Workshop on Shimura varieties, representation theory and related topics, 2019

Date July 15–19, 2019.

Location 4-501, Science building #4, Department of Mathematics, Hokkaido University

Organizers Noriyuki Abe (The University of Tokyo), Hiraku Atobe (Hokkaido University), Yoichi Mieda (The University of Tokyo).

Schedule

	10:00–11:00	11:30–12:30	14:30–15:30	16:00–17:00
15 (Mon)	Lan	Johansson	Shimizu	Yoshikawa
16 (Tue)	Oki	Richarz	Chan	Shin
17 (Wed)	Li	Xue	Free discussion	
18 (Thu)	Oi	Ichino	Suzuki	Fintzen
19 (Fri)	Chida	Liu	Ito	Free discussion

Abstracts

Charlotte Chan (Princeton University)

Flag varieties and representations of p-adic groups

Deligne–Lusztig varieties are subvarieties of flag varieties whose cohomology encodes the representations of reductive groups over finite fields. In this talk, we discuss recent progress towards geometric realizations of representations of p-adic groups in three arcs: affine flag varieties, semi-infinite flag varieties, and a finite-ring variant arising from the Moy–Prasad filtration of parahoric subgroups. This is joint work with Alexander Ivanov.

Masataka Chida (Tokyo Denki University)

Special values of p-adic L-functions on Shimura curves over totally real fields

Bertolini-Darmon-Prasanna proved a formula which relate special values of anticyclotomic p-adic L-functions for elliptic modular forms with the image of generalized Heegner cycles under the p-adic Abel-Jacobi map. In this talk, we will report a generalization of their result to totally real case. This is a joint work with Ming-Lun Hsieh.

Jessica Fintzen (University of Michigan)

Representations of p-adic groups

In the 1990s Moy and Prasad revolutionized the (complex and mod ℓ) representation theory of p-adic groups by showing how to use Bruhat-Tits theory to assign invariants to representations of p-adic groups. The tools they introduced resulted in rapid advancements in both representation theory and harmonic analysis – areas of central importance in the classical Langlands program. A crucial ingredient for many results is an explicit construction of (types for) representations of p-adic groups. In this talk I will indicate why, survey what constructions are known and present recent developments based on a refinement of Moy and Prasad's invariants.

Atsushi Ichino (Kyoto University)

The automorphic discrete spectrum of $\mathrm{Mp}(4)$

In his 1973 paper, Shimura established a lifting from half-integral weight modular forms to integral weight modular forms. After that, Waldspurger studied this in the framework of automorphic representations and classified the automorphic discrete spectrum of the metaplectic group $\mathrm{Mp}(2)$, which is a nonlinear double cover of $\mathrm{SL}(2)$, in terms of that of $\mathrm{PGL}(2)$. We discuss a generalization of his classification to the metaplectic group $\mathrm{Mp}(4)$ of rank 2. This is joint work with Wee Teck Gan.

Tetsushi Ito (Kyoto University)

Updates on the Tate conjecture for products of two K3 surfaces over finite fields

The Tate conjecture claims the surjectivity of the cycle class map into the Frobenius invariants of l-adic cohomology. Recently, this conjecture was proved for divisors on K3 surfaces as well as for codimension 2 cycles on the squares of K3 surfaces. In this talk, we try to explain the current status of the Tate conjecture for products of two K3 surfaces over finite fields. We hopefully give a proof of some new cases. The keys are to control the CM types appearing in the CM liftings and to invoke the local-global principle of Hermitian forms. We also explain the difficulties in the remaining cases. (This talk is based on a joint work with Kazuhiro Ito and Teruhisa Koshikawa.)

Christian Johansson (University of Gothenburg)

A vanishing theorem for Hodge type Shimura varieties at infinite Γ_1-level

I will discuss a new vanishing theorem for compactly supported cohomology of Hodge type Shimura varieties with group split at p, after taking a direct limit over the tower with $\Gamma_1(p^n)$-level structure. This is joint work in progress with Ana Caraiani and Dan Gulotta.

Kai-Wen Lan (University of Minnesota)

De Rham comparison and Poincaré duality for rigid varieties

I will give an overview on the de Rham comparison isomorphisms for p-adic étale local systems over smooth algebraic varieties (and their suitable rigid analytic analogues) that are not necessarily proper, in the context of p-adic Riemann–Hilbert correspondences, which are compatible with the formation of cohomology with (partial) compact supports and with Poincaré duality (among such cohomology). If time permits, I will also mention some application to the study of the cohomology of general Shimura varieties with nontrivial coefficients. (This is joint work with Ruochuan Liu and Xinwen Zhu.)

Wen-Wei Li (Peking University)

On some regular holonomic D-modules in harmonic analysis

Differential systems with regular singularities are indispensable tools in the representation theory of real Lie groups. They also appear in the relative setting, where one considers (absolutely) spherical homogeneous spaces. I will introduce various notions of admissibility and apply them to show that some frequently encountered D-modules are regular holonomic, including the localization of a Harish-Chandra module on a spherical homogeneous space, the D-module generated by K-finite generalized matrix coefficients, and that generated by a relative character with respect to two spherical subgroups. The proof follows Ginzburg's strategy and is based on the horocycle correspondence. It complements the recent result of Aizenbud-Gourevitch-Minchenko

on the holonomicity of relative characters.

Yifeng Liu (Yale University)

Arithmetic level raising for unitary groups and Beilinson-Bloch-Kato conjecture

In this talk, we will introduce the recent progress on Beilinson-Bloch-Kato conjecture for Rankin-Selberg motives of arbitrary rank. We will discuss an important technique used in the proof, namely, the arithmetic level raising for unitary groups of even rank. This is based on a joint work with Y. Tian, L. Xiao, W. Zhang, and X. Zhu.

Masao Oi (Kyoto University)

Towards the twisted endoscopic character relation for regular supercuspidal representations

In recent striking works of Kaletha, he constructed L-packets consisting of supercuspidal representations having a certain regularity condition, and proved several desired properties for those L-packets. Especially, by showing the endoscopic character relation for standard endoscopy, he checked that his L-packets are consistent with the standard endoscopic lifting, which is one of the most important cases of the Langlands functoriality.

In this talk, I will explain an attempt to establish the twisted version of his work, namely, the character relation for those regular supercuspidal representations in the case of twisted endoscopy.

Yasuhiro Oki (The University of Tokyo)

On supersingular loci of Shimura varieties for quaternion unitary groups of degree 2

For a Shimura variety of PEL type with certain level, Kottwitz and Rapoport-Zink associated a scheme over $\mathbb{Z}_{(p)}$, which is a moduli space of abelian varieties with additional structures. The supersingular locus is defined as the largest closed subscheme of its special fiber which parameterizes supersingular abelian varieties. Making an explicit description of the structure of the supersingular locus is motivated by arithmetic intersections. In this talk, we discuss this question for certain Shimura varieties for quaternion unitary similitude groups of degree 2 over ramified odd primes. Moreover, we also mention the similar results on the underlying reduced subschemes of the relating Rapoport-Zink spaces.

Timo Richarz (Technische Universität Darmstadt)

Cohen-Macaulayness of parahoric local models

The singularities arising in the mod-p-reduction of Shimura varieties with parahoric level structure can be described in terms of linear algebra via so called local models. By the work of G. Pappas and X. Zhu, these models are known to be normal with reduced special fibre in many cases of interest, and they conjecture that these models are Cohen-Macaulay as well. In the special case of unramified groups with Iwahori level structure, the conjecture was proven by X. He. In my talk, I report on joint work with T. Haines where we prove the conjecture in general if $p > 2$.

Koji Shimizu (University of California, Berkeley)

p-adic local systems in p-adic geometry

An etale p-adic local system on a rigid analytic variety can be regarded as a family of p-adic Galois representations parametrized by the variety. In this talk, we will explain several recent

results including the constancy of generalized Hodge-Tate weights. The latter is one instance of rigidity phenomena of geometric families of Galois representations in contrast to arithmetic families in the Galois deformation theory.

Sug Woo Shin (University of California, Berkeley)

On certain supercuspidal types

We will explain how to use certain supercuspidal types arising in the construction of supercuspidal representations by Adler and Yu to produce congruences of automorphic forms.

Miyu Suzuki (Kyoto University)

On some distinguished representations and base change lift for unitary groups

Flicker and Rallis conjectured a relation between distinguished cuspidal automorphic representations of GL and base change lift for unitary groups. In this talk, we consider a slight generalization of their conjecture. For GL(2), we can show this using a relative trace formula. We also consider p-adic analogue of this conjecture by classifying distinguished generic representations.

Ting Xue (The University of Melbourne)

Character sheaves for graded Lie algebras

We describe a strategy for classifying character sheaves in the setting of graded Lie algebras. The character sheaves for ungraded Lie algebras were classified by Lusztig. They are IC-sheaves associated to irreducible representations of various Coxeter groups. In our setting, representations of Hecke algebras associated to complex reflection groups at roots of unity enter the story. We explain how to carry out the strategy for $\mathbb{Z}/2$-graded classical Lie algebras. In particular, we explain a key geometric construction of nearby cycle sheaves associated to the adjoint quotient map. This is based on joint work with Kari Vilonen and partially joint with Misha Grinberg.

Sho Yoshikawa (Gakushuin University)

modularity of elliptic curves over certain totally real fields

Modularity lifting theorems (MLT) are a useful method to prove that a given Galois representation arise from a modular form. So far, many powerful MLTs have been developed, especially for 2-dimensional representations of the absolute Galois group of a totally real field. However, there seem still many (Galois representations attached to) elliptic curves over a totally real field whose modularity has not been known yet. In this talk, we discuss the conditions on the base field such that (hopefully all) elliptic curves over the field are proved to be modular via the known MLTs. In my previous work, I proved a modularity result of elliptic curves especially when the base field is abelian over the rationals. After explaining this abelian result, I will explain an approach and some new results beyond this case.