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Reconstruction of a moving boundary from Cauchy data in one dimensional heat equation

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RECONSTRUCTION OF A MOVING BOUNDARY FROM CAUCHY DATA IN ONE DIMENSIONAL HEAT EQUATION

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ABSTRACT. In this paper, we propose a new numerical method for determining a moving boundary from Cauchy data in one dimensional heat equation. The numerical scheme is based on the use of fundamental solutions of the heat equation as basis functions. In order to regularize the ill-conditioned linear system of equations resulted by collocating boundary data, we apply successfully the Tikhonov regularization with the generalized cross validation parameter choice rule to obtain a stable numerical approximation to a moving boundary.

1. INTRODUCTION

The boundary identification problem for the Laplace equation or a heat equation arises in the ironmaking blast furnace where we are requested to monitor the corroded thickness of the accreted refractory wall based on the measurement of temperature and heat flux on an accessible part of boundary or some internal positions. This kind of problem is ill-posed in Hadamard's sense (e.g., [39]). That is, any small change on the input data can result in a dramatic change to the solution. Hence, a special regularization technique is necessary for stabilizing the computations [15, 19, 39]. Reconstruction of a corroded boundary from the Laplace equation has been investigated in some papers [1, 2, 4, 8, 10, 20, 27, 30, 37, 38]. For a heat conducting solid, a number of numerical methods for determining a portion of steady state boundary where the temperature is fixed, have been proposed in references [3, 5, 6, 7, 9, 11, 12, 13, 40]. However, for estimating a time-varying

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boundary of the heat conduction problem, as we know, not many papers can be found [17, 31]. Most of the papers above mentioned used an iterative method to reconstruct an unknown boundary, *i.e.* one starts with an initial guess of boundary shape and adjusts it iteratively by minimizing a functional of defect between the calculated boundary data and the measured data. This kind of method could be time-consuming since at each iteration step a direct problem has to be solved. In paper [17], Fredman gave a direct method, called the method of lines, to calculate a moving boundary in one dimensional heat conduction problem, where the initial values of temperature should be used. We note that Manselli and Vesella proved the continuous dependence of moving boundary on noncharacteristic Cauchy data under an a priori information even without using the initial temperature [31]. Thus, for the boundary identification problem of heat equation, the initial condition is not necessary. In this paper we proposed a meshless approach, called the method of fundamental solutions (MFS) for estimating a moving boundary of the heat equation by using only Cauchy data on a part of boundary without using any initial temperature. To our knowledge, the proposed approach has not previously been used to solve a boundary identification problem.

The main idea of the MFS is to approximate an unknown solution by a linear combination of fundamental solutions whose singularities are located outside the solution domain. The coefficients in linear combination will be determined by solving a linear system of algebraic equations which is obtained by fitting the specified data on boundary. The MFS has recently been used extensively for solving various direct problems of linear elliptic equations. Details can be found in the review papers of Fairweather and Karageorghis [16] and Golberg and Chen [18] as well as the references therein. In the studies of inverse problems for elliptic equations, we refer to papers using the MFS combined with the Tikhonov regularization to solve a Cauchy problem: for Helmholtz-type equations , see Marin [33] and Marin and Lesnic [35] and for a biharmonic equation and the three dimensional elastostatics case, see Marin and Lesnic [34], Marin [32]. Jin and Zheng used the MFS combining the truncated singular value decomposition method to treat with the Cauchy problem of a Helmholtz equation [29]. Recently, Hon and Wei have extended the MFS to solve the inverse heat conduction problems [25, 26] and Mera employed the

MFS to solve a backward heat conduction problem [36]. In this paper, we apply the MFS to determine the moving boundary in a heat conduction problem. The MFS is a kind of spatial-time continuation technique which extends the solution to a desired boundary. Then we can determine an unknown boundary as a level set of the solution. This approach is different from most existing numerical schemes in solving dynamical problems where the finite difference quotient will be used to discretize the time variable. Our method is simple and feasible for treating various homogeneous heat conduction problems.

For an ill-posed problem, the linear system of equations resulting from the boundary collocation is highly ill-conditioned. Some regularization techniques usually give better controls on levels of numerical accuracy to the original problem but requires a good choice of regularization parameter for the optimal and stable performance. In this paper, we use the Tikhonov regularization with the generalized cross validation parameter choice rule for stabilizing the solution.

2. Formulation of the problem

Consider a heat conduction equation in one dimensional case with a moving boundary $s(t)$

$$\frac{\partial u(x,t)}{\partial t} = a^2 \frac{\partial^2 u(x,t)}{\partial x^2}, \qquad 0 < x < s(t), \quad 0 < t < T. \tag{2.1}$$

Cauchy data are specified at the left boundary $x = 0$, i.e.,

$$u(0,t) = u_0(t), \quad 0 < t < T, \tag{2.2}$$

$$\tfrac{\partial u(0,t)}{\partial x} = q_0(t), \quad 0 < t < T, \tag{2.3}$$

where $u(x,t)$ is the temperature distribution and T represents the maximum time of interest for the time evolution of the problem. The boundary identification problem of heat equation is then the determination of the boundary movement function $s(t)$ from a Dirichlet boundary condition

$$u(s(t),t) = u_s(t), \tag{2.4}$$

where $u_s(t)$ is a given function. In some case, $u_s(t) \equiv u_s$ is a constant and the fusion point of the medium under consideration. In general, this problem is severely ill-posed because it is involved with an ill-posed Cauchy problem (2.1)-(2.3). On the instability of the boundary identification to the Cauchy data, refer to paper [31]. We can shown the uniqueness of moving boundary reconstructed from Cauchy data for a general case, see to this paper's appendix.

The following sections will explain the basic idea on the application of the MFS in order to solve this kind of ill-posed problem. For obtaining a stable numerical result, the standard Tikhonov regularization technique and the generalized cross validation choice rule [22] are adopted for the resultant ill-conditioned linear systems.

3. The method of fundamental solutions

The fundamental solution of heat equation (2.1) is

$$G(x,t) = \frac{1}{2a\sqrt{\pi t}} e^{-\frac{x^2}{4a^2 t}} H(t), \tag{3.1}$$

where $H(t) = 1$ if $t \geq 0$ and $H(t) = 0$ if $t < 0$.

Take some source points (x_j^*, t_j^*), $j = 1, 2, \cdots n$ and a positive real number τ such that $\tau > \max_j\{t_j^*\}$. All the source points are pairwise distinct in the spatial-time space.

Following the idea of the method of fundamental solutions, we assume that an approximate solution to the inverse problem for (2.1) can be expressed by the following linear combination of basis functions:

$$u_n(x,t) = \sum_{j=1}^{n} \lambda_j G(x - x_j^*, t - t_j^* + \tau), \tag{3.2}$$

where λ_j are unknown coefficients to be determined. It is noted that the approximate solution u_n has already satisfied heat equation (2.1) for $t > 0$.

Take collocation points $\{(x_i, t_i), i = 1, 2, \cdots, n_D, n_D + 1, \cdots, n_D + n_N\}$ on the boundary $x = 0$. Let $n = n_D + n_N$, by fitting the boundary condition, we obtain a linear system

$$\mathbf{A}\lambda = \mathbf{b}, \tag{3.3}$$

where $\mathbf{A}$ is an $n \times n$ matrix:

$$\mathbf{A} = \begin{pmatrix} G(x_i - x_j^*, t_i - t_j^* + \tau) \\ \frac{\partial G}{\partial x}(x_k - x_j^*, t_k - t_j^* + \tau) \end{pmatrix}_{1 \leq i \leq n_D, n_D+1 \leq k \leq n, 1 \leq j \leq n}, \tag{3.4}$$

and λ, $\mathbf{b}$ are n-vectors:

$$\mathbf{b} = \begin{pmatrix} u_0(t_1) \\ \vdots \\ u_0(t_{n_D}) \\ q_0(t_{n_D+1}) \\ \vdots \\ q_0(t_n) \end{pmatrix}, \quad \lambda = \begin{pmatrix} \lambda_1 \\ \lambda_2 \\ \vdots \\ \lambda_n \end{pmatrix}. \tag{3.5}$$

In the next section, we successfully apply both the Tikhonov regularization technique and the generalized cross validation method and obtain a stable numerical solution to the linear system (3.3). The approximate temperature distribution (3.2) can then be obtained by substituting the coefficient vector λ into equation (3.2).

4. Regularization method

For the ill-posed problem, the matrix resulting by a discretization will have a large condition number. Hence, most standard numerical methods cannot achieve good accuracy in solving linear system (3.3). Several regularization methods have been developed for solving an ill-conditioned system [21, 22, 23]. In our computations we adapt the Tikhonov regularization technique to solve the equations (3.3). The Tikhonov regularized solution λ_α for (3.3) is defined by the solution of the following least square problem:

$$\min_{\lambda} \{\|\mathbf{A}\lambda - \mathbf{b}\|^2 + \alpha^2\|\lambda\|^2\}, \tag{4.1}$$

where $\|\cdot\|$ denotes the usual Euclidean norm and $\alpha > 0$ is called a regularization parameter.

The determination of a suitable value for the regularization parameter α is crucial and is still under intensive researches (e.g., [15, 19]). In our computations we use the generalized cross validation (GCV) method to determine a suitable value of α. The GCV method was firstly investigated by Wabba et al in [14, 41] and more recently Hansen et al [22, 24] gave the formulation of computations based on the singular value decomposition (SVD).

The GCV is a strategy which gives a good regularization parameter α_g by minimizing the following GCV function

$$G(\alpha) = \frac{\|\mathbf{A}\ \lambda_\alpha - \mathbf{b}\|^2}{(trace(I_n - \mathbf{A}\mathbf{A}^I))^2}, \quad \alpha > 0, \tag{4.2}$$

where $\mathbf{A}^I = (\mathbf{A}^T\mathbf{A}+\alpha^2 I_n)^{-1}\mathbf{A}^T$ is a matrix which produces the regularized solution when multiplied with $\mathbf{b}$, i.e., $\lambda_\alpha = \mathbf{A}^I\ \mathbf{b}$.

In our computations, we used the Matlab code developed by Hansen [23] for solving the discrete ill-conditioned system (3.3). Denote the regularized solution of (3.3) by λ^{α_g}. The approximate solution u_{n,α_g} for problem (2.1) is then given by

$$u_{n,\alpha_g}(x,t) = \sum_{j=1}^{n} \lambda_j^{\alpha_g} \phi(x - x_j^*, t - t_j^* + \tau). \tag{4.3}$$

Furthermore, find the values s_j such that $u_{n,\alpha_g}(s_j, \bar{t}_j) = 0$ for $\bar{t}_j = jh, h = T/m, j = 0, 1, \cdots, m$, then we can obtain an approximation boundary $\{(\bar{t}_j, s_j)\}$ to the moving boundary $s = s(t)$.

The numerical results in the following section indicate that the proposed scheme is stable, feasible, and efficient.

5. Numerical verification

For simplicity, we assume that the heat conduction coefficient is $a = 1$ and the maximum time is $T = 1$.

For the numerical error estimation, we compute the root mean square error by the following formula

$$\varepsilon(s) = \sqrt{\frac{1}{m}\sum_{i=1}^{m} (s(\bar{t}_i) - s_i)^2}, \tag{5.1}$$

where m is the total number of uniformly distributed test points on time internal $[0, T]$. In our computations, we always take $m = 21$. The numbers of collocation points and source points are $n_D = 51$ and $n_N = 51$ for all tests and the parameter τ is fixed at 5.2. The source points are always taken as $x_i^* = 0 - ds$, $t_i^* = 1/(N_D - 1) * (i - 1)$, $i = 1, 2, \cdots, N_D$ and $x_j^* = 1 + ds$, $t_j^* = 1/(n_N - 1) * (j - n_D - 1)$, $j = n_D + 1, n_D + 2, \cdots, n_D + n_N$ where $ds = 1$ is the distance of the source point to the boundary except another statement. Refer to Figure 1 for the collocation and source points.

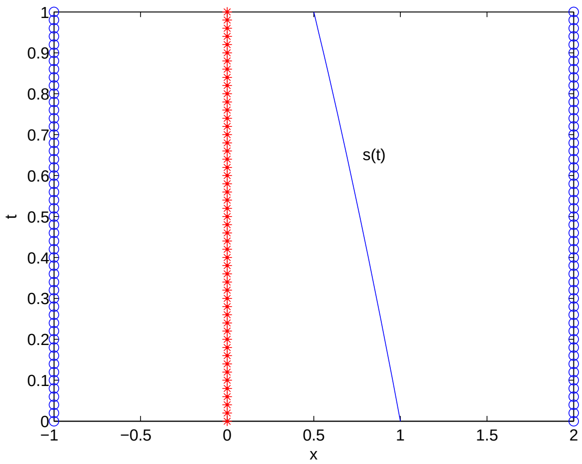

FIGURE 1. Dots $(\cdot)$ are collocation points for Dirichlet data; Stars (*) are collocation points for Neumann data and circles (o) are source points

When the measured Cauchy data include some random noises, we use noisy data

$$\widetilde{u}_0(t_i) = u_0(t_i)(1 + \delta \cdot \mathrm{rand}(i))$$

and

$$\widetilde{q}_0(t_i) = q_0(t_i)(1 + \delta \cdot \mathrm{rand}(i)),$$

where $u_0(t_i)$ and $q_0(t_i)$ are the exact data; $\mathrm{rand}(i)$ is a random number uniformly distributed in $[-1, 1]$ and the magnitude δ indicates a relative noise level.

Numerical experiments for five examples are investigated as follows.

Example 1: Let the exact solution for the problem (2.1)-(2.4) be

$$u(x, t) = 1 - x. \tag{5.2}$$

and take a steady state boundary $s(t) \equiv 1, u_s(t) = 0$. The Cauchy data can be calculated as $u_0(t) = 1$ and $q_0(t) = -1$. Numerical results versus various levels δ of relative noises are presented in Figure 2. It is seen that the numerical approximation are very well and the proposed approach is effective and stable.

Example 2: The exact solution for the problem (2.1)-(2.4) is chosen as

$$u(x, t) = \left(x + \frac{5}{4}\right)^2 + 2\left(t - \frac{81}{32}\right) \tag{5.3}$$

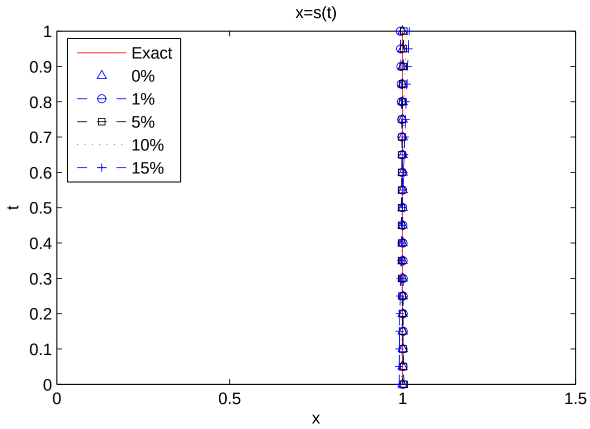

FIGURE 2. Steady state boundary and its approximations for Example 1 in the cases of noise levels $0, 1, 5, 15\%$.

and the moving boundary is a monotonously decreasing function of t given by

$$s(t) = \left(\frac{81}{16} - 2t\right)^{1/2} - \frac{5}{4}.$$

In this case, we have $u_s(t) = 0$. The Cauchy data can be calculated as

$$u_0(t) = \frac{25}{16} + 2\left(t - \frac{81}{32}\right), \quad q_0(t) = 2\left(x + \frac{5}{4}\right).$$

Numerical results versus various levels δ of relative noises are presented in Figure 3. We can see that the proposed scheme works very well even for a little large noise level.

Example 3: Take an exact solution for the problem (2.1)-(2.4) as

$$u(x,t) = \left(x - \frac{13}{4}\right)^2 + 2\left(t - \frac{81}{32}\right) \tag{5.4}$$

and the moving boundary is a monotonously increasing function of t given by

$$s(t) = \left(\frac{81}{16} - 2t\right)^{1/2} + \frac{13}{4}.$$

For this example, $u_s(t) = 0$. The Cauchy data can be calculated as

$$u_0(t) = \frac{169}{16} + 2\left(t - \frac{81}{32}\right), \quad q_0(t) = 2\left(x - \frac{13}{4}\right).$$

Numerical results with respect to various levels δ of relative noises are presented in Figure 4. It is observed that the numerical solution is accurate when using exact Cauchy data and become a little bad for noisy data in the time interval $0.5 \le t \le 1$.

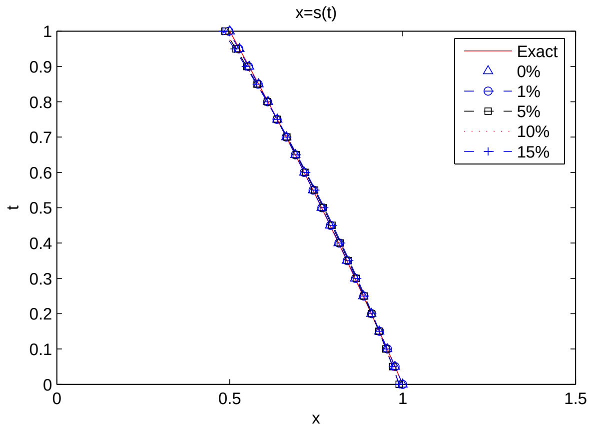

FIGURE 3. Moving boundary and its approximations for Example 2 in the cases of noise levels $0, 1, 5, 15\%$.

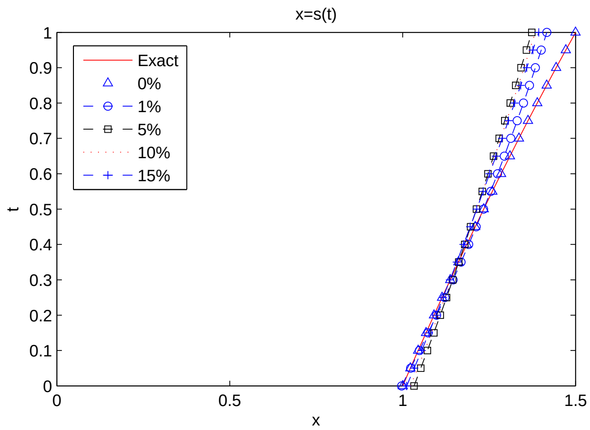

FIGURE 4. Moving boundary and its approximations for Example 3 in the cases of noise levels $0, 1, 5, 15\%$.

Example 4: The moving boundary is a piecewise smooth function given by

$$s(t) = \begin{cases} 1 - t, & \text{for} \quad 0 < t \leq \frac{1}{3}, \\ \frac{2}{3}, & \text{for} \quad \frac{1}{3} < t \leq \frac{2}{3}, \\ \frac{4}{3} - t, & \text{for} \quad \frac{2}{3} < t \leq 1, \end{cases}$$

and $u_s(t) \equiv 0$. The Neumman data is $q_0(t) = 1$ and the Dirichlet data at the end $x = 0$ is obtained by solving a direct problem from the Neumann condition

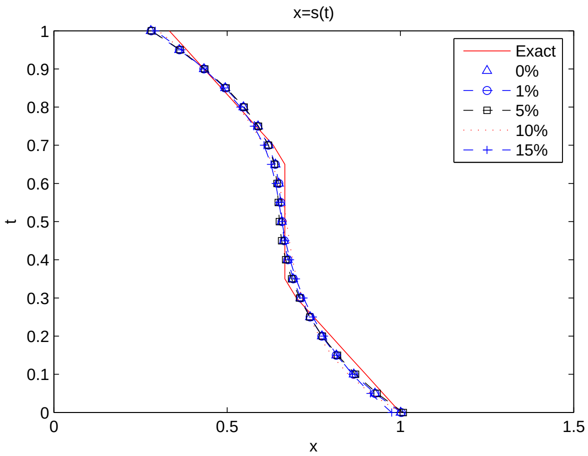

FIGURE 5. Moving boundary and its approximations for Example 4 in the cases of noise levels $0, 1, 5, 15\%$.

$\frac{\partial u(0,t)}{\partial x} = 1$ and the Dirichlet condition $u(s(t), t) \equiv 0$ as well as an initial condition $u(x, 0) = x - 1$ by the finite difference method.

The estimated moving boundary versus the various relative noise levels are shown in Figure 5. It is shown that the calculated boundary matches the exact one quite well everywhere except the small neighborhood of two non-smooth points.

Example 5: Suppose that the moving boundary is

$$s(t) = 1 - 0.3\cos(\pi(t - 0.5))$$

and $u_s(t) = 0$. The Neumman data at the end $x = 0$ is $q_0(t) = 1$ and the Dirichlet data is obtained by solving a direct problem from the Neumann condition $\frac{\partial u(0,t)}{\partial x} = 1$ and the Dirichlet condition $u(s(t), t) = 0$ as well as an initial condition $u(x, 0) = x - 1$ by the finite difference method.

The estimated moving boundary with respect to the different levels δ of relative noises are presented in Figure 6. Numerical approximation are accurate in the time interval $0 \le t \le 0.8$ and then became useless for $t > 0.8$.

In Figure 7, we display the root mean square errors $\varepsilon(s)$ given by (5.1) for Example 2 with respect to the distance ds of source points in which we fixed a relative noisy level $\delta = 1\%$ and $\tau = 5.2$. It can be seen that numerical accuracy keeps stable to small values ds and then becomes increaseing with respect to values of ds. It is still a open problem for choosing the optimal source points.

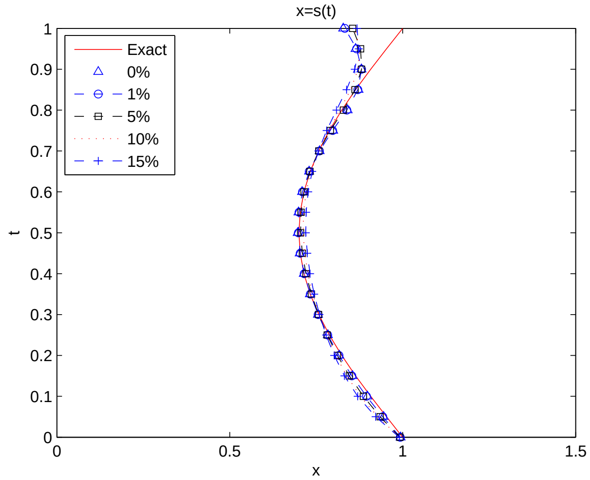

FIGURE 6. Moving boundary and its approximations for Example 5 in the cases of noise levels $0, 1, 5, 15\%$.

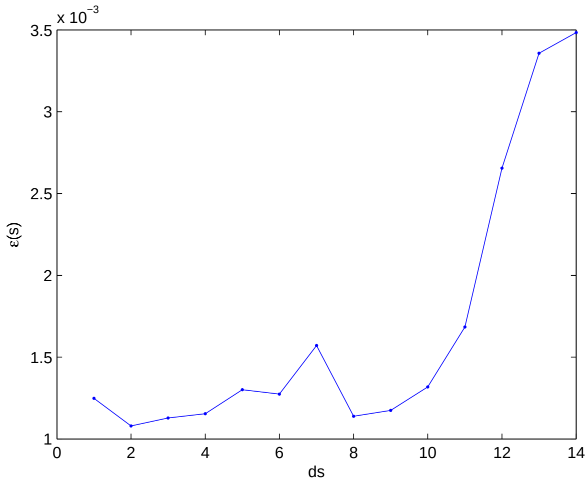

FIGURE 7. Root mean square errors for Example 2 versus the distances ds

In Figure 8, we display the root mean square errors $\varepsilon(s)$ for Example 2 with respect to parameters τ in which we fixed a relative noisy level $\delta = 1\%$ and $ds = 2$. It can be seen that the numerical accuracy decreases quickly as τ increase and then keeps stable to the values of $\tau > 12$. It indicates that we need to use a little large parameter τ to get an accurate solution.

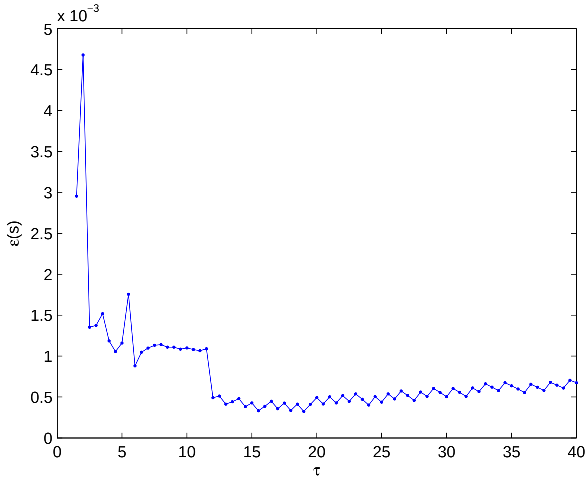

FIGURE 8. Root mean square errors for Example 2 versus parameters τ

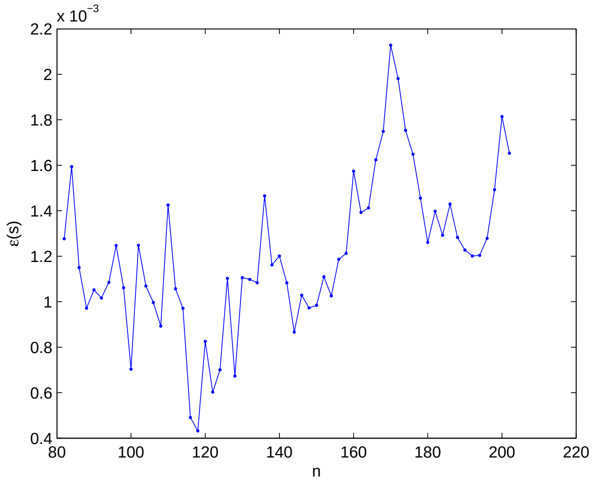

FIGURE 9. Root mean square errors for Example 2 versus the numbers of source points n

In Figure 9, we show the root mean square errors $\varepsilon(s)$ for Example 2 with respect to the numbers of source points n. We note that numerical errors oscillate in a certain range with respect to the number of source points. It indicates that there is no need to use a lot of source points in computation.

6. Conclusion

In this paper, we study the application of the method of fundamental solutions to reconstruct a moving boundary for a heat problem based on the Tikhonov regularization with the generalized cross validation choice strategy for the regularization parameter. Examples for various moving boundaries are presented. The numerical results show that the proposed method is efficient and stable.

7. Appendix

Theorem 7.1. *For $j = 1, 2$, we set $Q_j = \{(x,t) \,|\, 0 < x < s_j(t),\ 0 < t < T\}$, where $s_j \in C^2[0,T]$, $s_j(t) > 0$ for $0 < t \le T$. Let $u_j = u_j(x,t) \in C^2(Q_j) \cap C(\overline{Q_j})$ satisfy*

$$\frac{\partial u_j}{\partial t}(x,t) = a^2 \frac{\partial^2 u_j}{\partial x^2}(x,t), \qquad (x,t) \in Q_j \tag{7.1}$$

and

$$u_j(0,t) = u_0(t), \qquad 0 < t < T, \tag{7.2}$$

$$u_j(s_j(t),t) = 0, \qquad 0 < t < T. \tag{7.3}$$

We assume that

$$u_j(x,0) \ge 0, \qquad 0 \le x \le s_j(0), \tag{7.4}$$

and

$$u_0(t) \ge 0, \quad 0 < t < T \quad \text{and} \quad u_0(t) \not\equiv 0. \tag{7.5}$$

If there exist $t_1, t_2 \in (0,T)$ such that

$$\frac{\partial u_1}{\partial x}(0,t) = \frac{\partial u_2}{\partial x}(0,t), \qquad t_1 < t < t_2,$$

then $s_1(t) = s_2(t)$, $0 < t < T$.

In Chapko, Kress and Yoon [11] and [12], the uniqueness is proved under the assumption that the initial value is zero. However in many applications such as on-line processes related with continuous casting, we do not know the initial conditions, while we can know a priori that the temperature $u(x,t)$ is higher than the fusion point which is assumed to be zero.

Proof. Let $s_1 \not\equiv s_2$ in $(0, T)$. Then there exists $t_0 \in (0, T)$ such that $s_1(t_0) \neq s_2(t_0)$. Without loss of generality, we may assume that $x_0 = s_2(t_0) < s_1(t_0)$. By the unique continuation for $u_1 - u_2$, we see that $u_1 = u_2$ on $\overline{Q_1 \cap Q_2}$. In particular, $u_1(x_0, t_0) = u_2(x_0, t_0) = u_2(s_2(t_0), t_0) = 0$ by (7.3). By (7.4) and (7.5) we apply the maximum principle, we see that

$$u_1 \geq 0 \quad \text{on } \overline{Q_1}. \tag{7.6}$$

We choose $t_1', t_2', \alpha, \beta$ such that

$$0 < t_1' < t_0 < t_2' < T, \quad \alpha < x_0 < \beta, \quad [\alpha, \beta] \subset (0, s_1(t)),\ t_1' < t < t_2'. \tag{7.7}$$

Let $U(t, x, y)$ be the fundamental solution for $\frac{\partial}{\partial t} - a^2 \frac{\partial^2}{\partial x^2}$ in (α, β) with the zero Dirichlet boundary condition. Then we have

$$\frac{\partial U}{\partial \xi}(t, x, \alpha) \geq 0, \quad \frac{\partial U}{\partial \xi}(t, x, \beta) \leq 0, \quad U(t, x, y) \geq 0, \quad x, y \in (\alpha, \beta),\ t > 0. \tag{7.8}$$

(e.g., Itô [28]). Moreover

$$\begin{aligned} u_1(x, t) = &\textstyle\int_\alpha^\beta U(t - t_1', x, y) u_1(y, t_1') dy + \int_{t_1'}^t \frac{\partial U}{\partial \xi}(t - s, x, \alpha) u_1(\alpha, s) ds \\ &+ \textstyle\int_{t_1'}^t -\frac{\partial U}{\partial \xi}(t - s, x, \beta) u_1(\beta, s) ds, \quad \alpha < x < \beta,\ t > t_1'. \end{aligned} \tag{7.9}$$

Substituting $x = x_0$ and $t = t_0$ in (7.9) and using $u_1(x_0, t_0) = 0$, $u_1(\alpha, s) \geq 0$ and $u_2(\beta, s) \geq 0$ for $0 \leq s \leq T$ by (7.6), we see from (7.8) that

$$\int_\alpha^\beta U(t_0 - t_1', x_0, y) u_1(y, t_1') dy = 0.$$

By $U(t_0 - t_1', x_0, y) > 0$ for $\alpha < y < \beta$ (e.g., Itô [28]) and $u_1(y, t_1') \geq 0$ for $\alpha \leq y \leq \beta$, we have $u_1(y, t_1') = 0$ for $\alpha \leq y \leq \beta$. Since t_1' can be arbitrary provided that (7.7) holds, we have $u_1(y, t) = 0$ for $\alpha \leq y \leq \beta$ and $t_0 - \delta < t < t_0$ with some $\delta > 0$. By the unique continuation, we obtain $u_1 \equiv 0$ in Q_1, which contradicts (7.5). Thus the proof is completed. □

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