Down-side risk minimization for a hidden Markov factor model

Yûsuke Watanabe (Osaka University)

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We consider a market model consisting of one bank account S_t^0 and N risky securities $S_t^1, \ldots, S_t^N$. We assume that the mean returns of risky security prices depend nonlinearly upon "hidden economic factors," which evolve as a continuous-time Markov chain with finite state space. "Hidden" means that the factors are only partially observable through the information of security prices.

Let $V_T(h)$ be an investor's wealth at time T, correponding to an investment strategy $h = (h_t)_{t \geq 0}$. Set

$$X_T(h) := \log \frac{V_T(h)}{S_T^0}.$$

For a given level $k \in \mathbb{R}$, we want to minimize a down-side risk probability

$$P\Big(\frac{X_T(h)}{T} \leq k\Big)$$

over a large time interval $[0, T]$. More specifically, we consider the long-time average of a minimized down-side risk

$$\Pi_1(k) = \varliminf_{T \to \infty} \frac{1}{T} \inf_h \log P\Big(\frac{X_T(h)}{T} \leq k\Big),$$

and also the minimized long-time average of a down side risk

$$\Pi_2(k) = \inf_h \varliminf_{T \to \infty} \frac{1}{T} \log P\Big(\frac{X_T(h)}{T} \leq k\Big).$$

To treat these problems, we first consider the following risk-sensitive portfolio optimization problems (1) and (2), for a given "risk-averse" parameter $\gamma \in (-\infty, 0)$:

Finite time horizon problem:

$$\inf_h \log E\Big[\exp\Big\{\gamma X_T(h)\Big\}\Big], \tag{1}$$

and its long time average

$$\chi_1(\gamma) = \varliminf_{T \to \infty} \frac{1}{T} \inf_h \log E\Big[\exp\Big\{\gamma X_T(h)\Big\}\Big].$$

Infinite time horizon problem:

$$\chi_2(\gamma) = \inf_h \varliminf_{T \to \infty} \frac{1}{T} \log E\Big[\exp\Big\{\gamma X_T(h)\Big\}\Big]. \tag{2}$$

Suppose that we have "solved" the optimization problems (1) and (2). Then, in view of the large deviations principle, we expect that the following duality relation holds:

$$\Pi_\nu(k) = -\inf_{k' \in (-\infty, k]} \chi_\nu^*(k'), \qquad \nu = 1, 2,$$

where $\chi_\nu^*(\cdot)$ is the Legendre transform of $\chi_\nu(\cdot)$:

$$\chi_\nu^*(k) = \sup_{\gamma \in (-\infty, 0)} \{k\gamma - \chi_\nu(\gamma)\}, \qquad \nu = 1, 2.$$

We will formalize the above duality relation and then sketch the proof.