

Algebraic Lie Theory and Representation Theory 2025

日時 2025 年 5 月 23 日 (金) から 26 日 (月) まで

場所 石川県青少年総合研修センター 〒 920-0834 石川県金沢市常盤町 212-1

世話人 阿部 紀行 (東京大学), 桑原 敏郎 (筑波大学)

スケジュール

5 月 23 日 (金)

- 14:00–14:50 勝田篤 (九州大学/慶応大学)
Nilpotent Floquet-Bloch Theory and the Orbit method
- 15:00–15:50 元良直輝 (富山大学)
Equivalent definitions of W-algebras and Poisson geometry
- 16:00–16:50 渡邊英也 (立教大学)
第二種振れループ代数の有限次元既約表現
- 17:00–18:00 河野隆史 (早稲田大学)
Special classes of the equivariant quantum K-ring of the flag manifold in type C
- 19:00–20:00 Duc-Khanh Nguyen (OIST)
A generalization of the Murnaghan-Nakayama rule for K-k-Schur and k-Schur functions

5 月 24 日 (土)

- 09:00–10:00 Xuanzhong Dai (京都大学)
Chiral differential operators on G/N and quasi-lisse vertex algebras
- 10:15–12:15 山崎雅人 (東京大学)
簾ヤンギアンをめぐる数学と物理学
- 13:15–14:15 上田衛 (東京大学)
Affine Yangians and non-rectangular W-algebras
- 14:30–15:30 森脇湧登 (理化学研究所)
Conformal field theory and Operads
- 15:45–16:45 Hao Lie (京都大学)
Virasoro tensor category at the negative rational level
- 17:00–18:00 廣田竣介 (京都大学)
奇 Verma の定理
- 19:00–20:00 八尋耕平 (京都大学)
2D パーシステンス加群の表現空間の結晶構造

5月25日(日)

- 09:00–10:00 Louise Sutton (OIST)
Cyclotomic KLR algebras in types A_∞ and C_∞
- 10:15–11:15 村田遼人 (東京大学)
Affine highest weight structures on module categories over quiver Hecke algebras
- 11:30–12:30 星野真生 (東京大学)
A classification of equivariant quantum full flag manifolds
- 13:30–15:30 松本久義 (東京大学)
実簡約表現の既約表現研究の回想
- 15:45–16:45 Pablo Sanchez Ocal (OIST)
Deformations of Frobenius algebras and noncommutative 2d topological quantum field theories
- 17:00–18:00 山口航平 (名古屋大学)
Equivariant K-homology of symplectic affine Grassmannian

5月26日(月)

- 09:00–09:50 石橋典 (東北大学)
Cyclic quantum Teichmüller theory
- 10:00–11:00 安達晶平・小池隼人 (早稲田大学)
Semi-infinite Lakshmibai-Seshadri paths of twisted affine type
- 11:15–12:05 廣恵一希 (千葉大学)
微分方程式の不確定特異点の開折とルート系

概要

勝田篤（九州大学/慶応大学）

Nilpotent Floquet-Bloch Theory and the Orbit method

The Floquet-Bloch theory is a widely used tool for investigating materials with periodic structures. For instance, it can demonstrate that the spectrum of periodic Schrödinger operators exhibits band structures. Within this context, this theory has been applied to the following problems over several decades in the case of abelian extensions: (1) A geometric analog of the Chebotarev density theorem for prime closed geodesics in a compact Riemannian manifold with negative curvature. (2) A long time asymptotic expansion of the heat kernels on covering manifolds of compact Riemannian manifolds.

In this talk, we explain our version of the generalized Floquet-Bloch theory for discrete nilpotent groups and apply it to the above problems for nilpotent extension. Since the 1960s, primarily through the work of Glimm, direct handling of these groups has been considered difficult due to their non-type I property and the complexity of their unitary dual structure.

One of our contributions is to relate finite-dimensional representations of a discrete torsion-free nilpotent group and infinite-dimensional representations of its Malcev completion, the simply connected nilpotent Lie group containing the former discrete group as its lattice. We obtain a branching formula of restriction to the lattice of the above infinite representations, providing approximations of a finite-dimensional representation by infinite-dimensional ones and vice versa.

元良直輝（富山大学）

Equivalent definitions of W-algebras and Poisson geometry

Premet originally introduced finite W-algebras as generalizations of Kac and Lynch's works to study modular representations of Lie algebras. Later, Gan and Ginzburg showed that finite W-algebras have many different equivalent definitions by using the geometry of Slodowy slices. (Affine) W-algebras were originally defined by Feigin and Frenkel and generalized by Kac, Roan, and Wakimoto. We show that W-algebras also have many different equivalent definitions as analogs of finite cases. This is joint work with Thibault Juillard.

渡邊英也（立教大学）

第二種捩れループ代数の有限次元既約表現

第二種捩れループ代数 (twisted loop algebra of the second kind) とは、ある無限次元リー代数のクラスで、(捩れ) ループ代数、カレント代数、一般化オンサーガー代数をサブクラスに持つ。従って、その有限次元表現は数学的、物理的に重要な研究対象である。Neher–Savage–Senesi (2012) は、同変写像代数 (equivariant map algebra) という、第二種捩れループ代数より広いクラスの無限次元リー代数の有限次

元既約表現を分類した。本講演では、これとは独立に、より代数的な分類理論を紹介する。

河野隆史 (早稲田大学)

Special classes of the equivariant quantum K-ring of the flag manifold in type C

Finding good representatives of Schubert classes in the equivariant quantum K-ring of the flag manifold is one of the main problems of the Schubert calculus. We construct an explicit quotient ring of a certain Laurent polynomial ring, which is isomorphic to the equivariant quantum K-ring of the flag manifold in type C. This isomorphism is called the Borel-type presentation. In addition, we describe Laurent polynomials that represent special Schubert classes in the Borel-type presentation. As an application, we obtain a factorization formula of the Schubert class corresponding to the longest element of the Weyl group. This talk is based on a joint work with Takeshi Ikeda and Satoshi Naito.

Duc-Khanh Nguyen (OIST)

A generalization of the Murnaghan-Nakayama rule for K-k-Schur and k-Schur functions

The K-k-Schur functions and k-Schur functions appeared in the study of K-theoretic and affine Schubert Calculus as polynomial representatives of Schubert classes. In this work, we introduce a new family of symmetric functions $F_{\lambda}^{(k)}$, that generalizes the constructions via the Pieri rule of K-k-Schur functions and k-Schur functions. Then we obtain the Murnaghan-Nakayama rule for the generalized functions. The rule is described explicitly in the cases of K-k-Schur functions and k-Schur functions, with concrete descriptions and algorithms for coefficients. Our work recovers the result of Bandlow, Schilling, and Zabrocki for k-Schur functions, and explains it as a degeneration of the rule for K-k-Schur functions. In particular, many other special cases and connections promise to be detailed in the future.

Xuanzhong Dai (京都大学)

Chiral differential operators on G/N and quasi-lisse vertex algebras

The chiral differential operator (CDO) is defined as a sheaf of vertex algebras on nonsingular algebraic varieties or smooth manifolds. It is known that the global section of the CDO on any smooth affine variety is quasi-lisse, as its associated variety is canonically isomorphic to the cotangent bundle. In this talk, we consider the CDO on the base affine space G/U , where G is a complex connected semisimple group of ADE type, and U is its maximal unipotent subgroup. We begin with a brief review of the construction of CDO and then show that, in the case of type A , the associated variety of the CDO on G/U is isomorphic to the affine closure of its cotangent bundle. This is a joint work in progress with Tomoyuki Arakawa and Bailin Song.

山崎雅人（東京大学）

簾ヤングアンをめぐる数学と物理学（概説講演）

簾ヤングアン（およびそれらの三角型、楕円型拡張）は近年導入された全く新しい代数であり、A 型のアフライン・ヤングアンを特殊例として含んでいるほか、頂点作用素代数（W 代数）との関連も議論されている。また簾ヤングアンは BPS 代数であると考えられており、その上三角部分として簾に付随したコホモロジカル・ホール代数を含んでいると期待されている。本講演では、簾ヤングアンの定義や基本的な性質に加え、この代数が超対称場の理論や超弦理論のどのような物理学的な動機から現れるかを説明し、幾何、代数、表現論を巻き込んだ新しい数学を生み出していく可能性について議論したい。

上田衛（東京大学）

Affine Yangians and non-rectangular W-algebras

Brundan and Kleshchev wrote down a finite W -algebra of type A as a quotient algebra of the shifted Yangian. The shifted Yangian contains a finite Yangian of type A as a subalgebra. De Sole, Kac, and Valeri constructed a homomorphism from this subalgebra to the finite W -algebra of type A by using the Lax operator.

In this talk, I will explain how to construct a homomorphism from the affine Yangian of type A to a non-rectangular W -algebra of type A , which can be regarded as an affine version of the result of De Sole-Kac-Valeri. This homomorphism is expected to lead to a generalization of the AGT conjecture.

森脇湧登（理化学研究所）

Supersymmetric 2d conformal field theory and mirror symmetry

カラビヤウ多様体のミラー対称性は複素構造とケーラー構造が入れ替わるミステリアスな操作であるが、その発見の元となった超対称 2d 共形場理論のレベルでは、リー代数の外部自己同型に対応する自明な操作である。本講演では arXiv:2504.09919 に基づき、共形場理論のミラー対称性を解説し、ミラー・カラビヤウ多様体の存在についての新しい証明手法を提案する。

Hao Lie（京都大学）

Virasoro tensor category at the negative rational level

The representation theory of minimal series principal W -algebras is well understood; in particular, Arakawa proved C_2 -cofiniteness and rationality for their modular-invariant representations. In contrast, general principal W -algebras often lack such properties. Recently, there has been substantial progress in understanding the tensor categories of $W^k(\mathfrak{sl}_2)$ -modules for general $k \in \mathbb{C} \setminus \{-2\}$. In this talk, I will report on joint work with McRae and Yang concerning the tensor category of $V^{13+6p/q+6q/p}(\text{Vir})$ -modules. We obtain several good categorical properties in this setting, which in

turn provide new tools for studying extensions of the Virasoro algebra via tensor category methods.

廣田峻介（京都大学）

奇 Verma の定理

Kac-Moody Lie 超代数や正標数の Kac-Moody Lie 代数においては、Borel 部分代数の取り方が本質的に一意でないことが古より知られて居たが、Heckenberger-山根 (2008) による Weyl groupoid は、それらの root 系もどきや frieze pattern を統一的に一般化する形で、普遍的 Hopf 代数的対象である Nichols 代数の分類を Lie 理論的に説明した。有限次元 Kac-Moody Lie 超代数の表現論は、圏化理論の成功や例外型の rank の低さなどのおかげで既約指標がほとんど理解されるなど、標準的な Borel 部分代数だけ固定しているにも関わらず著しい進展があった一方、Borel 部分代数の取り替えに執着する研究が多くはなかったように思われる。実際、Verma 加群間の準同型を記述することでよく知られる Verma の定理の自然な奇鏡映版というべきものが、有限次元 Kac-Moody Lie 超代数、より一般に Nichols 代数の設定で、Weyl groupoid の弱い性質を用いることで得られた。定式化には応用数学でよく研究される rainbow connected graph を強くした辺彩色グラフのクラスの導入が要請され、それは root 系, Weyl groupoid の良い一般化と考えられる。応用として、Verma 加群の射影次元の有限性の判定や、 A_2 型前射影代数やその一般化を実現する semibrick の構成がある。それとは別の方向として rank1 の Lie 超代数の主ブロックを実現する semibrick も構成した。

八尋耕平（京都大学）

2D パーシステンス加群の表現空間の結晶構造

パーシステンス加群は位相的データ解析で使われる、ある特殊な関係式付き籠の表現である。1D パーシステンス加群は A_n 型籠の表現であり、表現の同型類とパーシステンス図と言われるものが一対一に対応する。2D パーシステンス加群はほとんどの場合にワイルドな表現型でありそのようなことは期待できないので、別の方法で離散的な不変量を取り出す方法が要求される。この講演では、柏原と斉藤による $B(\infty)$ の構成と類似の構成により、2D パーシステンス加群の表現空間の既約成分全てのなす集合に結晶構造が入ることを説明する。柏原作用素を具体的な記述が得られている 2×2 の場合についても説明する。この講演の内容は京都大学の平岡裕章氏との共同研究に基づく。

Louise Sutton (OIST)

Cyclotomic KLR algebras in types A_∞ and C_∞

In this talk, I will begin with an overview of the representation theory of the cyclotomic KLR algebras in types A and C. The representation theory of the cyclotomic KLR algebras in type C is relatively unknown, unlike that in type A, and has seen a growing interest in its study. I will then present joint work with Chris Bowman, Rob Muth and Liron Speyer on an isomorphism theorem connecting the cyclotomic KLR algebras of types A_∞ and C_∞ .

村田遼人 (東京大学)

Affine highest weight structures on module categories over quiver Hecke algebras

Quiver Hecke algebra is a graded algebra associated with a symmetrizable Kac-Moody algebra, and its module category provides a categorification of the quantum group. In this talk, I will explain that the subcategory corresponding to the quantum unipotent subgroup determined by an element of the Weyl group has a structure of an affine highest weight category. This generalizes earlier results in the cases where the Kac-Moody Lie algebra is finite-dimensional or of symmetric affine type, due to Kato, Brundan, Kleshchev, McNamara and Muth. Our approach differs from these prior studies in that we explicitly construct the standard modules using determinantal modules and analyze them via distinguished homomorphisms known as R-matrices.

星野真生 (東京大学)

A classification of equivariant quantum full flag manifolds

I will explain a classification result on semisimple left module C^* -categories over the representation category of $U_q(su_n)$ with the torus-like fusion rule. In light of the duality theorem due to De Commer and Yamashita, these categories can be thought as equivariant quantizations of $TSU(n)$ as a smooth manifold. In fact the classification says that they are parametrized by a certain class of Poisson structures on $TSU(n)$, which concludes that there is no exotic examples in this case. The proof depends on an explicit construction inspired by the parabolic induction and a diagrammatic category which has a full functor to the representation category of $U_q(sl_n)$.

松本久義 (東京大学)

実簡約表現の既約表現研究の回想 (概説講演)

表現論の目的は既約表現の分類と理解並びに (重要な表現の) 既約分解を与えると言われるがここでは前者の話題についての研究の歴史について振り返りたい。既約表現については分類も重要だが指標の決定や、Beilinson-Bernstein 対応や軌道法のような幾何的な対象と結びつけて理解することがなされてきた。これらの話題について概観を試みる。

Pablo Sanchez Ocal (OIST)

Deformations of Frobenius algebras and noncommutative 2d topological quantum field theories

In this talk I will present an attempt to define noncommutative 2d topological quantum field theories using deformations of Frobenius algebras. First, we will overview the importance and uses of 2d topological quantum field theories, as well as their equivalence to commutative Frobenius algebras. Then, we will consider the deformations given by cotwisted tensor products, characterize when these

are Frobenius algebras, and explain their deficiencies for our goal. Afterwards, I will introduce the notion of warped tensor products of Frobenius algebras and characterize when these are Frobenius algebras. This notion enables us to construct a family of bifunctors that could potentially yield nonsymmetric monoidal structures on the category of Frobenius algebras, which would then deserve to be called noncommutative 2d topological quantum field theories. This is work with Rohan Das and Julia Plavnik.

山口航平 (名古屋大学)

Equivariant K-homology of symplectic affine Grassmannian

Let G be the symplectic group $Sp_{2n}(\mathbb{C})$, and Gr_G the affine Grassmannian of G . The purpose of the study is to provide a realization of the torus equivariant K-homology ring $K_*^T(Gr_G)$ in terms of symmetric functions. Nakagawa and Naruse introduced the K-theoretic (equivariant) dual P-function as a polynomial representing the Schubert class of the infinite dimensional Lagrangian Grassmannian. When considering the ring generated by these functions, we prove that the subring generated by the “Pieri elements” is isomorphic to $K_*^T(Gr_G)$. Via this isomorphism, the function corresponding to the structure sheaf of a Schubert variety of Gr_G can be computed explicitly using the Demazure operators. This talk is based on joint work of Takeshi Ikeda and Mark Shimozono.

石橋典 (東北大学)

Cyclic quantum Teichmüller theory

Based on the pioneering ideas of Kashaev, we present a fully explicit construction of a finite-dimensional projective representation of the dotted Ptolemy groupoid when the quantum parameter q is a root of unity, which reproduces the central charge of the $SU(2)$ Wess–Zumino–Witten model. A basic ingredient is the cyclic quantum dilogarithm. We introduce the quantum intertwiner associated with a mapping class as a composite of cyclic quantum dilogarithm operators, whose trace defines a quantum invariant. Its relation to the quantum hyperbolic field theory of Baseilhac–Benedetti will be discussed. This talk is based on the arXiv preprint 2501.02316.

安達晶平・小池隼人 (早稲田大学)

Semi-infinite Lakshmibai-Seshadri paths of twisted affine type

Kashiwara introduced extremal weight modules and proved the existence of their crystal bases. In particular, the structure of level-zero extremal weight modules is more complicated than in the positive- and negative-level cases. Ishii, Naito, and Sagaki introduced semi-infinite Lakshmibai-Seshadri (LS) paths and proved that they are isomorphic, as crystals, to the crystal bases of level-zero extremal weight modules over the quantum affine algebra of untwisted type. For the twisted case, we introduced a

semi-infinite length function that is compatible with the quantum Bruhat graph. The semi-infinite LS paths constructed using this function provide a combinatorial realization of the level-zero extremal weight crystal for the quantum affine algebra of twisted type.

廣惠一希（千葉大学）

微分方程式の不確定特異点の開折とルート系

Gauss の超幾何微分方程式の特異点の合流によって, Kummer の合流型超幾何微分方程式や Bessel, Hermite-Weber, Airy の微分方程式といった不確定特異点を持つ微分方程式が得られることが古典的に知られている. こうした観察から, 「Riemann 球面上に不確定特異点を持つ微分方程式は, 必ず Fuchs 型 (確定特異点のみを持つ) 微分方程式の合流として得られるか」という素朴な問題を考えられる. 本講演ではこの問題を, 大島利雄氏のアイデアに基づいてルート系を用いた組み合わせ的問題として定式化することで, 一定の解決が得られることを解説する.