Hayama Symposium on Complex Analysis in Several Variables XXIII

July 23–26, 2022 at LecTore Hayama, Room 201 July 27–28, 2022 at the University of Tokyo, Room 002 "Z" indicats Zoom talk

Program

SATURDAY, JULY 23

15:00–15:50 **Siqi Fu** (Rutgers University–Camden) Spectral stability of the complex Laplacians

16:00–16:50 **Hajime Tsuji** (Sophia University) Approximation of invariant measures by Bergman kernels and other invariant measures

SUNDAY, JULY 24

9:00–9:50^{*Z*} **Mei-Chi Shaw** (University of Notre Dame) *Extendability and the Cauchy–Riemann operator on the Hartogs triangle*

10:00–10:50^{*Z*} **Dror Varolin** (Stony Brook University) *BLS fields*

11:20–12:10 **Finski Siarhei** (École Polytechnique) Semiclassical Ohsawa–Takegoshi extension theorem and its applications

15:00–15:50 **Hideyuki Ishi** (Osaka Metropolitan University) *Complex domains admitting homogeneous Kähler metrics*

16:00–16:50 **Miroslav Engliš** (Czech Academy of Sciences) *M-harmonic Bergman kernels*

19:30–20:30^{*Z*} Short Communications

Monday, July 25

9:00–9:50 Friedrich Haslinger (Universität Wien) Real holomorphic vector fields and the ∂ -complex

10:00–10:50 **Yoshinori Hashimoto** (Osaka Metropolitan University) Donaldson's quantisation: extremal Kähler metrics and Fano manifolds

11:20–12:10 **Junyan Cao** (Université Côte d'Azur) Subharmonicity of direct images and applications

15:00–15:50 **Tomoyuki Hisamoto** (Tokyo Metropolitan University) On some asymptotic construction of optimal degeneration for a Fano manifold

16:00–16:50 Xiaonan Ma (Université de Paris) Bergman kernels on punctured Riemann surfaces

19:30-20:30 Short Communications

TUESDAY, JULY 26

10:00–10:50 **Chin-Yu Hsiao** (Academia Sinica) Semi-classical Bergman kernels on complex manifolds with pseudoconcave boundary

11:20–12:10 **Zbigniew Błocki** (Jagiellonian University) *Rearrangemnts and the Monge–Ampère equations*

WEDNESDAY, JULY 27

10:00–10:50 **Masanori Adachi** (Shizuoka University) A residue formula for meromorphic connections and applications to stable sets of foliations

11:00–11:50^{*Z*} **Genkai Zhang** (Chalmers University of Technology) Wehrl-type inequality for Bergman spaces on bounded domains in \mathbb{C}^n

Thursday, July 28

10:00–10:50 **Joe Kamimoto** (Kyusyu University) Asymptotic analysis of the Bergman kernel on pseudoconvex model domains

11:00–11:50^{*Z*} **Zhitong Mi** (Chinese Academy of Sciences) Concavity property of minimal L^2 integrals

14:00–14:50 **Takeo Ohsawa** (Nagoya University) Convexity properties of complete Kähler domains

15:10–16:00^{*Z*} Xiangyu Zhou (Chinese Academy of Sciences) Converse of L^2 existence, Nakano positive vector bundles and Lempert's problem

16:10–17:00^Z **Bo-Yong Chen** (Fudan University) Some aspects of the p-Bergman theory

Abstracts

Masanori Adachi (Shizuoka University)

A residue formula for meromorphic connections and applications to stable sets of foliations

We define the residues of a holomorphic connection of a holomorphic line bundle along a simple normal crossing divisor in a complex manifold by only assuming that its curvature extends across the divisor. We then prove a residue formula that localizes the first Chern class to the singular locus of the given holomorphic connection. As applications, we discuss proofs for a nonexistence theorem of Levi flat hypersurfaces with transversely affine Levi foliation in compact Kähler surfaces and Brunella's conjecture about exceptional minimal sets of codimension one holomorphic foliations with ample normal bundle. This is joint work with S. Biard and J. Brinkschulte.

Zbigniew Błocki (Jagiellonian University) *Rearrangemnts and the Monge–Ampère equations*

Comparison results for rearrangements of solutions to elliptic equations are known for linear equations (Talenti) and for the real Monge–Ampère equation (Talenti, Tso). The main tool in the proof is always an appriopriate isoperimetric inequality. These results often lead to optimal estimates, as they reduce the problem to radially symmetric solutions. We will discuss a similar question for the complex Monge–Ampère equation. Some of those problems are in fact related to lower bounds for the Bergman kernel in terms of pluripotential theory.

Junyan Cao (Université Côte d'Azur) Subharmonicity of direct images and applications

The positivity of direct images plays an important role in complex geometry. One important analytic approach to this type problem is to study the variation of Bergman kernels, established by the works of Berndtsson, Maitani, Paun, Yamaguchi etc. In this talk, we will explain some generalizations for the direct images of twisted bundles which is not necessary positive in all directions. As an application, we obtain an algebraic criteria for holomorphic foliations which partly confirms a conjecture of Pereira–Touzet. It is a joint work with Campana and Paun.

Bo-Yong Chen (Fudan University) Some aspects of the *p*-Bergman theory

We discuss some recent progress on the p-Bergman theory, including regularity problem of the p-Bergman kernel, curvature properties of the p-Bergman (Finsler) metric, and function theory of the p-Bergman space. This talk is based on joint works with Liyou Zhang and Yuanpu Xiong respectively.

Miroslav Engliš (Czech Academy of Sciences) *M-harmonic Bergman kernels*

M-harmonic functions on the complex unit ball are those that are annihilated by the Moebiusinvariant Laplacian. While holomorphic and harmonic Bergman kernels with respect to standard weights on the ball have by now been well known objects already for decades, their M-harmonic counterparts turn out to be significantly more elusive. We survey this subject starting with its beginnings in the work of Rudin, Folland, and other authors, through the invariant potential theory of Stoll and related aspects, and concluding with the recent advances for the unit ball as well as with a description of the differencies that occur in the more general situation when the unit ball is replaced by a bounded symmetric domain.

Finski Siarhei (École Polytechnique)

Semiclassical Ohsawa–Takegoshi extension theorem and its applications

Ohsawa and Takegoshi in 1987 gave a sufficient condition under which a holomorphic section of a vector bundle on the submanifold extends to the holomorphic section over the ambient manifold.

In this talk, in the semiclassical setting, i.e. when the section is taken from a sufficiently big tensor power of a positive line bundle, we provide an explicit asymptotic formula for the optimal extension operator. In this asymptotic regime, we derive several consequences among which are the transitivity of the extension operator with respect to a tower of submanifolds, calculation of the optimal constant in Ohsawa–Takegoshi extension theorem and equivalence of various canonical metrics on the space of holomorphic jets along submanifolds.

Siqi Fu (Rutgers University–Camden) Spectral stability of the complex Laplacians

In physical sciences, exact values are oftentimes difficult—in some cases, impossible—to obtain and approximate values are observed and utilized instead. It is thus important to know whether a quantify under study remains stable when other parameters are slightly perturbed. In this talk, we will discuss stability of the spectrum of complex Laplacians when the underlying analytic or geometric structures are perturbed. In particular we study spectral stability of the Kohn Laplacian and related it to stability of embeddability of CR manifolds. This talk is based on joint work with Howard Jacobowitz and Weixia Zhu.

Yoshinori Hashimoto (Osaka Metropolitan University)

Donaldson's quantisation: extremal Kähler metrics and Fano manifolds

The Bergman kernel plays a key role in the theorem of Donaldson which states that a Kähler metric of constant scalar curvature can be approximated by a sequence of Fubini–Study metrics called balanced metrics if the manifold has a discrete automorphism group, which is often called Donaldson's quantisation. After reviewing some background materials, we discuss some results due to the author and others on its generalisation to extremal Kähler metrics, and some recent results concerning its version for Fano manifolds by Berman and Witt Nyström.

Friedrich Haslinger (Universität Wien)

Real holomorphic vector fields and the ∂ -complex

We study certain densely defined unbounded operators on the Segal–Bargmann space. These are the annihilation and creation operators of quantum mechanics. In several complex variables we have the ∂ -operator and its adjoint ∂^* acting on (p, 0)-forms with coefficients in the Segal–Bargmann space. We consider the corresponding ∂ -complex and the corresponding complex Laplacian $\tilde{\Box} = \partial \partial^* + \partial^* \partial$. In addition we investigate the existence of real-valued weight functions with real holomorphic gradient fields on Kähler and conformally Kähler manifolds and their relationship to the ∂ -complex on weighted Bergman spaces having similar duality properties as the Segal–Bargmann space. Finally we discuss basic estimates for the generalized complex Laplacian on the Segal–Bargmann space and compactness of the inverse operators of the generalized complex Laplacian.

Tomoyuki Hisamoto (Tokyo Metropolitan University)

On some asymptotic construction of optimal degeneration for a Fano manifold

Optimal degeneration is a test configuration which optimally destabilizes a given polarized manifold. For a Fano manifold we study Kähler-Ricci flow and its quantization so that obtain an asymptotic construction of the optimal degeneration.

Chin-Yu Hsiao (Academia Sinica)

Semi-classical Bergman kernels on complex manifolds with pseudoconcave boundary

Let M be a complex manifold with pseudoconcave boundary and let L be a positive line bundle over \overline{M} . Suppose that \overline{M} admits a holomorphic \mathbb{R} -action and the \mathbb{R} -action can be lifted to L. In this work, we show that the Bergman kernel admits an asymptotic expansion with respect to a high power of L. This is a joint work with Xiaoshan Li and George Marinescu.

Hideyuki Ishi (Osaka Metropolitan University) Complex domains admitting homogeneous Kähler metrics

A Kähler metric on a complex manifold is said to be homogeneous if the Kähler automorphism group acts on the complex manifold transitively. The fundamental theorem of homogeneous Kähler manifold tells us that, if a simply connected complex domain admits a homogeneous Kähler metric, then the domain is biholomorphic to the product of a complex vector space and a bounded homogeneous domain. In this talk, we show that such a domain is realized as an orbit in the Siegel–Jacobi domain under the action of a subgroup of the Jacobi group. Moreover, we determine whether the metric comes from the reproducing kernel of an invariant Hilbert space of holomorphic functions on the domain.

Joe Kamimoto (Kyusyu University) Asymptotic analysis of the Bergman kernel on pseudoconvex model domains

In order to investigate the boundary behavior of the Bergman kernel for weakly pseudoconvex domains, it is necessary to understand the geometrical structure of degeneracy of the boundary of a given domain. The "Newton polyhedron" contains important information of the above structure and, moreover, it plays useful roles in the analysis of the behavior of the Bergman kernel. In this talk, we will investigate the boundary behavior of the Bergman kernel in the sense of asymptotic expansion for a certain class of weakly pseudoconvex model domains by using Newton polyhedra.

Xiaonan Ma (Université de Paris) Bergman kernels on punctured Riemann surfaces

We will review our recent works on Bergman kernel on punctured Riemann surfaces. We consider a punctured Riemann surface endowed with a Hermitian metric that equals the Poincaré metric near the punctures, and a holomorphic line bundle that polarizes the metric. We will explain the Bergman kernel can be localized around the singularities and its local model is the Bergman kernel of the punctured unit disc endowed with the standard Poincaré metric. We will explain that the quotient of the Bergman kernel of high tensor powers of the line bundle and of the Bergman kernel of the Poincaré model near the singularity tends to one up to arbitrary negative powers of the tensor power.

This is a joint work with Hugues Auvray and George Marinescu.

Zhitong Mi (Chinese Academy of Sciences) Concavity property of minimal L^2 integrals

In this talk, we survey the progress on the concavity property of minimal L^2 integrals.

Takeo Ohsawa (Nagoya University) Convexity properties of complete Kähler domains

An approximation theorem will be proved for the space of L^2 holomorphic sections of vector bundles on certain Zariski open sets of weakly 1-complete manifolds. As an existence result on such manifolds, a solution of the bundle-valued version of the Levi problem will be given by a variant of a method of Hörmander.

Mei-Chi Shaw (University of Notre Dame) Extendability and the Cauchy–Riemann operator on the Hartogs triangle

The Hartogs triangle T serves as an important example in several complex variables. In this talk, we discuss the extendability of Sobolev spaces on the Hartogs triangle. Furthermore, we show that the weak and strong maximal extensions of the Cauchy–Riemann operator agree on T. These results can be used to study the Dolbeault cohomology groups with Sobolev coefficients on the complement of T. We also discuss some recent progress for the Cauchy–Riemann equations on Hartogs triangles in complex projective space of dimension two (joint work with A. Burchard, J. Flynn and G. Lu).

Hajime Tsuji (Sophia University)

Approximation of invariant measures by Bergman kernels and other invariant measures

In this talk, we discuss approximation of invariant measures (such as Kähler-Einstein volume forms) by Bergman kernels and other invariant measures. We also discuss some applications of these approximations.

Dror Varolin (Stony Brook University) *BLS fields*

We introduce a notion of complex analytic structure for the smooth Hilbert fields of Lempert– Szőke. This complex analytic structure, which takes the form of a $\bar{\partial}$ -operator, was first defined by Berndtsson in special but important cases, and thus we call these Hilbert fields *Berndttson– Lempert–Szőke fields*, or simply *BLS fields*. The BLS structure allows one to define a unique Chern connection and its curvature. A number of natural Hilbert fields arising from deformation theory were investigated by a number of mathematicians, going back to Griffiths. In previous results, conditions that force the Hilbert field to be locally trivial were always assumed. Our structures are in general not vector bundles nor even sheaves. As a first application, we give a new proof, and a generalization, of Berndtsson's celebrated result on the direct image of a family of adjoint bundles. The proof gives an explicit curvature formula that is different from (but of course algebraically equivalent to) the formulas of Berndtsson (for families of line bundles) and of Liu–Yang (for families of vector bundles).

Genkai Zhang (Chalmers University of Technology)

Wehrl-type inequality for Bergman spaces on bounded domains in \mathbb{C}^n

We prove a Wehrl-type inequality for L^p -Bergman spaces on bounded domains in \mathbb{C}^n and for Bergman space of vector valued functions on bounded symmetric domains. We prove that the maximizers of the inequality are the Bergman kernels. This generalizes earlier results of of Lieb–Solovej.

Xiangyu Zhou (Chinese Academy of Sciences) Converse of L^2 existence, Nakano positive vector bundles and Lempert's problem

We present some our recent result on the converse of L^2 existence theorem due to Hörmander and Demailly, and some applications including our recent affirmative solution of Lempert's problem on the Nakano positive vector bundles which asking whether a C^2 -smooth hermitian metric, which locally is a limit of an increasing sequence of C^2 -smooth Nakano semi-positive hermitian metrics, is still Nakano semi-positive.

Abstracts of Short Communications

Gopal Datt

Meromorphically normal families in several variables

Meromorphic normality is a notion of sequential compactness in the meromorphic category introduced by Fujimoto. In this talk, we shall discuss some sufficient conditions of meromorphic normality for families of meromorphic mappings taking values in a complex projective space. As a consequence of these sufficient conditions we shall, finally, see a meromorphic version of the Montel–Carathéodory theorem.

Andrea Galasso

Toeplitz operators on CR manifolds

Let X be a connected orientable compact CR manifold with non-degenerate Levi form of constant signature. In a joint project with Chin-Yu Hsiao, we study the algebra of Toeplitz operators on X and we establish the star product for a certain class of symbols on X. The main application is a construction of deformation quantization of compact "quantizable" pseudo-Kähler manifolds of constant signature.

Wei-Chuan Shen

On the second coefficient of the asymptotic expansion of Boutet de Monvel-Sjöstrand

The study of reproducing kernel in several complex variables is fundamental in complex and CR geometry. In this talk, we will focus on the Szegő kernel on embeddable compact strongly pseudoconvex CR manifolds. We will recall the asymptotic expansion obtained by Boutet de Monvel and Sjöstrand via complex-valued phase Fourier integral operator, and discuss how to determine its second coefficient (in the part of pole type singularities). This is a joint work with Chin-Yu Hsiao.