On optimal periodic dividend strategies for Lévy risk processes

Kei Noba (Kyoto University), José Luis Pérez (CIMAT),
Kazutoshi Yamazaki (Kansai University) and Kouji Yano (Kyoto Universtiy)

1 Introduction

This talk is based on [4] and [5]. In this talk, we revisit the optimal periodic dividend problem, in which dividend payments can only be made at the jump times of an independent Poisson process. In the dual (spectrally positive Lévy) model, recent results have shown the optimality of a periodic barrier strategy, which pays dividends at Poissonian dividend-decision times, if and only if the surplus is above some level. In this talk, we show the optimality of this strategy for a spectrally negative Lévy process whose dual has a completely monotone Lévy density. We also consider the version with bail-outs where the surplus must be non-negative uniformly in time. There are many previous studies of spectrally negative cases. Loeffen([3]) and Kyprianou et al.([2]) showed the optimality of a barrier strategy in the classical case and that of a threshold strategy under the absolutely continuous assumption on the dividend strategy, respectively. Avram et al.([1]) and Pérez et al.([7]) showed the optimality of those of the version with bail-outs. In this draft, we give the main results for the models without bail-outs.

2 Preliminary facts and main results

Let X be a spectrally negative Lévy process. Suppose that the Lévy measure of $-X$ has a completely monotone with respect to the Lebesgue measure. Let N^r be the Poisson process with rate $r > 0$ which are independent from X. Let $\mathbb{F}$ be the filtration generated from X and N^r. In this setting, a strategy $\pi = \{L^\pi_t : t \geq 0\}$ is a non-decreasing, right-continuous, and $\mathbb{F}$-adapted process such that the cumulative amount of dividends L^π admits the form

$$L^\pi(t) = \int_{[0,t]} \nu^\pi(s) dN^r(s), \tag{2.1}$$

for some $\mathbb{F}$-adapted càglàd process ν^π. The surplus process U^π after dividends are deducted is such that

$$U^\pi(t) = X(t) - L^\pi(t) \tag{2.2}$$

where $\sigma^\pi_0 = \inf\{t > 0 : U^\pi(t) < 0\}$ is the corresponding ruin time. We assume that the payment cannot exceed the available surplus and hence

$$0 \leq \nu^\pi(s) \leq U^\pi(s-), \qquad s \geq 0. \tag{2.3}$$

We fix $q > 0$ which is the discount rate. We define the expected net present value of dividends paid until ruin as the following:

$$v_\pi(x) = \mathbb{E}_x\left[\int_{[0,\sigma^\pi_0]} e^{-qt} dL^\pi(t)\right]. \tag{2.4}$$

Let $\mathcal{A}_r$ be the set of all admissible strategies. The problem is to compute the value function

$$v_{\pi^*}(x) = v(x) := \sup_{\pi \in \mathcal{A}_r} v_\pi(x). \tag{2.5}$$

For $b \geq 0$, the periodic barrier strategy π^b is the strategy which satisfies the

$$\nu^{\pi^b}(t) = (U_r^{\pi^b}(t-) - b) \vee 0, \qquad t > 0. \tag{2.6}$$

The strategy π^b was constructed by [6]. The expected NPV v^{π^b} was computed by [6, Corollary 4.4] using the scale functions.

Let Φ be the inverse Laplace exponent of X and $W^{(q)}$ be the q-scale function of X. We denote

$$Z^{(q)}(x, \Phi(q+r)) = r \int_0^\infty e^{-\Phi(q+r)z} W^{(q)}(z+x) dz, \qquad x \in \mathbb{R}, \tag{2.7}$$

$$h(b) = e^{-\Phi(q+r)b} \left(r W^{(q)\prime}(b) - \Phi(q+r) Z^{(q)\prime}(b, \Phi(q+r)) \right), \quad b > 0. \tag{2.8}$$

We define

$$b^* = \inf\{b > 0 : h(b) \leq 0\}. \tag{2.9}$$

Then we have the following theorem:

Theorem 2.1. *For $x > 0$, we have $v_{\pi^{b^*}}(x) = v(x)$.*

References

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