Workshop on Shimura varieties, representation theory and related topics, 2024

Date 7 (Mon)–11 (Fri), October, 2024.

Location Lecture hall, Graduate School of Mathematical Sciences, The University of Tokyo.

Organizers Noriyuki Abe (Tokyo), Naoki Imai (Tokyo), Tetsushi Ito (Kyoto), Yoichi Mieda (Tokyo)

Schedule

	10:00–11:00	11:15–12:15	14:00–15:00	15:15–16:15	16:30–17:30
7 (Mon)	Boyer	Bando	Lourenco	Zou	Takaya
8 (Tue)	Wang	Takanashi	Schwein	Heyer	Wakatsuki
9 (Wed)	Gardner	Kim	Nakamura	Liu	Ito
10 (Thu)	Mikami	Rodríguez Camargo		free discussion	
	9:30–10:30	10:45–11:45	13:30–14:30	14:45–15:45	16:00–18:00
11 (Fri)	Yasuda	Koskivirta	Lan	Shin	free discussion

Abstracts

Katsuyuki Bando (University of Tokyo)

Derived Satake category and Affine Hecke category in mixed characteristics

In this talk, we will explain a new affine Grassmannian which connects an equal characteristic affine Grassmannian and Zhu's Witt-vector affine Grassmannian. As a result, we deduce the mixed characteristic version of the Bezrukavnikov-Finkelberg's derived Satake equivalence. By the same argument, we also obtain the mixed characteristic version of the Bezrukavnikov's equivalence between two categorifications of an affine Hecke algebra.

Pascal Boyer (Université Paris 13)

$\overline{\mathbb{Z}}_l$-nearby cycles complex for KHT Shimura

Let $X \to \operatorname{Spec} \mathbb{Z}_p$ be a generically smooth scheme whose special fiber is too ramified for a geometric approach to describe the nearby cycles complex Ψ. When X is a KHT Shimura variety we will see how the representation theory using derivative for the mirabolic group, allows to completely elucidate Ψ over $\mathbb{Q}_l$, $\mathbb{Z}_l$ and $\mathbb{F}_l$.

Zachary Gardner (Boston College)

Moduli of prismatic (G, μ)-apertures and applications

Following a suggestion of Drinfeld and building off of much recent work on prismatic Dieudonne theory and "stacky" prismatic cohomology, we introduce the notion of (truncated) (G, μ)-apertures for G a smooth affine group scheme over $\mathbb{Z}_p$ and μ a 1-bounded cocharacter of G defined over an unramified extension of $\mathbb{Z}_p$. These objects serve as group-theoretic generalizations of (truncated) p-divisible groups, and in fact there is an explicit relationship between the two when G is a

general linear group. We study the moduli of (truncated) prismatic (G, μ)-apertures and resolve several conjectures of Drinfeld on the resulting (derived) p-adic formal stacks $\mathrm{BT}_n^{G,\mu}$ (where n encodes the level of truncation). In particular, we show that the stacks $\mathrm{BT}_n^{G,\mu}$ have good smoothness, finiteness, and representability properties and construct analogues of Dieudonne theory and Grothendieck-Messing theory in this setting. These results rest crucially on the formalism of (animated higher) frames and 1-bounded derived stacks. If time allows, we will discuss how this formalism can be used to construct Rapoport-Zink spaces.

This is joint work with Keerthi Madapusi and Akhil Mathew.

Claudius Heyer (University of Paderborn)

A 6-functor formalism for smooth mod p representations

The yoga of six operations was introduced by Grothendieck in order to show that many phenomena in the étale cohomology of schemes can be formally deduced from a small set of axioms. Since then these six operations have been constructed in many other contexts such as D-modules, motives and rigid-analytic geometry. But only recently has there been a formal definition of a 6-functor formalism, mainly due to Liu–Zheng and then further simplified by Mann in his PhD thesis. Also in Fargue–Scholze's geometrization of the local Langlands correspondence the six operations are a guiding theme. In this talk I will report on joint work with Lucas Mann where we construct a full 6-functor formalism in the setting of smooth representations of p-adic Lie groups with mod p coefficients (in fact, we allow arbitrary discrete coefficient rings). As an application we use the formalism to construct a canonical anti-involution on derived Hecke algebras generalizing earlier work by Schneider–Sorensen.

Tetsushi Ito (Kyoto University)

Towards the Tate and the Hodge standard conjectures for self-products of irreducible symplectic varieties over finite fields

Theory of Shimura varieties has applications to the arithmetic of K3 surfaces. Recently, the Tate and the Hodge standard conjectures for self-products of K3 surfaces are proved by Ito-Ito-Koshikawa by similar but more refined techniques, pioneered by Madapusi. Irreducible symplectic varieties (aka hyperkaehler varieties) are higher dimensional generalizations of K3 surfaces, which have nice Hodge theoretic properties by the work of Verbitsky and Markman. In this talk, I report on an attempt to prove arithmetic results on codimension 2 cycles on self-products of irreducible symplectic varieties over finite fields by similar techniques. This talk is based on a joint work with Kazuhiro Ito (Tohoku) and Teruhisa Koshikawa (Kyoto, RIMS).

Dongryul Kim (Stanford University)

Igusa stacks for Hodge type Shimura data

Igusa stacks are geometric objects that parametrize points of Shimura varieties up to "quasi-isogeny", whose existence was conjectured by Scholze. In a joint work with Daniels, van Hoften, and Zhang, we construct Igusa stacks for all Hodge type Shimura data, extending a previous result of Zhang in the PEL case. I will discuss the construction of the Igusa stack as well as its geometric properties, and talk about how its cohomology relates to the cohomology of Shimura varieties.

Jean-Stefan Koskivirta (Saitama University)

Trivial-type automorphic forms

The motivation of the talk is to understand the relation between automorphic forms in characteristic p and the stack of G-zips of Pink-Wedhorn-Ziegler. We introduce the notion of trivial-type automorphic forms for this purpose. As an application, we prove the Cone Conjecture for weights of automorphic forms in characteristic p for Siegel modular varieties.

Kai-Wen Lan (University of Minnesota)

Boundary cohomology of well-positioned subschemes of integral models of Shimura varieties

I will first review what we know about the toroidal and minimal compactifications of Shimura varieties and their integral models, and the well-positioned subschemes of these integral models. Then I will explain some p-adic analogues of Harris and Zucker's work on the boundary cohomology of Shimura varieties and of well-positioned subschemes of their integral models (when defined). (Based on thesis works of Peihang Wu and Shengkai Mao, and on joint work with David Sherman on p-adic log Riemann-Hilbert functors in the ideally log smooth case.)

Joao Lourenco (Universität Münster)

p-adic central sheaves and Arkhipov–Bezrukavnikov

We study the nearby cycles functor for the p-adic Hecke stack attached to a Iwahori group scheme over the corresponding ring of integers. Namely, we show that this lifts to the Drinfeld center and is t-exact for the perverse t-structures. This proceeds by constructing a Wakimoto filtration on these p-adic central sheaves. We will explain how this helps in proving normality of local models (and therefore also integral models of Shimura varieties). Afterwards, we will discuss how to upgrade the nearby cycles functor to a Bezrukavnikov functor and explain how to prove the Arkhipov–Bezrukavnikov equivalence at least for GL_n. If time permits, we will discuss future directions and possible applications in categorical local Langlands.

Yifeng Liu (Zhejiang University)

Anticyclotomic Iwasawa main conjecture for Rankin-Selberg motives

I will explain some recent results on the anticyclotomic Iwasawa main conjecture for Rankin-Selberg motives for $\mathrm{GL}(n) \times \mathrm{GL}(n+1)$. I will then discuss a main ingredient in the proof, namely, the generalization of Ribet's theorem on the level raising for modular curves at good primes to unitary Shimura varieties. This is a joint work with Yichao Tian and Liang Xiao.

Yutaro Mikami (University of Tokyo)

(φ, Γ)-modules over relatively discrete algebras

In this talk, we introduce (φ, Γ)-modules over rings that are "combinations of discrete algebras and affinoid $\mathbb{Q}_p$-algebras", and present basic results, such as the existence of a fully faithful functor from the category of Galois representations, the deperfection of (φ, Γ)-modules over perfect period rings, and the dualizability of the cohomology of (φ, Γ)-modules. We also discuss the relationship between this work and the categorical p-adic Langlands correspondence for locally analytic representations, as proposed by Emerton-Gee-Hellmann, and the GL_1 case, as formulated and proved by Rodrigues Jacinto-Rodríguez Camargo.

Kentaro Nakamura (Kyushu University)

A local sign decomposition for symplectic self-dual Galois representations

Around 1987, Rubin envisioned a signed Iwasawa theory for CM elliptic curves at supersingular primes p over the anticyclotomic $\mathbb{Z}_p$-extension of the CM field, conjectural on a fundamental sign decomposition of the local Iwasawa cohomology. This conjecture was resolved by A. Burungale, K. Ota and S. Kobayashi in 2021. We generalize this decomposition to arbitrary families of two dimensional symplectic self dual Galois representations of $\mathbb{Q}_p$ using Kato's local epsilon conjecture, which was resolved by the speaker. This is a joint work (in progress) with A. Burungale, K. Ota, and S. Kobayashi.

Juan Esteban Rodríguez Camargo (Columbia University)

Integral Hodge-Tate decompositions of p-adic Siegel modular forms

Joint work in progress with Ana Caraiani and James Newton.

There are two natural sources of families of p-adic modular forms. One arises from the coherent cohomology of Igusa varieties and the other one from the completed cohomology of Shimura varieties. In this talk I will explain how both constructions can be compared via the Hodge-Tate structure of completed cohomology thanks to the recent higher Hida theory of Boxer and Pilloni.

David Schwein (Universität Bonn)

Non-basic rigid packets for discrete L-parameters

We propose a new way to organize discrete compound L-packets of a p-adic group G. Our packets include representations not only of inner twists of G but also of its elliptic twisted Levi subgroups, conjecturally realizing some version of loop Deligne–Lusztig induction or Yu's construction. The main idea is to replace Kottwitz's set with the non-basic cohomology of the Kaletha gerbe. This new cohomology set is much richer but turns out to carry many of the same structures as $B(G)$, such as the Kottwitz and Newton maps. On the Galois side, we define a rigid enhancement for L-parameters and show that the resulting refined local Langlands conjectures reduce, à la Bertoloni Meli–Oi, to the basic rigid conjectures of Kaletha. This work is joint with Peter Dillery.

Sug Woo Shin (UC Berkeley)

The local intertwining relation

The local intertwining relation is an identity that gives precise information about the action of normalized intertwining operators on parabolically induced representations. It plays a central role in Arthur's endoscopic classification for quasi-split classical groups. I will explain new results on the local intertwining relation and their implication on the status of Arthur's endoscopic classification. Joint work with Hiraku Atobe, Wee Teck Gan, Atsushi Ichino, Tasho Kaletha, and Alberto Minguez.

Yugo Takanashi (University of Tokyo)

On the formal degree conjecture for G_2

In 2008, Hiraga, Ichino and Ikeda proposed a conjecture on the explicit description of the Plancherel measures of reductive groups over local fields assuming the existence of conjectural local Langlands correspondences. This conjecture is called the formal degree conjecture. In 2021,

Beuzart-Plessis announced the proof of this conjecture for classical groups, using the Arthur's endoscopic character relations for classical groups.

In this talk, we will explain how the endoscopic character relations for triality $PGSO_8$ and G_2 imply the formal degree conjecture for G_2.

Yuta Takaya (University of Tokyo)

Second adjointness and cuspidal supports at the categorical level

The categorical local Langlands correspondence aims to realize the local Langlands correspondence as the geometric Langlands correspondence over the Fargues-Fontaine curve. In this program, geometric Eisenstein series play the role of parabolic inductions. In this talk, we will discuss their properties from two perspectives: second adjointness and cuspidal supports. We will explain the relationship between geometric Eisenstein series and the usual representation-theoretic parabolic inductions.

Seidai Yasuda (Hokkaido University)

On fine structures of two-dimensional crystalline representations

Reductions of two-dimensional crystalline representations of the absolute Galois group of the field of p-adic numbers are studied by many researchers with several methods. In this talk, I will present several results on finer structures on such representations. Most of the results are based on my joint work with Go Yamashita.

Yiyang Wang (Kyoto University)

Formal degrees and parabolic induction: the maximal generic case

The formal degree conjecture of Hiraga-Ichino-Ikeda expresses the formal degree of a discrete series of a reductive p-adic group in terms of arithmetic invariants of its Langlands parameter. A natural question is to study the compatibility of this conjecture with the parabolic induction process. In this talk I will explain how to use local harmonic analysis to deal with the simplest nontrivial case, together with some future problems.

Satoshi Wakatsuki (Kanazawa University)

Asymptotic behavior for traces of Hecke operators for holomorphic and quaternionic cusp forms

In this talk, I discuss the asymptotic behavior of traces of Hecke operators on spaces of cusp forms with respect to the weight aspect. As our main result, I will explain the asymptotic formulas for the holomorphic Siegel cusp forms of general degree and the quaternionic cusp forms of G_2, and their proofs and applications. After that, I discuss an optimal bound of the reminder for holomorphic and quaternionic cusp forms in general. Our method is based on several recent studies for Maass forms. This is a joint work with Shingo Sugiyama and Masao Tsuzuki.

Konrad Zou (Universität Bonn)

Categorical local Langlands for GL_n in the generic case for integral coefficients

Fargues and Scholze recently proposed a geometrization of the (ℓ-adic) local Langlands conjectures. At the heart of this lies an equivalence of categories between coherent sheaves on the stack of L-parameters and sheaves on the moduli stack of G-bundles on the Fargues-Fontaine curve. We discuss a proof of this conjecture after localization to parameters of Langlands-Shahidi

type. In this case we even obtain t-exactness of the equivalence for the natural t-structure on the coherent sheaves and the perverse t-structure on sheaves on the stack of G-bundles. We will mainly focus on the case of irreducible parameters and sketch the extension to Langlands-Shahidi type parameters if time permits. We will discuss some easy applications of this result.