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THE EXTINCTION PROPERTY FOR SUPERPROCESSES AND THE SUPERDIFFUSION WITH BRANCHING RATE FUNCTIONAL

超過程の消滅性と分枝率汎関数を伴う超拡散

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Let $(\Omega, \mathcal{F}, P)$ be a basic probability space. Let $p > d$, and we define the reference function ϕ_p by

$$\phi_p(x) = (1 + |x|^2)^{-p/2}, \quad x \in \mathbb{R}^d.$$

When $C \equiv C(\mathbb{R}^d)$ is the space of all continuous functions on $\mathbb{R}^d$, we define

$$C_p = \{f \in C : \ |f| \leqslant C_f \cdot \phi_p, \ \exists C_f > 0 : \text{some constant}\}.$$

Let $M_p \equiv M_p(\mathbb{R}^d)$ denote the set of all non-negative measures μ on $\mathbb{R}^d$, satisfying $\langle \mu, \phi_p \rangle < \infty$, and the member of M_p is called a p-tempered measure, where $\langle \mu, f \rangle$ indicates the integral of f with respect to $d\mu$. Let us define the second order elliptic operator L by

$$L = \frac{1}{2} \sum_{i,j=1}^{d} \frac{\partial}{\partial x_i} \left(a_{ij}(x) \frac{\partial}{\partial x_j} \right) + \sum_{i=1}^{d} b_i(x) \frac{\partial}{\partial x_i}$$

with symmetric positive definite matrix $a(x) = (a_{ij}(x))$, $i, j = 1, \dots, d$, and $a_{ij}, b_i \in C^{1,\eta} \equiv C^{1,\eta}(\mathbb{R}^d)$, where $C^{1,\eta}$ is the space of Hölder continuous functions with index η. Let $\Xi = \{\xi, \Pi_{s,a}, \ s \geq 0, a \in \mathbb{R}^d\}$ be an L-diffusion process, and K is a positive continuous additive functional (CAF) of ξ in Dynkin's sense, cf. E.B. Dynkin (1997). $M_F \equiv M_F(\mathbb{R}^d)$ denotes the set of all finite measures on $\mathbb{R}^d$. Then $\mathbb{X} = \{X, \mathbb{P}_{s,\mu}, \ s \geq 0, \ \mu \in M_F\}$ is said to be a superdiffusion with branching rate functional K, if $X = \{X_t\}$ is a continuous M_F-valued time-inhomogeneous Markov process with Laplace functional

$$\mathbb{E}_{s,\mu} e^{-\langle X_t, \varphi \rangle} = e^{-\langle \mu, v(s,t) \rangle}, \quad 0 \leqslant s \leqslant t, \mu \in M_F, \varphi \in C_b^+$$

where C_b denotes the set of all bounded continuous functions $f \in C$, and C_b^+ denotes all the positive menbers g in C_b. Here the function v is uniquely determined by the log-Laplace equation

$$v(s,a) = \Pi_{s,a} \varphi(\xi_t) - \Pi_{s,a} \int_s^t v^2(r, \xi_r) K(dr), \quad 0 \leqslant s \leqslant t, a \in \mathbb{R}^d.$$

Especially when $d = 1$, for $0 < \nu < 1$, $p > 1/\nu$, and the Lebesgue measure λ on $\mathbb{R}$, let $(\gamma, \mathbb{P})$ be the stable random measure with index ν on $\mathbb{R}$ with Laplace functional

$$\mathbb{E} e^{-\langle \gamma, \varphi \rangle} = \exp \left\{ - \int \varphi^\nu(x) \lambda(dx) \right\}, \quad \varphi \in C_b^+.$$

Then for a.a. ω realization $\gamma \equiv \gamma(\omega) \in M_p$, we define

$$K[s,t] \equiv K_\gamma[s,t] = \int_s^t dr \int \gamma(db)\delta_b(\xi_r).$$

Theorem 1. If K_γ is locally admissible, then there exists a superdiffusion

$$\mathbb{X}^\gamma = \{X^\gamma, \mathbb{P}^\gamma_{s,\mu}, s \geq 0, \mu \in M_F\}$$

with branchingrate functional K_γ, $\mathbb{P}$-a.a. ω.

Let $\mathbb{C} = C(\mathbb{R}_+, \mathbb{R}^d)$ with topology of uniform convergence on compact subsets of $\mathbb{R}_+$. To each $w \in \mathbb{C}$ and $t \geq 0$, $w^t \in \mathbb{C}$ denotes the stopped path of w. While, $\mathbb{C}^t$ is the totality of all these paths stopped at time t. To every $w \in \mathbb{C}$, we associate the corresponding stopped path trajectory $\tilde{w}$ by setting $\tilde{w}_t = w^t$, $t \geq 0$. The image of L-diffusion w under the map : $w \mapsto \tilde{w}$ is called the L-diffusion path process. Set

$$\mathbb{R}_+ \hat{\times} \mathbb{C}^{\cdot} = \{(s,w) : s \in \mathbb{R}_+, w \in \mathbb{C}^s\},$$

and $M(\mathbb{R}_+ \hat{\times} \mathbb{C}^{\cdot})$ denotes the set of all measures η on $\mathbb{R}_+ \hat{\times} \mathbb{C}^{\cdot}$ which are finite, if restricted to a finite time interval. Then for $d \geq 1$ and a positive CAF K of ξ, we can construct a historical superprocess (cf. Dynkin (1991))

$$\tilde{\mathbb{X}} = \{\tilde{X}, \tilde{\mathbb{P}}_{s,\mu}, s \geq 0, \mu \in M_F(\mathbb{C}^s)\}$$

with state $\tilde{X}_t \in M_F(\mathbb{C}^t)$, $t \geq s$. When we denote, for $K \geq 1$,

$$\mathbb{C}_K = \{w \in \mathbb{C} : \ |w_s| < K, \forall s \geq 0\},$$

then by the compact support property of Dawson-Li-Mueller (1985), we have

$$\lim_{K\to\infty} \inf_{t\geq 0} \tilde{\mathbb{P}}^\gamma_{0,\mu} \left(\text{supp}(\tilde{X}^\gamma_t) \subseteq \mathbb{C}_K \right) = 1$$

for each $t \geq 0$, $\mathbb{P}$-a.s. Then by the comparison argument of extinction probabilities (cf. Dawson et al. (2000)) it follows that

Proposition 2. For a fixed sample γ,

$$\lim_{t\to\infty} \tilde{\mathbb{P}}^\gamma_{0,\mu} \left(\tilde{X}^\gamma_t \neq 0 \quad \text{and} \quad \text{supp}(\tilde{X}^\gamma_t) \subseteq \mathbb{C}_K \right) = 0.$$

Finally, through the projection technique (cf. Dawson-Perkins (1991), Dôku (2003)) we obtain

Theorem 3. (Finite Time Extinction) Let $d = 1$ and fix $\mu \in M_F$ with compact support. Then $\mathbb{P}$-a.a. γ,

$$\mathbb{P}^\gamma_{0,\mu} (X^\gamma_t = 0 \quad \text{for some} \quad t > 0) = 1.$$