

# Workshop on Shimura varieties, representation theory and related topics

**Date** November 21–25, 2016

**Location** Department of Mathematics, Kyoto University, Faculty of Science Bldg No.3, Lecture Room 110

**Organizers** Noriyuki Abe (Hokkaido University), Naoki Imai (University of Tokyo), Tetsushi Ito (Kyoto University), Yoichi Mieda (University of Tokyo)

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## Schedule

	10:00–11:00	11:15–12:15	14:00–15:00	15:30–16:30	17:00–18:00
21 (Mon)	Abe	Kohlhaase	Tokimoto	Tsushima	Zhu
22 (Tue)	Hauseux	Hu	Oi	Atobe	Morimoto
23 (Wed)	Mieda	Hansen	Koshikawa	Harashita	Kret
24 (Thu)	Patrikis	Nakamura			
	9:30–10:30	10:40–11:40	11:50–12:50	14:15–15:15	15:30–16:30
25 (Fri)	Ito	Matsumoto	Lan	Hattori	Shin

## Abstracts

*Noriyuki Abe (Hokkaido University)*

### On mod $p$ Satake transforms

We give an explicit formula of the inverse of the mod  $p$  Satake transform. This immediately gives the “change of weight theorem” which plays an important role to prove a classification theorem of irreducible admissible mod  $p$  representations of reductive groups. This is joint work with Florian Herzig and Marie-France Vignéras.

*Jan Kohlhaase (Universität Duisburg-Essen)*

### On the construction of supersingular representations of $p$ -adic reductive groups

Let  $G$  be a  $p$ -adic reductive group with maximal compact subgroup  $K$ , let  $\rho$  be an irreducible smooth representation of  $K$  over an algebraically closed field  $E$  of characteristic  $p$ , and let  $M$  denote the smooth compact induction of  $\rho$  from  $K$  to  $G$ . The question of whether  $M$  admits any admissible irreducible quotients which are supersingular is related to the behavior of a Koszul complex associated with  $M$ . Although the corresponding complex of pro- $p$  Iwahori invariants is known to be acyclic this information does not seem to be sufficient to classify the supersingular quotients of  $M$  and their associated pro- $p$  Iwahori-Hecke modules. We will report on a refined approach to this problem involving the Koszul complex of an injective resolution of  $M$ .

*Kazuki Tokimoto (University of Tokyo)*

### **Affinoids in the Lubin-Tate perfectoid space and special cases of the local Langlands correspondence**

The non-abelian Lubin-Tate theory asserts that the local Langlands correspondence for  $GL(n)$  and the local Jacquet-Langlands correspondence are realized in the cohomology of the Lubin-Tate tower. Motivated by this fact, Boyarchenko-Weinstein and Imai-Tsushima constructed a family of affinoid subspaces in the Lubin-Tate perfectoid space (a certain limit space of the tower) and proved that the cohomology of the reduction of (a formal model of) each affinoid realizes the two correspondences for certain representations.

In this talk we will discuss a similar result for some other representations, assuming that the base field is of equal-characteristic.

*Takahiro Tsushima (Chiba University)*

### **Non-abelian Lubin-Tate theory for general linear groups in some cases**

The non-abelian Lubin-Tate theory asserts that the middle cohomology of Lubin-Tate space realizes the local Langlands correspondence (shortly, LLC) and the local Jacquet-Langlands correspondence for general linear groups  $GL_n$  over a non-archimedean local field  $F$ . This is proved by Harris-Taylor and Boyer by using global automorphic representations. This theory is originally proved by Deligne when  $F = \mathbb{Q}_p$  and  $p \neq 2$ , and conjectured by Carayol. The theory is a generalization of the Lubin-Tate theory, which explicitly constructs maximal abelian extension of  $F$  by using torsion points of formal groups. The Lubin-Tate theory gives a local proof of the local class field theory (shortly, LCFT). The LLC is regarded as a generalization of the LCFT. We hope a local proof of the non-abelian Lubin-Tate theory. Let  $\text{char } F$  denote the characteristic of  $F$ . By analyzing some kind of affinoids in the Lubin-Tate space, we can give a purely local proof when the char  $F$  is odd prime and  $n = 2$ .

We introduce a research in progress generalizing this to the case where  $n$  is a general prime number different from char  $F$ . This research is a joint work with Y. Mieda.

*Xinwen Zhu (California Institute of Technology)*

### **On some Tate cycles on Shimura varieties mod $p$**

I will first describe a large class of conjectural Tate classes in the middle dimensional étale cohomology of many Shimura varieties mod  $p$  (e.g. Hilbert and Picard modular surfaces at inert primes). According to the Tate conjecture, there should exist corresponding algebraic cycles. Then I will explain a uniform construction of these cycles using some ideas from the geometric Langlands program. This is based on a joint work with Liang Xiao.

*Julien Hauseux (King's College London)*

### **Parabolic induction and extensions**

Let  $G$  be a  $p$ -adic reductive group. The irreducible admissible smooth mod  $p$  representations of  $G$  have been classified by Abe, Henniart, Herzig and Vignéras in terms of supersingular representations of Levi subgroups of  $G$  and parabolic induction. However the supersingular representations remain a complete mystery in general. In this talk, I will describe how to compute the extensions between admissible smooth mod  $p$  representations of  $G$  in terms of extensions

between supersingular representations of Levi subgroups and parabolic induction. A key ingredient is the computation of Emerton's delta-functor of derived ordinary parts on parabolically induced representations of  $G$ .

*Yongquan Hu (Chinese Academy of Sciences)*

### **On the Cohen-Macaulayness of crystabelline Galois deformation rings**

In this talk, we will prove that certain crystabelline Galois deformation rings of two dimensional residual representations of  $\text{Gal}(\overline{\mathbb{Q}_p}/\mathbb{Q}_p)$  are Cohen-Macaulay, when  $p > 3$  and the residual representation has only scalar endomorphisms. This is joint work with V. Paškūnas.

*Masao Oi (University of Tokyo)*

### **On the endoscopic lifting of simple supercuspidal representations of classical groups**

Recently, Arthur proved the local Langlands correspondence for quasi-split classical groups over  $p$ -adic fields. He characterized the correspondence via the endoscopic character relation, which is an equality between the characters of representations of a classical group and the twisted characters of their liftings to a general linear group. On the other hand, Gross and Reeder defined a some special class of supercuspidal representations which they call the simple supercuspidal representations. In this talk, I will show that simple supercuspidal representations constitutes L-packets, and determine their endoscopic liftings to general linear groups by computing the endoscopic character relation explicitly.

*Hiraku Atobe (Kyoto University)*

### **On the non-vanishing of local theta liftings**

The global theta lift is a general lifting which contains many classical liftings, e.g., the Shimura correspondence and the Saito-Kurokawa lifting. There is a local-global criterion of non-vanishing of global theta lifts established by Yamana and Gan-Qiu-Takeda. In this talk, we give a criterion of non-vanishing of local theta lifts of unitary dual pairs in terms of the local Langlands correspondence.

*Kazuki Morimoto (Kobe University)*

### **Refined Gross-Prasad conjecture on special Bessel periods for $\text{SO}(2n+1) \times \text{SO}(2)$ .**

The Gross-Prasad conjecture relates the non-vanishing of central values of  $L$ -functions for  $\text{SO}(2n+1) \times \text{SO}(2m)$  and the non-vanishing of certain Bessel periods. Ichino and Ikeda refined the Gross-Prasad conjecture in co-rank 1 case, and recently Liu extended their work to any  $n, m$ . In this talk, under certain assumptions, we prove refined Gross-Prasad conjecture in the  $\text{SO}(2n+1) \times \text{SO}(2)$ -case when the automorphic representation of  $\text{SO}(2)$  is trivial. As an application, we prove the conjecture by Böcherer on an explicit formula of central critical values of imaginary quadratic twists of spinor L-function for holomorphic Siegel cusp forms of degree 2. This is a joint work with Masaaki Furusawa.

*Yoichi Mieda (University of Tokyo)*

### **Compactly supported cohomology of affinoid perfectoid spaces and their reductions**

Under some conditions, I will compare the compactly supported  $\ell$ -adic cohomology of an affinoid perfectoid space and that of the reduction of its formal model. I will also give some applications to the local Langlands correspondence and the non-abelian Lubin-Tate theory.

*David Hansen (Columbia University)*

### **The geometry of $p$ -adic period domains**

The rigid generic fiber of a Rapoport-Zink space admits a remarkable étale map to a rigid analytic flag variety; this is the so-called Grothendieck-Messing period map. Studying the geometry of this map and its image - the “admissible locus” - is a difficult problem, and our knowledge is poor outside of a few special cases. I’ll describe some recent progress on understanding these maps; in particular, I’ll explain some ideas for getting to grips with the complement of the admissible locus. We’ll look at some fun examples, in particular at the Rapoport-Zink space associated with the isoclinic  $p$ -divisible group over  $\overline{\mathbb{F}}_p$  of dimension 2 and height 7; here the relevant flag variety is 10-dimensional, and I’ll try to convince you that the inadmissible locus is 3-dimensional and naturally stratified into three disjoint strata. This is joint work with Jared Weinstein.

*Teruhisa Koshikawa (RIMS, Kyoto University)*

### **Hypersurfaces and the weight-monodromy conjecture**

The weight-monodromy conjecture is a generalization of the Weil conjecture (or the purity) for smooth proper varieties over  $p$ -adic local fields. Scholze proved the conjecture in certain cases, in particular for all smooth hypersurfaces, by developing the theory of perfectoid spaces.

Scholl observed that the purity for smooth hypersurfaces over finite fields implies the purity for all smooth proper varieties. It is tempting to consider an analogue for the weight-monodromy conjecture. I will explain difficulties and some related ideas.

*Shushi Harashita (Yokohama National University)*

### **On $p$ -divisible groups with saturated Newton polygons**

I will talk on a “classification” of isogeny classes of  $p$ -divisible groups with saturated Newton polygons. This topic can be regarded as an extension of Dieudonné-Manin classification over algebraically closed fields (and over perfect fields), the generalization by Zink over regular rings (including the case of imperfect fields) and that by Oort-Zink over normal noetherian rings. The preceding studies always assumed that a family of  $p$ -divisible groups has a constant Newton polygon, but we shall weaken it to the assumption that the family has saturated Newton polygons.

*Arno Kret (University of Amsterdam)*

### **Galois representations for the general symplectic group**

In a recent preprint with Sug Woo Shin (<https://arxiv.org/abs/1609.04223>) I construct Galois representations corresponding for cohomological cuspidal automorphic representations of general symplectic groups over totally real number fields under the local hypothesis that there

is a Steinberg component. In this talk I will explain some parts of this construction that involve the eigenvariety.

*Stefan Patrikis (University of Utah)*

### **Residual irreducibility of compatible systems**

We show that a compatible system of absolutely irreducible  $\ell$ -adic representations of the Galois group of a number field is, for a density one set of  $\ell$ , also absolutely irreducible modulo  $\ell$ . The theorem generalizes previous work of Barnet-Lamb, Gee, Geraghty, and Taylor (for Hodge-Tate regular Galois representations) and of Zarhin (for Tate modules of abelian varieties). This is joint work with Andrew Snowden and Andrew Wiles.

*Kentaro Nakamura (Saga University)*

### **A construction of the local epsilon isomorphisms using Colmez's multiplicative convolution**

For any de Rham representation of  $\text{Gal}(\overline{\mathbb{Q}_p}/\mathbb{Q}_p)$ , one can define a canonical trivialization, which is called the local epsilon isomorphism, of the determinant of its Galois cohomology (precisely, a trivialization of the fundamental line) using the Bloch-Kato's fundamental exact sequence and the local  $\varepsilon$ -constant of the associated Weil-Deligne representation. In the late 90's, Kazuya Kato proposed a conjecture called the local epsilon conjecture predicting that such a trivialization can be interpolated to all the families of  $p$ -adic representations of  $\text{Gal}(\overline{\mathbb{Q}_p}/\mathbb{Q}_p)$ . Up to now, such a trivialization is known only in some special cases, e.g. the trianguline case and the rank two case (in the latter case, we use  $p$ -adic local Langlands correspondence for  $\text{GL}_2(\mathbb{Q}_p)$ ). In my talk, I propose a conjecture on the construction of the local epsilon isomorphisms for any families by using Colmez's multiplicative convolution defined on the associated  $(\phi, \Gamma)$ -modules.

*Kazuhiro Ito (Kyoto University)*

### **Existence of supersingular reduction for families of K3 surfaces with large Picard number in positive characteristic**

We focus on non-isotrivial families of K3 surfaces in characteristic  $p$  whose geometric generic fibers satisfy  $\rho = 22 - 2h$  and  $h \geq 3$ , where  $\rho$  is the Picard number and  $h$  is the height of the formal Brauer group. We show that, under a mild assumption on the characteristic of the base field, they have potentially supersingular reduction. Our methods depend on Maulik's results on families of K3 surfaces and the construction of sections of Hodge bundles due to van der Geer and Katsura. For some  $p$  and  $h$ , we construct non-isotrivial families of K3 surfaces satisfying  $\rho = 22 - 2h$ . Using Taelman's results, we also give conditional construction of such families for any  $p$  and  $h$  if we assume semistable reduction.

*Yuya Matsumoto (Nagoya University)*

### **Degeneration of K3 surfaces and automorphisms**

We discuss relations between automorphisms and degeneration of K3 surfaces. We prove that K3 surfaces with non-symplectic automorphisms of certain orders do not degenerate. This is a joint work with Yuji Odaka.

*Kai-Wen Lan (University of Minnesota)*

### **Compactifications of subschemes of integral models of Shimura varieties**

I will explain that many “well-positioned” subschemes of mixed characteristics models of Shimura varieties admit good (partial) toroidal and minimal compactifications, with familiar boundary stratifications and formal local structures, as if they were Shimura varieties in characteristic zero. If time permits, I will also explain some generalizations of Koecher’s principle and the relative vanishing of subcanonical extensions for coherent sheaves, and of Pink’s and Morel’s formulae for étale sheaves, to the context of such subschemes. (This is joint work with Stroth.)

*Shin Hattori (Kyushu University)*

### **On a properness of the Hilbert eigenvariety at integral weights**

Let  $p$  be a rational prime and  $G$  a reductive algebraic group over  $\mathbb{Q}$ . The eigenvariety for  $G$  is a rigid analytic variety which  $p$ -adically interpolates Hecke eigensystems of finite slopes appearing in the space of automorphic forms of a fixed tame level on  $G$ . Though it plays an important role in number theory, its geometry is not well-understood yet. In this talk, I will explain how to show a properness of Andreatta-Iovita-Pilloni’s eigenvariety of Hilbert cuspidal eigenforms at integral weights for some cases, using the theory of canonical subgroups of abelian varieties.

*Sug Woo Shin (UC Berkeley)*

### **Non-emptiness of newton strata**

I will explain a group theoretic result (joint work with Mark Kisin and Keerthi Madapusi Pera) on Kottwitz isocrystals and Hodge cocharacters for a connected reductive group  $G$  over  $\mathbb{Q}_p$ , which holds under a technical hypothesis on  $G$  (which is always satisfied if  $G$  is quasi-split and sometimes satisfied if  $G$  is not quasi-split). This result implies, under the same hypothesis, that the Newton strata in the mod  $p$  fiber of a Hodge-type Shimura variety is non-empty.