Representation theory of reductive Lie groups and algebras

in honor of Hisayosi Matumoto on the occasion of his 60th birthday

Date March 27–29, 2019.

Location Room 002 (27th), Room 123 (28th, 29th), Graduate School of Mathematical Sciences, the University of Tokyo.

Organizers Syu Kato, Noriyuki Abe, Katsuyuki Naoi, Kohei Yahiro, Takuma Hayashi.

Schedule

	10:30–11:30	11:45–12:45	14:15–15:15	15:30–16:30	16:45–17:45
27 (Wed)		Kobayashi	Evens	Y. Oshima	Trapa
28 (Thu)	Ikeda	Huang	T. Oshima	Vogan	Matumoto
29 (Fri)	Ochiai	Barchini	Hiraga	Oda	Ariki

Abstracts

Susumu Ariki (Osaka University)

Block algebras of Hecke algebras of classical type and the cellularity

After explaining how I determined the representation type of block algebras of Hecke algebras of classical type, I show some examples of Morita classes in derived equivalence classes arising from the block algebras. The cellularity plays an important role throughout.

Leticia Barchini (Oklahoma State University)

On the characteristic variety of Harish-Chandra modules

The characteristic variety and the associated variety are delicate invariants of Harish-Chandra modules. They have been used in an essential manner, for example, by Adams-Barbasch-Vogan in their approach to the Arthur conjectures; by Kobayashi in his theory of discrete decomposability and by Matumoto in his study of algebraic Whittaker vectors. I will survey basic facts about these invariants. In the context of $U(p,q)$-modules I will describe an explicit example of a modules with reducible leading term cycle by analyzing a 4-dimensional singular variety in

$\mathbf{C}^8$. Kashiwara-Saito study of an eight dimensional singular variety of $\mathbf{C}^{16}$, produced the first example of a highest weight module with reducible characteristic variety. Later Williamson, in the context Schubert varieties, found an example of a highest weight module with reducible leading term cycle. I will relate our singularity to the work by Kashiwara-Saito and Williamson.

(The talk is partially based on joint work with Petr Somberg and Peter Trapa.)

Samuel Evens (University of Notre Dame)

The Belkale-Kumar cup product and the wonderful compactification of a symmetric space

I will explain some joint work with Bill Graham interpreting a family of products on the cohomology of G/P in terms of relative Lie algebra cohomology. This family was introduced by Belkale and Kumar, and gives an irredundant solution to the geometric Horn problem, by work of Ressayre.

I will also explain an extension of this family to certain non-Kahler homogeneous spaces, which is motivated by a construction using the wonderful compactification.

Kaoru Hiraga (Kyoto University)

On endoscopy for n-fold covering group of SL(2)

This is a joint work with T. Ikeda. In "L-indistinguishability for SL(2)", Labesse and Langlands established the theory of endoscopy for SL(2) and its anisotropic inner form D^1. Though covering group is not an algebraic group, for some of the covering groups of SL(2) and D^1, we showed the existence of correspondences analogous to the endoscopy. In this talk, I will explain such correspondences and how they vary depending on the degree n.

Jing-Song Huang (HKUST)

Dirac Series, Coherent Continuation and Unipotent Representations

Classifying irreducible unitary representations of real reductive Lie groups is a central problem in representation theory, which is well-known as the unitary dual problem. The orbit method establishes a correspondence between irreducible unitary representations of a Lie group and its coadjoint orbits. Diximer's work on primitive ideals gave one of the earliest indications of such a correspondence. The theory was established by Kirillov for nilpotent groups and it was later extended by Kostant and Auslander to solvable groups. Vogan proposed that the orbit method should serve as a unifying principle in the description of the unitary dual of real reductive groups.

In Vogan's formulation of the orbit method for real reductive groups, the correspondence from the coadjoint orbits to irreducible unitary representations is divided into three steps according to the Jordan decomposition of a linear functional on Lie algebras into hyperbolic, elliptic and nilpotent components. The hyperbolic step and elliptic step are well understood, while the nilpotent step to construct unipotent representations from nilpotent orbits has been a focus of active research in many related areas.

The aim of this talk is to show that our recent work joint with Pandzic and Vogan on classifying unitary representations by their Dirac cohomology shed light on understanding unipotent representations. In particular, the coherent continuation relates the Dirac series (irreducible unitary representations with nonzero Dirac cohomology) to unipotent representations.

Tamotsu Ikeda (Kyoto University)

An explicit formula for the Siegel series

Degenerate principal series plays an important role in the theory of automorphic forms. For example, the Fourier coefficients of the Siegel Eisenstein series are expressed by Siegel series, which are degenerate Whittaker functions. In this talk, I will talk about an explicit formula for the Siegel series by using the Gross-Keating invariant of a quadratic form.

Toshiyuki Kobayashi (The University of Tokyo)

Homomorphisms between Verma Modules and Symmetry Breaking Operators

It is classically known that homomorphisms between Verma modules induce differential intertwining opeartors for principal series representations. We extend this observation to a more general setting of branching problems for the restriction of representations for a pair of groups, and also from local operators to non-local ones such as integral symmetry breaking operators. If time permits, we discuss explicit constructions and classification of such symmetry breaking operators in conformal geometry.

Hisayosi Matumoto (The University of Tokyo)

On homomorphisms between scalar generalized Verma modules for complex simple Lie algebras of type B and C

In the classification of the homomorphisms between scalar generalized Verma modules for $\mathfrak{gl}(n, \mathbb{C})$, the translation principle in the mediocre region plays an important role. In this talk, I try to apply similar idea to the corresponding problem on $\mathfrak{sp}(n, \mathbb{C})$ and $\mathfrak{so}(2n+1, \mathbb{C})$.

Hiroyuki Ochiai (Kyushu University)

Nilpotent orbits in complex simple Lie algebras and prehomogeneous vector spaces

We examine the example given by Barchini-Zierau in 2008, that the conormal bundle of a closed K-orbit on a partial flag variety G/Q associated with an appropriate parabolic subalgebra Q is not prehomogeneous, by constructing relative and absolute invariants of the related representations.

Hiroshi Oda (Takushoku University)

Spherical functions for small-dimensional K-types

Let G be a connected real semisimple Lie group with a maximal compact subgroup K. In this talk we introduce a notion of *minuscule K-types*, which is a simultaneous generalization of fine K-types for real split semisimple Lie groups and minuscule representations for complex semisimple Lie groups. If (π, V) is such a K-type then the algebra $\boldsymbol{D}^\pi$ of invariant differential operators for the vector bundle $G \times_K V$ has very simple structure, so that we can expect in the harmonic analysis for $C^\infty(G \times_K V)$ as explicit results as in the classical study for $C^\infty(G/K)$. In fact, in many cases the elementary spherical functions ϕ^π_λ for $G \times_K V$ are expressed by Opdam's non-symmetric hypergeometric functions, from which we can deduce the Plancherel formulas explicitly. This talk is based on some joint works with Nobukazu Shimeno.

Yoshiki Oshima (Osaka University)

On the asymptotic support of Plancherel measures for homogeneous spaces

Let G be a real reductive group and X a homogeneous G-manifold. The Plancherel measure for X describes how $L^2(X)$ breaks up into irreducible unitary representations of G. We discuss asymptotics of the support of Plancherel measure and relate it with geometry of coadjoint orbits. In particular, we give a sufficient condition for the existence of discrete series. This is a joint work with Benjamin Harris (Cornell University).

Toshio Oshima (Josai University)

Analysis of hypergeometric systems via confluence and fractional derivative

We will explain our recent study of hypergeometric equations which may have unramified irregular singularities. Here examples of the equations are rigid Fuchsian ordinary differential equations, KZ equations and their confluences. Our main subject is the connection problem of their solutions.

Peter Trapa (University of Utah)

Cells for the twisted Lusztig-Vogan Hecke algebra action

Motivated partly by the study of unitary representation theory, a few years ago Lusztig and Vogan introduced a quasiplit Hecke algebra action on a quotient of the Grothendieck group of certain twisted Harish-Chandra modules. Using this action, one can define twisted cells. When the twist is trivial, the Hecke action reduces to the original Lusztig-Vogan action arising in the proof of the Kazhdan-Lusztig conjectures for real groups, and the corresponding untwisted cells encode information about tensoring Harish-Chandra modules with finite-dimensional representations. (For this reason, cells have many important applications, as is showcased in some of Matumoto's work.) The goal of this talk is to explain relationships between twisted and untwisted cells, and perhaps give some hints about representation theoretic applications.

David Vogan (MIT)

The local Langlands conjecture for finite groups of Lie type

In the late 1960s, Langlands' study of automorphic forms led him to a remarkable conjecture about the representation theory of reductive groups over local fields. The simplest and most fundamental case of this conjecture says that if F is any local field, then the (infinite-dimensional) irreducible representations of $\mathrm{GL}(n,F)$ are indexed (approximately) by the n-dimensional representations of the Galois group of F.

In the 1970s, Macdonald formulated and proved an analogue of Langlands' conjecture for the finite group $\mathrm{GL}(n,\mathbb{F}_q)$. I will explain how one can extend Macdonald's formulation to any finite group of Lie type; what results of Lusztig and Shoji offer toward proof of this extension; and how these questions are related to Langlands' (still unproven!) conjecture about local fields.