,
Mikio Furuta (The University of Tokyo)
A gerbe-like construction in gauge theory

Abstracts

Hyungryul Baik (KAIST)

Constructing 3-manifolds from 1-dimensional actions

Some groups acting on 1-dimensional spaces can be shown to be 3-manifold groups. I will explain how to construct 3-manifolds from those actions. This talk is based on a joint work with Hongtaek Jung and KyoungRo Kim.

Inhyeok Choi (KIAS)

Genericity of contracting elements in groups

Given a finitely generated group G, one can discuss the dynamics of each element of G via their action on the Cayley graph of G. When G is hyperbolic in the sense of Gromov, the action of an element of G is either elliptic or loxodromic, the latter case being generic. In this talk, I will explain the converse of this statement. Namely, we deduce the global hyperbolicity of G from the genericity of its contracting elements, which is a natural notion that generalizes loxodromic elements. This is based on joint work with Kunal Chawla and Giulio Tiozzo.

Mikio Furuta (The University of Tokyo)

A gerbe-like construction in gauge theory

Recently Mitsuyoshi Adachi obtained the following result: For a homotopy K3 surface X, the rank-7 real vector bundle $TX \oplus H^+(X)$ has a canonical spin structure. In this talk we explain his construction using the Seiberg-Witten monopole equations and its implications. Actually it is regarded as a construction of an isomorphism between appropriate two $\mathbb{Z}_2$-gerbes on $B\,\mathrm{Diff}(X)$. We also explain an idea to generalize the construction to Donaldson theory, which is a work in progress with M. Adachi and Y. Morita.

Kazuo Habiro (The University of Tokyo)

Johnson homomorphisms for handlebody groups

The Johnson filtrations and the Johnson homomorphisms are important tools in the study of the mapping class groups of surfaces. The handlebody group is the subgroup of the mapping class group consisting of mapping classes that extend to a fixed handlebody bounded by the surface. I plan to explain analogues of the Johnson filtration and homomorphisms for the handlebody group. This is a joint work with Gwenael Massuyeau.

Kazuki Kannaka (RIKEN)

Zariski dense discontinuous surface groups for reductive symmetric spaces

Let $G \supset H$ be two real reductive algebraic groups, and let Σ be an orientable closed surface of high genus. We study faithful and discrete representations of the fundamental group $\pi_1(\Sigma)$ into G such that the action of $\pi_1(\Sigma)$ on the homogeneous space G/H is properly discontinuous. Our main concern is when G and H are both non-compact. In this case, the assumption of proper discontinuity imposes a strong constraint. In this talk, I would like to discuss whether a Zariski dense representation exists among such representations. This talk is based on a joint work with Takayuki Okuda and Koichi Tojo.

Mai Katada (The University of Tokyo)

Stable rational cohomology of the IA-automorphism groups of free groups

The IA-automorphism group IA_n of the free group F_n is a normal subgroup of the automorphism group of F_n, which is an analogue of the Torelli groups of surfaces. We study the stable rational cohomology of IA_n, and its subgroup that is obtained as the image of the map induced by the abelianization map of IA_n. We also consider the stable rational cohomology of the Torelli groups of surfaces.

Yoshikata Kida (The University of Tokyo)

Orbit equivalence, treeings, and Baumslag-Solitar groups

Given two probability-measure-preserving (p.m.p.) actions of countable groups, we say that they are orbit equivalent if there is an orbit-preserving isomorphism between the spaces on which the groups act. The study of orbit equivalence dates back to 1950s, originally motivated by the study of operator algebras. Treeings are a key notion in this study, which is a counterpart of free generating systems of the free groups. I will focus on the background and show my recent result on p.m.p. actions of Baumslag-Solitar groups and related groups.

Minkyu Kim (KIAS)

Finite path integral model and toric code based on homological algebra

Finite path integral is a mathematical methodology to construct TQFT's (topological quantum field theories) from finite gauge theory. In three dimensions, it is generalized to state sum models closely related with finite Hopf algebra gauge theory. Toric code originated from topological quantum computation provides a combinatorial description of state sum models. This talk concerns a refinement of finite path integral models and toric code by restricting to bicommutative Hopf algebras. The category of bicommutative Hopf algebras is abelian so that we can consider homological algebra based on bicommutative Hopf algebras. By replacing Hopf algebra gauge theory with homological algebra based on bicommutative Hopf algebras, we give a framework to obtain toric code from chain complexes; and TQFT from homology (more generally, Mayer-Vietoris functors). It turns out that the ground state space of our toric code is isomorphic to homology (valued in the

category of bicommutative Hopf algebras), which extends to a projective TQFT. We also give our results about vanishment of the projectiveness.

Se-Goo Kim (Kyunghee University)

Topological and smooth concordances of knots

A slice knot in the 3-sphere bounds a slice disk, which is a 2-disk in the 4-ball whose boundary is the 3-sphere. Depending on whether a slice disk can be embedded locally flatly or smoothly, a slice knot is said to be topologically or smoothly slice, respectively. Smoothly slice knots are topologically slice. However, there are many topologically slice knots that are not smoothly slice. The first example was derived from Freedman's work on topological surgery and Donaldson's gauge theoretic approach to four-manifolds. Since Ozsváth and Szabó introduced the theory of Heegaard Floer homology, there have been a surge on studying this topic. We survey recent results.

Anderson Vera (IBS Center for Geometry and Physics)

Around the Le-Murakami-Ohtsuki invariant

The Le-Murakami-Ohtsuki invariant is a powerful invariant of 3-manifolds (universal among finite type invariants) taking values in a space of Jacobi diagrams. Its original definition uses the Kontsevich integral of links, the so-called iota maps and several projection maps between different quotients of spaces of Jacobi diagrams. In this talk we explain how to omit one of such projection maps without losing the invariance property. (This talk is based on a joint work with Benjamin Enriquez)