Discussion: Emergence of the concept of the basic reproduction number from mathematical demography

1. Introduction

Nishiura et al. (2006) recently reported the earliest explicit documentation of the basic reproduction number, \( R_0 \), proposed by Dr. Theophil Lotz in 1880. Theoretical explanations by Dr. Lotz concerning the geometric increase in smallpox cases were highly successful, and his impressive description of herd immunity was most appropriate for his time (Lotz, 1880). In the present paper, we introduce two earlier studies to facilitate the discussion regarding the explicit documentation of the basic reproduction number, time (Lotz, 1880). In the present paper, we introduce two earlier studies to facilitate the discussion regarding the emergence of the concept of \( R_0 \) in mathematical demography, where \( R_0 \) is defined as the average number of offspring an individual in a population will produce in his/her lifetime. Although Theophil Lotz and Adolf Gottstein explicitly used \( R_0 \) in describing the geometric progression of directly transmitted diseases in the late 19th century (Lotz, 1880; Gottstein, 1897), the concept of \( R_0 \) in demography is believed to have emerged independently at a similar time (Böckh, 1886; Farr, 1880; Lewes, 1984; Dietz, 1993; Heesterbeek, 2002) and these two closely related measures remained unconnected until later researchers, most notably Alfred Lotka (1880–1949), discussed the mathematical theory in both fields (Sharpe and Lotka, 1911; Lotka, 1923; Heesterbeek, 2002). While the discussions of \( R_0 \) in demography most likely started in the late 19th century, the relevant conceptual frameworks, which are useful for enhancing understanding of the emergence of \( R_0 \), were conducted 50–100 years earlier by the French mathematicians Drs. Emmanuel-Etienne Duvillard (1755–1832) and Francis Corbaux (1769–1843).

2. Duvillard's survival probability of marriage

Dr. Emmanuel-Etienne Duvillard was a mathematician who contributed to the development of theories in finance, physics, epidemiology and demography (Duvillard, 1787, 1806, 1825). He was born on April 2, 1755, in Geneva to French Protestants who moved to Switzerland before his birth. Dr. Duvillard himself returned to Paris in January 1775, when he was employed as an administrator in the Comptroller General of Finance, France. Although he returned to his native town of Geneva after the expiration of the appointment, he once again moved to Paris in August 1786, after receiving the approval of the French Academy of Science (Biondi, 2003). From 1805, he was appointed head of the statistical department of population in the office of the French Ministry of the Interior and, thereafter, continued to work on mathematical issues of population science (Quiquet, 1934; Bourdelais, 1977).

There are two famous contributions by Dr. Duvillard to the foundation of theories in economics and demography: (i) a calculation of the modified internal rate of return with respect to annuity (Duvillard, 1787; Biondi, 2003) and (ii) one of the earliest uses of a mortality table in demography in relation to smallpox deaths in France (Duvillard, 1806; Duplaquier, 1985). In the latter work, Dr. Duvillard mainly explored the law of mortality (i.e., age-specific force of death), resting on the assumption of a stationary population (Dietz and Heesterbeek, “Epidemics: the discovery of their dynamics”, in preparation), and applied a model of smallpox to the observed death-registry data in France, the idea of which was originally inspired by a preceding work by Daniel Bernoulli (1700–1782) (Bernoulli, 1766; Duvillard, 1806; Dietz and Heesterbeek, 2000, 2002). Although the mortality table relevant to smallpox research in 1806 was relatively well known among specialists, the age-specific force of death also appeared in an earlier work, entitled “Recherches sur les rentes, les emprunts et les remboursements” (Research on annuities, loans and refunding), where the demographic information was used as a basis for his discussions on annuities (Duvillard, 1787; see online supporting material 1 for the original). Moreover, as a relevance to Dr. Duvillard’s argument on the mortality table, he estimated the survival probability of marriage using the observed data in Geneva.

First, Dr. Duvillard estimated the age- and sex-specific survivorship of the entire population based on the registered mortality data in the town of Geneva during the mid- and late-18th century. Fig. 1 shows the obtained age-specific survivorship by sex. By carefully studying this figure, he discussed the following:

These curves show that there are more females than males, even though there are more males at birth, considering that the sex ratio at birth is approximately 22 to 21. From these curves, which explain the law of mortality, a conclusion can immediately be made: this results in a fate that a great number of females who marry will become widows someday; i.e., because, according to the natural order, males aged 25, 30 and
35 years get married to females 5 years younger, aged 20, 25 and 30 years old (p. 57, authors’ translation).

Dr. Duvillard noted that not all marriages would contribute to reproduction, considering the age-specific survivorship. Next, he started to discuss the numerical example based on the population in Geneva:

Suppose that all males at 25 years of age and females at 20 years of age marry at once. The population size of 25-year-old males in Geneva is 10,948. Also, there are 15,223 females aged 20. After 5 years, there are 10,182 males aged 30 and 14,485 females aged 25 (p. 57).

A numerical illustration of the age- and sex-specific survivorship is given in the first 4 columns of Table 1. Next, Dr. Duvillard examined the survival probability of marriage using simple equations (i.e., age-specific survivorship of marriage):

Let us assume that women will not marry again after becoming widows. I consider 100 males aged 25 and 100 females aged 20 who marry at exactly these ages. Let \( Y \) and \( y \) represent the numbers of males who survived and died after marriage. And let \( Z \) and \( z \) represent the numbers of females in the same way, except that the calculation for females starts from 20 years of age. Finally, let \( M = 100 \), the number of marriages at the beginning, and I get:

\[
M = Y + y \quad \text{and} \quad M = Z + z.
\]

Also, I get:

\[
MM = YZ + yZ + Yz + yz
\]

and therefore,

\[
M = \frac{YZ}{M} + \frac{yZ}{M} + \frac{Yz}{M} + \frac{yz}{M}.
\]

According to the principles of the calculation of probabilities, \( YZ/M \) would be the number of marriages in which the husbands and wives are both alive. \( yz/M \), \( y(1-Z/M) \) or \( z(1-Y/M) \) denote the marriages in which both the husbands and wives have died. \( YZ/M \) or \( Y(1-Z/M) \) is the number of existing widowers and \( yZ/M \) or \( Z(1-Y/M) \) is the number of widows. By means of this calculation, one can see that the marriages would have only half the couples alive at 23 years after marriage and both dead would exceed 50% at 47 years after marriage (p. 58–59, see also Table 1).

Based on this argument, Dr. Duvillard reached the conclusion that it is extremely important to take into account the survival probability of marriage in order to properly discuss the fertility. In a later unpublished work (Duvillard, Rapports des Naissances, the Academy of Sciences, 1796, unpublished; Quiquet, 1934), he used the above theory to validate an estimate of marital fertility. First, by dividing the annual number of births by the annual number of marriages, he obtained an estimate of approximately 3.5 births per marriage. Second, by dividing the annual number of births by a product of the existing number of marriages which lasted for 10 years and an inverse of the survival probability of 10 years after marriage (i.e., \( 1/0.78 \)), he obtained a similar estimate (i.e., 3.5 births), from which he concluded that there would be an average of 3.5 births assignable to each marriage. In other words, he obtained a similar estimate of marital fertility by using the survival probability of marriage.

Although the survival probability of marriage as well as the model of smallpox death were rather remarkable in the progress of mathematical theory in demography, and despite the fact that his work was one of the pioneering uses of a mortality table, his contributions were sharply criticized by other specialists in the 19th century, thereby dampening the reputation of Dr. Duvillard (Thuillier, 1997). This was partly attributable to the fact that Dr. Duvillard’s arguments were based on a mixture of observed data in Geneva and Paris, such that the numerical exercises were confusing for political demography at that time. Moreover, he published the mortality table in relation to a quarrel against the management of the Bureau of Statistics in France (Dupaquier, 1985).

3. Corbaux’s marital fertility

More than 40 years after Dr. Duvillard’s work, further numerical exercises were performed by Dr. Corbaux on the
Table 1
Age-specific survivorship of males and females surviving after marriage, based on the mortality table for Geneva during the first 74 years of the 19th century (from Duvillard (1787, pp. 60–61)

<table>
<thead>
<tr>
<th>Age (years)</th>
<th>Males</th>
<th>Females</th>
<th>Remaining and dead husbands</th>
<th>Remaining and dead wives</th>
<th>Existing marriages</th>
<th>Widowers</th>
<th>Widows</th>
<th>Extinct marriages</th>
</tr>
</thead>
<tbody>
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<td></td>
<td>Age</td>
<td>Population</td>
<td>Years after marriage</td>
<td>Y (alive)</td>
<td>z (dead)</td>
<td>Years after marriage</td>
<td>Z (alive)</td>
<td>Z (dead)</td>
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<td>0</td>
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<td>75</td>
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<td>100,000</td>
<td>80</td>
<td>0,066</td>
<td>99,934</td>
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</table>


359
issue of fertility, which were documented in his contribution entitled “On the natural and mathematical laws concerning population, vitality, and mortality” published in London (Corbaux, 1833; see online supporting materials 2 and 3). Before this publication, Dr. Corbaux was actively involved in applied mathematics, publishing mathematical studies on physics, macroeconomics and annuity (Corbaux, 1812, 1824, 1825). Although it is known that Dr. Corbaux was born in France and thereafter moved to England, the details of his educational background and exact time of immigration are, to the best of our knowledge, unavailable at the present time. He is, however, known to be the father of the famous painter Fanny Corbaux [1812–1883], whose formal name was Marie Francoise Catherine Doetter Corbaux (Matthew and Harrison, 2004). Although Dr. Corbaux’s work led him to become a Fellow of the Royal Society in 1834, he withdrew on March 20, 1841, stating that he was “precluded by infirmities and other circumstances”. According to his literature and the Election Certificate of the Royal Society, he lived in Winchelsea, East Sussex, UK, in 1824 (Corbaux, 1824) and London and Paris in 1833 (Archive Catalogue, The Royal Society, 1834; Corbaux, 1833).

His literature on mathematical demography is concerned with various aspects of the entire population of France based on data obtained in the early 19th century, especially from 1817 to 1830. He mainly covered mortality with time, paying particular attention to the numerical exercises concerning heterogeneous patterns by region and/or country, and age and sex. Unfortunately, Dr. Corbaux’s analytical work was not based on firm knowledge of the stationary population, and thus, his contributions remain relatively unknown or neglected to date. Nevertheless, in two chapters, Dr. Corbaux discussed detailed observations of fertility that he implicitly deemed an important measure of the population (Corbaux, 1833). Specifically, Dr. Corbaux explored marital fertility in each year in France from 1817 to 1830. After a brief introduction of his specific questions concerning the law of mortality, he documents his motivations to numerically examine marital fertility:

Other questions, hitherto unsettled, concerning the relative quantities of births of males and females, the quantities of periodic marriages compared with the amount of any populations, and also the average quantity of births generally attributable to each marriage, are carefully investigated from incontestable data; and it is shown that all observed variations of such results proceed from ascertainable causes, amongst which the population’s special distribution into relative quantities of each sex and existing within certain intervals of age; a distribution materially influenced by such a population being either stationary, or else in progress of increase or of decrease (preface, p. xii).

Dr. Corbaux’s estimates of what he called “the average quantity of births attributable to each marriage” are shown in Table 2. As his estimates fluctuated slightly, he further attempted to calculate the average of the average quantity, which he called “the periodic averages”, from 1817. Observing that both estimates gradually decreased over time, he discussed the potential reason as follows:

An increasing population determines a more elevated proportion of early marriages, and therefore of corresponding births; both of which circumstances are consequent on the proportion of youthful population—in the distribution of the whole—being then superior to what it is in a stationary population; and an apparent decrease of the births, compared with the

<table>
<thead>
<tr>
<th>Year</th>
<th>Marriages during each year</th>
<th>Births during each year</th>
<th>Average births per marriage</th>
<th>Marriages during intervals of five consecutive years</th>
<th>Births during intervals of five consecutive years</th>
<th>Average births per marriage during intervals of five years</th>
<th>Periodic average births, from 1817</th>
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marriages, will follow any diminution of the ratio according to which the population progressed (p. 143).

In other words, he attributed the decline in his estimates to an increasing population, in which estimates are influenced by a large proportion of young generation who produce more marriages and births than those of a stationary population. Dr. Corbaux then critically reviewed similar estimates in previously published studies and discussed differences resulting from the population structures of other European countries (Dr. Duvillard’s work in France was also one of the subjects of Dr. Corbaux’s critiques). Unfortunately, Dr. Corbaux neither stratified his estimates by maternal age nor explained the mechanisms of population growth based on these estimates. The preface and relevant chapters are given in supporting online materials 2 and 3.

4. Discussions over the history of $R_0$ in demography

The above-described two impressive previous works by French mathematicians specifically measured marital fertility. It should be noted that Drs. Duvillard and Corbaux were not the originators of marital fertility. To the best of our knowledge, Willem Kersseboom (1691–1771), a Dutch mathematician, is known as the first person to suggest using the ratio of annual births to annual marriages as an implication of the population growth (Kersseboom, 1738; Behar, 1987) and, indeed, both Drs. Duvillard and Corbaux referenced Kersseboom’s work conducted during the early 18th century. Moreover, similar implicit use of this measure was followed by Drs. Johann Peter Süssmilch (1707–1767) and Thomas Robert Malthus (1766–1834), both known as founders of mathematical demography (Süssmilch, 1740; Malthus, 1803; Behar, 1987; Hecht, 1987). Although we could not sufficiently trace the influences of Drs. Duvillard and Corbaux on later works in France, marital fertility using the same calculation came to be used frequently by French demographers in the late 19th and early 20th century. A theoretical flaw in relying on marriage and this crude measure was later criticized (Cole, 1996).

To digest the contributions of the two previous works appropriately, we next discuss this issue with an explicit mathematical definition of $R_0$. Let $p(a)$ and $β(a)$ denote the probability of a woman surviving to age $a$ and the rate of giving birth to a girl for an individual of age $a$, respectively. The basic reproduction number, $R_0$, also known as the net reproductive ratio (NRR), in demography is given by (Coale, 1972; Pollard, 1973; Keyfitz, 1977; Dietz, 1993):

$$R_0 = \int_0^\infty p(a)β(a)da.$$  \hspace{1cm} (1)

As far as we understand, an explicit description of $R_0$ originated from Richard Böckh (1824–1907) (Böckh, 1886; Dietz, 1993; Heesterbeek, 2002), who initially designated the ratio “the total reproduction of the population”. Thereafter, Kuczynski (1932) and Dublin and Lotka (1925) developed mathematical theories following and not following the idea of Böckh, respectively. We need to clarify the differences between Böckh and the two earlier French studies as well as the key points to deem the measure of Böckh as an explicit pioneering work on $R_0$.

First, rather than $R_0$, the theoretical concept of marital fertility given in the two earlier studies is specifically closer to what is presently referred to as the period total fertility rate (PTFR) at time $t$, $Λ(t)$:

$$Λ(t) = \int_0^\infty β(t,a)da = \int_0^\infty \frac{B(t,a)}{n(t,a)} da,$$  \hspace{1cm} (2)

where $B(t,a)$ and $n(t,a)$ are the density of newborns, whose mothers are aged $a$, at time $t$ and the total female population at age $a$ and time $t$, respectively (Alho & Spencer, 2005). Here, it should be noted that the denominator, $n(t,a)$, is not the number of marriages but the number of females. In other words, although the registered number of marriages may have been useful during the late 18th and early 19th century, the definition of this key function was not appropriate for discussing fertility in relation to population growth (e.g., there were substantial numbers of illegitimate births). Even provided that the use of marriage was acceptable, the explicit mathematical definition of $R_0$ in demography requires the age-specific fertility $β(a)$ and age-specific survivorship $p(a)$. Although Dr. Duvillard’s work was conducted on the basis of understanding the age-specific survivorship, the age-specific fertility was not seen in either study and, indeed, was seldom discussed until the late 19th century, with the exception of a few rigorous studies (Duncan, 1864, 1867a, b; Clark, 1949).

Second, accordingly, they could not attribute the estimates to the ratio according to which the population progresses. From a theoretical viewpoint, this may be the most significant difference between Böckh and the two earlier studies in France. In other words, the difference is highlighted not only in the mathematical notations (i.e., the differences between Eqs. (1) and (2)) but also by an explicit statement to explain the progress of the population. Consequently, the two earlier works by Drs. Duvillard and Corbaux do not permit us to estimate the population growth or Malthusian parameter.

Third, we have to address the issues of social history with regard to communications and/or inspirations of knowledge between researchers. Unfortunately, the two studies in France were not referenced later as being of relevance to $R_0$ by the originators, namely Böckh (1886), Kuczynski (1932), and Dublin and Lotka (1925). Therefore, there is no particular evidence suggesting that the invention of $R_0$ was significantly influenced by their discussions on marital fertility. With the exception of some later works in France, the two previous studies were not effectively associated with later demographers in other countries. This may be very typical among early studies during the late 18th and 19th century, leading to independent progress among the
It is worth noting that the mass action principle, first suggested by William Hamer (1862–1936) (1906), was also unknown to later researchers, such as Anderson Gray McKendrick (1876–1943) and Ronald Ross (1857–1932), in infectious disease epidemiology (Heesterbeek, 2005).

5. Relationship between \( R_0 \) and marital fertility

To appropriately understand the reason why the idea of marital fertility could be seen as a prelude to \( R_0 \), we briefly sketch other histories in the 20th century. In fact, under appropriate circumstances, we can calculate \( R_0 \) based on the knowledge of marital fertility.

More modern discussions on reproduction indices that can take marriage phenomena into account were enhanced during World War II (Karmel, 1947). Since World War II promoted or delayed marriages and the birth rate trends varied irregularly, the \( R_0 \) of Lotka and Kuczynski became recognized as insufficient as an index of reproduction and, consequently, the relevant demographers started to explore potential alternatives, incorporating the process of marriage and further improvements (Pollard, 1948). Although these nuptiality-fertility related studies have not necessarily been well succeeded among later western demographers, Japanese studies have continued to pay much attention to the effect of marriage on reproduction indices, since there is a strong tradition of monogamous marriage in Japan and ex-nuptial phenomena (cohabitation and childbearing outside of legal marriage) are still rare (the proportion of illegitimate births is less than 2% of the total births). Under the assumption that only first marriage couples produce newborns, Ito (1978) and Inaba (1992, 1995) proposed a calculation formula for \( R_0 \) as follows:

\[
R_0 = \gamma \int_0^\infty S(a)\phi(a)p(a)da, \tag{3}
\]

where \( \gamma \) denotes the proportion of female newborns, \( S(a) \) is the expected total number of children produced per marriage for those married at age \( a \), and \( \phi(a) \) is the probability of the first marriage occurring at age \( a \).

Although we omit the explicit formula of \( S(a) \) here, it can be calculated from the duration-dependent marital fertility rate for those married at age \( a \) and the force of dissolution of couples (Inaba, 1995). If the duration-dependent marital fertility rate is constant, \( S(a) \) is the product of the constant marital fertility rate and the duration of marriage for those who married at age \( a \).

If we further assume that \( S(a) \) is independent of marriage at age \( a \) and the effect of the survival rate \( p(a) \) is negligible, it follows from Eq. (3) that \( R_0 \) is the product of the expected total number of female children produced per marriage given by \( \gamma S \) and the proportion of ever marrying, \( \phi(a) \), calculated from the annual number of births by the annual number of marriages.

Furthermore, a recent study reported that among European countries in the Mediterranean littoral, a formula similar to Eq. (3) is very useful for understanding changes in below-replacement fertility (Billari et al., 2000).

6. Conclusion

In summary, this paper presented two French studies on marital fertility and discussed the emergence of \( R_0 \) in mathematical demography. The key issues in the genesis of \( R_0 \) were highlighted according to (i) the mathematical definition, (ii) an explicit note of the population growth and (iii) interactions between researchers and contributions to the relevant works. Although French mathematicians were very early to discuss a stationary population theory and apply it to observed data, the two studies presented in this paper were not explicit enough to attribute their measures to the population growth. Therefore, the next question arising from a historical viewpoint would be: Are there any unknown researchers or studies, at a similar time, which could possibly invent \( R_0 \)? Considering that Leonhard Euler (1707–1783) described the geometric progression of population growth during the 18th century (Euler, 1760), it is not surprising that some mathematicians or demographers understood the concept of \( R_0 \) either implicitly or explicitly, in the late 18th or early 19th century. Since the researchers of this time briefly presented their findings only at local meetings and communicated the technical details in personal letters (i.e., personal letters rather than publications in periodicals), such historical materials may be useful for shedding further light on the history of \( R_0 \). Since such a historical theory is described in a rudimentary fashion, we believe the relevant materials will also probably help toward understanding the basis of the demographic model in the field of theoretical biology.

7. Supporting online information

The following supporting information accompanies the paper on the journal’s website (www.elsevier.com/locate/jtbi).

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Appendix A. Supplementary materials

The online version of this article contains additional supplementary data. Please visit doi:10.1016/j.jtbi.2006.08.004.
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