UTMS 2007–13 July 30, 2007

Stability estimate for the hyperbolic inverse boundary value problem by local Dirichlet-to-Neumann map

by

M. Bellassoued, D.Jellali and M.Yamamoto

UNIVERSITY OF TOKYO

GRADUATE SCHOOL OF MATHEMATICAL SCIENCES

KOMABA, TOKYO, JAPAN

Stability Estimate for the hyperbolic inverse boundary value problem by local Dirichlet-to-Neumann map

M.BELLASSOUED[1], D.JELLALI and M.YAMAMOTO[2]

[1] Faculté des Sciences de Bizerte,

Dép. des Mathematiques, 7021 Jarzouna Bizerte, Tunisia.

mourad.bellassoued@fsb.rnu.tn

[2] Department of Mathematical Sciences,

The University of Tokyo, 3-8-1 Komaba, Meguro, Tokyo 153, Japan.

myama@ms.u-tokyo.ac.jp

Abstract

In this paper we consider the stability of the inverse problem of determining a function $q(x)$ in a wave equation $\partial_t^2 u - \Delta u + q(x)u = 0$ in a bounded smooth domain in $\mathbb{R}^n$ from boundary observations. This information is enclosed in the hyperbolic (dynamic) Dirichlet-to-Neumann map associated to the solutions to the wave equation. We prove in the case of $n \geq 2$ that $q(x)$ is uniquely determined by the range restricted to a subboundary of the Dirichlet-to-Neumann map whose stability is a type of double logarithm.

1 Introduction

Let $\Omega \subset \mathbb{R}^n$ be a bounded domain with $\mathcal{C}^\infty$ boundary $\Gamma = \partial\Omega$. Throughout this paper we assume that the spatial dimension $n \geq 2$. We consider the following initial boundary value problem for the wave equation,

$$\begin{cases} (\partial_t^2 - \Delta + q(x))u(t,x) = 0 & \text{in } \ Q \equiv (0,T) \times \Omega, \\ u(0,x) = 0, \quad \partial_t u(0,x) = 0 & \text{in } \ \Omega, \\ u(t,x) = f(t,x) & \text{on } \ \Sigma \equiv (0,T) \times \Gamma, \end{cases} \tag{1.1}$$

where a function $q(x)$ is assumed in $W^{1,\infty}(\Omega)$. It is well known (see [19], [21]) that if $f \in H^1(\Sigma)$ and $f(0,x) = 0$, there exists a unique solution $u \in C([0,T]; H^1(\Omega)) \cap C^1([0,T]; L^2(\Omega))$ with $\partial_\nu u \in L^2(\Sigma)$ to (1.1). Here $\nu(x)$ denotes the unit outward normal to Γ at x and we set $\partial_\nu u = \nabla u \cdot \nu$. We denote the solution to (1.1) by u_q. Therefore we can define the Dirichlet-to-Neumann map

$$\Lambda_q : H^1(\Sigma) \ \longrightarrow \ L^2(\Sigma)$$

$$f \quad \longmapsto \quad \partial_\nu u_q \tag{1.2}$$

Using an energy estimate, one can prove that Λ_q is continuous from $H^1(\Sigma)$ to $L^2(\Sigma)$ (e.g., [19]). The inverse problem is whether knowledge of the Dirichlet-to-Neumann map on a particular subset of the boundary determines a function q uniquely.

From the physical viewpoint, our inverse problem consists in determining the properties e.g., a dispersion term of an inhomogeneous medium by probing it with disturbances generated on the boundary. The data are responses of the medium to these disturbances which are measured on a suitable suboundary, and the goal is to recover $q(x)$ which describes the property of the medium. Here we assume that the medium is quiet initially, and f is a disturbance which is used to probe the medium. Roughly speaking, the data is $\partial_\nu u$ measured on a subboundary for different choices of f.

Rakesh and Symes [25] uses complex geometrical optics solutions concentrating near lines with any direction $\omega \in \mathbb{S}^{n-1}$ to prove that Λ_q determines $q(x)$ uniquely. In [25], Λ_q gives equivalent information to the responses on the whole boundary for all the possible input disturbances. Ramm and Sjöstrand [26] has extended the result in [25] to the case of q depending on x and t. Isakov [12] has considered the simultaneous determination of a zeroth order coefficient and a damping coefficient. A key ingredient in the existing results, is the construction of complex geometric optics solutions of the wave equation, concentrated along a line, and the relationship between the hyperbolic Dirichlet-to-Neumann map and the X-ray transform play a crucial role.

The uniqueness by a local Dirichlet-to-Neumann map is solved well (e.g., Belishev [1], Katchlov, Kurylev and Lassas [15], Kurylev and Lassas [18]). However the stability by a local Dirichlet-to-Neumann map is not discussed comprehensively. For it, see Isakov and Sun [14] where a local Dirichet-to-Neumann map yields a stability result in determining a coefficient in a subdomain. In the case where the Dirichlet-to-Neumann map is considered on the whole lateral boundary Σ, the stability is established in Cipolatti and Lopez [9], Stefanov and Uhlmann [28], Sun [29].

As for results by a finite number of data of Dirichlet-to-Neumann map, see Cheng and Nakamura [8], Cipolatti and Lopez [9], Rakesh [24]. There are very many works on Dirichlet-to-Neumann maps, and so our references are far from being perfect, and see Cardoso and Mendoza [7], Rachele [23], Romanov [27], Uhlmann [30] as related papers.

In this paper we prove a $\log\log$-type estimate which shows that a dispersion term q depends stably on the Dirichlet-to-Neumann map even when the boundary measurement is taken only on a subbundary which is slightly larger than the half of the boundary Γ.

Our inverse problem is formulated with many boundary measurements. On the other hand, as for a different formulation of inverse problems with a single measurement, the main methodology is based on an L^2-weighted inequality called a Carleman estimate, and was introduced by Bukhgeim and Klibanov [4]. Furthermore, as for applications of Carleman estimates to inverse problems, we can refer to Bellassoued [2], Imanuvilov and Yamamoto [11], Isakov [13], Klibanov [16], Klibanov and Timonov [17]. Most of those papers treat the determination of spatially varying functions by a single measurement. As for observability inequalities by means of a Carleman estimate, see [17].

In order to formulate our result, we need to introduce some notations. Henceforth we arbitrarily choose

$$\omega_0 \in \mathbb{S}^{n-1} \equiv \{\omega \in \mathbb{R}^n;\ |\omega| = 1\}$$

and fix $\varepsilon > 0$. By $(x \cdot y)$ we denote the scalar product of $x, y \in \mathbb{R}^n$ and set

$$\Gamma_{+,\varepsilon}(\omega_0) = \{x \in \Gamma;\ (\nu(x) \cdot \omega_0) > \varepsilon\}, \quad \Gamma_{-,\varepsilon}(\omega_0) = \Gamma \backslash \overline{\Gamma_{+,\varepsilon}(\omega_0)},$$

$$\Sigma_{+,\varepsilon}(\omega_0) = (0, T) \times \Gamma_{+,\varepsilon}(\omega_0), \quad \Sigma_{-,\varepsilon}(\omega_0) = \Sigma \backslash \overline{\Sigma_{+,\varepsilon}(\omega_0)}.$$

We also write $\Gamma_+(\omega_0) = \Gamma_{+,0}(\omega_0)$, $\Sigma_+(\omega_0) = \Sigma_{+,0}(\omega_0)$ as well as $\Gamma_-(\omega_0) = \Gamma_{-,0}(\omega_0)$ and $\Sigma_-(\omega_0) = \Sigma_{-,0}(\omega_0)$.
We introduce the local Dirichlet-to-Neumann map by

$$\begin{aligned} \Lambda_q' : H^1(\Sigma) &\longrightarrow L^2(\Sigma_{-,\varepsilon}(\omega_0)) \\ f &\longmapsto \Lambda_q'(f) = \partial_\nu u_q \Big|_{\Sigma_{-,\varepsilon}(\omega_0)}. \end{aligned} \tag{1.3}$$

By $\left\|\Lambda_{q_1}' - \Lambda_{q_2}'\right\|$ we denote the operator norm.

The main result of this paper can be stated as follows.

Theorem 1 *Assume that $T > \mathrm{diam}\,\Omega$. Let $q_1, q_2 \in H^\alpha(\Omega)$, $\alpha > \frac{n}{2} + 1$, such that $\|q_j\|_{H^\alpha(\Omega)} \leq M$. Then there exist constants $C > 0$ and $s_1, s_2 \in (0, 1)$ such that*

$$\|q_1 - q_2\|_{L^\infty(\Omega)} \leq C \left[\left\|\Lambda_{q_1}' - \Lambda_{q_2}'\right\|^{s_1} + \left(\log\left|\log\left\|\Lambda_{q_1}' - \Lambda_{q_2}'\right\|\right|\right)^{-s_2} \right], \tag{1.4}$$

where C depends on Ω, M, ε, n, α and s_1, s_2.

Our proof is inspired by techniques used by Bukhgeim and Uhlmann [5] which proves a uniqueness theorem from an inverse problem for an elliptic equation. Their idea in turn goes back to the work of Calderón [6]. Our problem turns out to be easier because geometric optics solutions interact with the interior of Ω in the hyperbolic case but not in the elliptic case. The main idea is to probe the medium by real geometric optics solutions of the wave equation, concentrated along a line, starting on one side of the boundary, and measure responses of the medium on other side of the boundary. A response gives a line integral of q.

The plan of this paper is as follows. Some basic lemmata are given in section 2. Section 3 is devoted to the proof of Theorem 1.

2 Preliminaries

In this section we collect some results from [3] which are needed in the proof of Theorem 1. The first one is the Carleman estimate for the hyperbolic operator $\partial_t^2 - \Delta + q(x)$. For fixed $\omega \in \mathbb{S}^{n-1}$, we introduce the functions ϕ_j, $j = 1, 2$, defined by

$$\phi_1(t, x; \omega) = x \cdot \omega + t, \quad \phi_2(t, x; \omega) = x \cdot \omega - (T - t), \quad \omega \in \mathbb{S}^{n-1}.$$

Then we have the following Carleman type estimate:

Lemma 2.1 *([3]) Let $q \in L^\infty(\Omega)$ such that $\|q\|_{L^\infty(\Omega)} \le M$. There exist constants $\lambda_0 > 0$ and $C > 0$ such that for $j = 1, 2$, the following estimate holds true:*

$$\begin{aligned} & \int_Q e^{-2\lambda\phi_j(t,x;\omega)}(\lambda^2 |u|^2 + |\nabla u|^2) dxdt + \lambda \int_{\Sigma_+(\omega)} (\omega\cdot\nu) e^{-2\lambda\phi_j(t,x;\omega)} |\partial_\nu u|^2 \, d\sigma dt \\ \le \quad & C \int_Q e^{-2\lambda\phi_j(t,x;\omega)} \left|\left(\partial_t^2 - \Delta + q(x)\right) u\right|^2 dxdt - \lambda \int_{\Sigma_-(\omega)} (\omega\cdot\nu) e^{-2\lambda\phi_j(t,x;\omega)} |\partial_\nu u|^2 \, d\sigma dt \end{aligned}$$

for every $u \in H^2(Q)$ with $u|_\Sigma = 0$, $u|_{t=0} = \partial_t u|_{t=0} = 0$ and $\lambda \ge \lambda_0$.

Using Lemma 2.1 and a Carleman estimate in Sobolev spaces of negative order proved in [3] we are able to construct real geometric optics solutions for the wave operator, which are crucial ingredients in the proof of Theorem 1. In this section, we precise and explain the existence of exponentially growing solutions. By selecting suitably small $\varrho > 0$, we assume that

$$T > \mathrm{diam}\,\Omega + 4\varrho. \tag{2.1}$$

Denote

$$\Omega_\varrho = \left\{x \in \mathbb{R}^n \backslash \overline{\Omega};\ \mathrm{dist}(x, \Omega) < \varrho\right\}.$$

Henceforth $q \in H^\alpha(\Omega)$ is regarded as a function in $H^\alpha(\mathbb{R}^n)$ with $\|q\|_{H^\alpha(\mathbb{R}^n)} \le C\|q\|_{H^\alpha(\Omega)}$ by the zero extension to $\mathbb{R}^n \setminus \overline{(\Omega \cup \Omega_\varrho)}$.

Let $\chi \in C_0^\infty(\Omega_\varrho)$. Then we have

$$\mathrm{supp}\,\chi \cap \Omega = \emptyset, \quad (\mathrm{supp}\,\chi \pm T\omega) \cap \Omega = \emptyset. \tag{2.2}$$

Let

$$\chi_1(t,x) = \chi(x + t\omega), \quad \chi_2(t,x) = \chi(x - (T-t)\omega).$$

Lemma 2.2 *([3]) Let $q \in H^\alpha(\Omega)$ such that $\|q\|_{H^\alpha(\Omega)} \le M$ and $\omega \in \mathbb{S}^{n-1}$. For λ large enough we can construct a special solution $u^{(j)}$ of*

$$(\partial_t^2 - \Delta + q(x))u(t,x) = 0 \quad \text{in } Q, \quad u|_{t=0} = \partial_t u|_{t=0} = 0 \quad \text{in } \Omega$$

in the form

$$u^{(j)}(t,x) = e^{\lambda\phi_j(t,x;\omega)}\left(\chi_j(t,x) + \psi_q^{(j)}(t,x;\lambda)\right), \quad j = 1, 2,$$

where $\psi_q^{(j)}$ satisfies

$$\left\|\psi_q^{(j)}(\cdot,\cdot;\lambda)\right\|_{L^2(0,T;H^k(\Omega))} \le \frac{C}{\lambda^{1-k}} \|\chi\|_{H^5(\mathbb{R}^n)}; \quad k = 0, 1, 2,$$

where $C > 0$ depends only on Ω, T and M.

We can similarly prove

Lemma 2.3 *([3]) Let $q \in H^\alpha(\Omega)$ such that $\|q\|_{H^\alpha(\Omega)} \leq M$ and $\omega \in \mathbb{S}^{n-1}$. For λ large enough we can construct a special solution $u^{(j)}$ of*

$$(\partial_t^2 - \Delta + q(x))u(t,x) = 0 \quad \text{in } Q, \quad u_{|t=T} = \partial_t u_{|t=T} = 0 \quad \text{in } \Omega$$

in the form

$$u^{(j)}(t,x) = e^{-\lambda\phi_j(t,x;\omega)} \left(\chi_j(t,x) + \psi_q^{(j)}(t,x;\lambda)\right); \quad j = 1,2,$$

where $\psi_q^{(j)}$ satisfies

$$\left\|\psi_q^{(j)}(\cdot,\cdot;\lambda)\right\|_{L^2(0,T;H^k(\Omega))} \leq \frac{C}{\lambda^{1-k}} \|\chi\|_{H^5(\mathbb{R}^n)}, \quad k = 0,1,2,$$

where $C > 0$ depends only on Ω, T and M.

We will apply this lemmas with $\phi_j(t,x,\omega)$ where ω varies in a neighbourhood around ω_0 on $\mathbb{S}^{n-1}$ and estimate the Fourier transform of $q_1 - q_1$ in a conic subset of $\mathbb{R}^n$. In order to extend the estimate on the conic subset to an estimate on the ball, we use an idea of Heck and Wang [10] and conditional stability for analytic continuation established by Vessella [31].

3 Stability Estimate

In this section, we complete the proof of Theorem 1. The key is the combination of the exponentially growing solutions of equation (1.1) and the X-ray transform. We shall use the following notations. For $\varepsilon > 0$ and $\omega_0 \in \mathbb{S}^{n-1}$, by

$$\mathcal{V}_\varepsilon(\omega_0) = \left\{\omega \in \mathbb{S}^{n-1};\ |\omega - \omega_0| < \frac{\varepsilon}{2}\right\}$$

we denote a neighbourhood around ω_0 on $\mathbb{S}^{n-1}$. Then for each $\omega \in \mathcal{V}_\varepsilon(\omega_0)$

$$\Sigma_{-,\frac{\varepsilon}{2}}(\omega) \subset \Sigma_{-,\varepsilon}(\omega_0).$$

3.1 Preliminary estimate

Lemma 3.1 *Let $q_1, q_2 \in H^\alpha(\Omega)$ such that $\|q_j\|_{H^\alpha(\Omega)} \leq M$ and $q = q_2 - q_1$. There exist $\beta > 0$, $C > 0$ such that for any $\omega \in \mathcal{V}_\varepsilon(\omega_0)$ and $\chi \in C_0^\infty(\Omega_\varrho)$ the following estimates holds true:*

$$\left|\int_{-T}^T \int_{\mathbb{R}^n} \chi^2(x) q(x+t\omega)\,dx\,dt\right| \leq C\left(\frac{1}{\sqrt{\lambda}}\|q\|_{L^\infty(\Omega)} + e^{\beta\lambda}\left\|\Lambda'_{q_1} - \Lambda'_{q_2}\right\|\right)\|\chi\|^2_{H^5(\mathbb{R}^n)}. \quad (3.1)$$

for any sufficiently large $\lambda > 0$. Here C depends only on Ω, T and M.

Proof . For λ sufficiently large, Lemma 2.2 guarantees the existence of the exponentially growing solutions $u_2^{(j)}$, $j = 1,2$, to

$$\left(\partial_t^2 - \Delta + q_2(x)\right) u(t,x) = 0 \quad \text{in } Q, \quad u(0,\cdot) = \partial_t u(0,\cdot) = 0 \quad \text{in } \Omega$$

in the form

$$u_2^{(j)}(t,x) = e^{\lambda\phi_j(t,x;\omega)}\left(\chi_j(t,x) + \psi_{q_2}^{(j)}(t,x,\lambda)\right), \tag{3.2}$$

where $\psi_{q_2}^{(j)}$ satisfies

$$\left\|\psi_{q_2}^{(j)}(\cdot,\cdot,\lambda)\right\|_{L^2(0,T;H^k(\Omega))} \le \frac{C}{\lambda^{1-k}}\left\|\chi\right\|_{H^5(\mathbb{R}^n)}, \quad k=0,1,2. \tag{3.3}$$

By $u_1^{(j)}$, $j=1,2$, we denote the solutions to

$$\begin{cases} (\partial_t^2 - \Delta + q_1(x))u_1^{(j)} = 0, & \text{in} \quad Q, \\ u_1^{(j)}(0,x) = \partial_t u_1^{(j)}(0,x) = 0, & \text{in} \quad \Omega, \\ u_1^{(j)}(t,x) = u_2^{(j)}(t,x) := f_{j,\lambda}(t,x), & \text{on} \quad \Sigma. \end{cases}$$

Defining

$$u^{(j)} = u_1^{(j)} - u_2^{(j)}, \quad q(x) = q_2(x) - q_1(x),$$

we have

$$\begin{cases} (\partial_t^2 - \Delta + q_1(x))u^{(j)} = q(x)u_2^{(j)}, & \text{in} \quad Q, \\ u^{(j)}(0,x) = \partial_t u^{(j)}(0,x) = 0, & \text{in} \quad \Omega, \\ u^{(j)}(t,x) = 0, & \text{on} \quad \Sigma. \end{cases}$$

For sufficiently large λ, Lemma 2.3 guarantees the existence of exponentially growing solutions $v^{(j)}$ to the backward wave equation

$$\left(\partial_t^2 - \Delta + q_1(x)\right)v(t,x) = 0, \quad (t,x)\in Q, \quad v(T,x) = \partial_t v(T,x) = 0, \quad x\in\Omega,$$

of the form

$$v^{(j)}(t,x) = e^{-\lambda\phi_j(t,x;\omega)}\left(\chi_j(t,x) + \psi_{q_1}^{(j)}(t,x,\lambda)\right), \tag{3.4}$$

corresponding to q_1 and ϕ_j, $j=1,2$, where $\psi_{q_1}^{(j)}$ satisfies

$$\left\|\psi_{q_1}^{(j)}(\cdot,\cdot;\lambda)\right\|_{L^2(0,T;H^k(\Omega))} \le \frac{C}{\lambda^{1-k}}\left\|\chi\right\|_{H^5(\mathbb{R}^n)}, \quad k=0,1,2. \tag{3.5}$$

Integrating by parts and using the Green's formula, we obtain

$$\begin{aligned} \int_Q \left[\left(\partial_t^2 - \Delta + q_1(x)\right)u^{(j)}(t,x)\right]v^{(j)}(t,x)dxdt &= \int_Q q(x)u_2^{(j)}(t,x)v^{(j)}(t,x)dxdt \\ &= \int_\Sigma \partial_\nu u^{(j)}(t,x)v^{(j)}(t,x)d\sigma dt. \end{aligned} \tag{3.6}$$

It follows from (3.2), (3.4) and (3.6) that

$$\int_Q q(x)\chi_j^2(t,x)dxdt + \int_Q q(x)\chi_j(t,x)(\psi_{q_1}^{(j)}(t,x;\lambda) + \psi_{q_2}^{(j)}(t,x;\lambda))dxdt$$

$$
\begin{aligned}
&+ \int_Q q(x)\psi_{q_1}^{(j)}(t,x;\lambda)\psi_{q_2}^{(j)}(t,x;\lambda)dxdt \\
&= \int_{\Sigma_{+,\varepsilon/2}(\omega)} \partial_\nu u^{(j)}(\chi_j(t,x)+\psi_{q_1}(t,x))e^{-\lambda\phi_j(t,x;\omega)}d\sigma dt \\
&\quad + \int_{\Sigma_{-,\varepsilon/2}(\omega)} \partial_\nu u^{(j)}(t,x)v^{(j)}(t,x)d\sigma dt. \qquad (3.7)
\end{aligned}
$$

Since (3.3) and (3.5) imply

$$
\left|\int_Q q(x)\chi_j(t,x)(\psi_{q_1}^{(j)}(t,x;\lambda)+\psi_{q_2}^{(j)}(t,x;\lambda))dxdt\right| \le \frac{C}{\lambda}\left\|\chi\right\|_{L^2(\mathbb{R}^n)}\left\|\chi\right\|_{H^5(\mathbb{R}^n)}
$$

and

$$
\left|\int_Q q(x)\psi_{q_1}^{(j)}(t,x;\lambda)\psi_{q_2}^{(j)}(t,x;\lambda)dxdt\right| \le \frac{C}{\lambda^2}\left\|\chi\right\|_{H^5(\mathbb{R}^n)}^2 .
$$

Furthermore we have

$$
\begin{aligned}
&\left|\int_{\Sigma_{-,\varepsilon/2}(\omega)} \partial_\nu u^{(j)}(t,x)v^{(j)}(t,x)d\sigma dt\right| \le \|\partial_\nu u^{(j)}\|_{L^2(\Sigma_{-,\varepsilon/2}(\omega))}\|v^{(j)}\|_{L^2(\Sigma_{-,\varepsilon/2}(\omega))} \\
&\le \left\|v^{(j)}\right\|_{L^2(0,T;H^1(\Omega))}\left\|\partial_\nu u^{(j)}\right\|_{L^2(\Sigma_{-,\varepsilon/2(\omega)})} \\
&\le Ce^{\beta_1\lambda}\left\|\chi\right\|_{H^5(\mathbb{R}^n)}\|\Lambda'_{q_1}(f_\lambda^j)-\Lambda'_{q_2}(f_\lambda^j)\|_{L^2(\Sigma_{-,\varepsilon/2}(\omega))} \qquad (3.8)
\end{aligned}
$$

for some positive constants C and β_1.

By the wave equation, we have

$$
\|\partial_t^2 u_2^{(j)}\|_{L^2(Q)} \le C\|u_2^{(j)}\|_{L^2(0,T;H^2(\Omega))},
$$

and so

$$
\|u_2^{(j)}\|_{H^2(Q)} \le C\|u_2^{(j)}\|_{L^2(0,T;H^2(\Omega))}.
$$

Hence (3.2) and (3.3) yield

$$
\|u_2^{(j)}\|_{H^2(Q)} \le C\lambda e^{\beta_1\lambda}\|\chi\|_{H^5(\mathbb{R}^n)}.
$$

Moreover, since $\omega\in\mathcal{V}_\varepsilon(\omega_0)$, we obtain $\Sigma_{-,\varepsilon/2}(\omega)\subset\Sigma_{-,\varepsilon}(\omega_0)$ and

$$
\begin{aligned}
\left\|\Lambda'_{q_1}(f_\lambda^j)-\Lambda'_{q_2}(f_\lambda^j)\right\|_{L^2(\Sigma_{-,\varepsilon/2}(\omega))} &\le \left\|\Lambda'_{q_1}(f_\lambda^j)-\Lambda'_{q_2}(f_\lambda^j)\right\|_{L^2(\Sigma_{-,\varepsilon}(\omega_0))} \\
&\le \|\Lambda'_{q_1}-\Lambda'_{q_2}\|\|f_\lambda^j\|_{H^1(\Sigma)} \\
&\le \|\Lambda'_{q_1}-\Lambda'_{q_2}\|\|u_2^{(j)}\|_{H^2(Q)} \\
&\le Ce^{\beta_2\lambda}\left\|\chi\right\|_{H^5(\mathbb{R}^n)}\|\Lambda'_{q_1}-\Lambda'_{q_2}\|. \qquad (3.9)
\end{aligned}
$$

Hence, by (3.7), we obtain

$$
\left|\int_Q q(x)\chi_j^2(t,x)dxdt\right| \le \frac{C}{\lambda}\|\chi\|_{H^5(\mathbb{R}^n)}^2 + C\left\|\chi\right\|_{H^5(\mathbb{R}^n)}\|e^{-\lambda\phi_j}\partial_\nu u^{(j)}\|_{L^2(\Sigma_{+,\varepsilon/2}(\omega))}
$$

$$+Ce^{\beta_3\lambda}\,\|\chi\|^2_{H^5(\mathbb{R}^n)}\,\|\Lambda'_{q_1}-\Lambda'_{q_2}\|. \tag{3.10}$$

By Lemma 2.1, we obtain

$$\begin{aligned}
&\varepsilon\lambda\int_{\Sigma_{+,\varepsilon/2}(\omega)}\left|\partial_\nu u^{(j)}\right|^2 e^{-2\lambda\phi_j(t,x;\omega)}dxdt \le \lambda\int_{\Sigma_+(\omega)}(\omega\cdot\nu)\left|\partial_\nu u^{(j)}\right|^2 e^{-2\lambda\phi_j(t,x;\omega)}d\sigma dt\\
&\le C\int_Q\left|q(x)u_2^{(j)}(x,t)\right|^2 e^{-2\lambda\phi_j(t,x;\omega)}dxdt + Ce^{\beta_4\lambda}\int_{\Sigma_-(\omega)}\left|\partial_\nu u^{(j)}\right|^2 d\sigma dt\\
&\le C\int_Q\left|q(x)\left(\chi_j(t,x)+\psi_{q_2}^{(j)}(t,x,\lambda)\right)\right|^2 dxdt + Ce^{\beta_5\lambda}\int_{\Sigma_-(\omega)}\left|\partial_\nu u^{(j)}\right|^2 d\sigma dt\\
&\le C\,\|\chi\|^2_{H^5(\mathbb{R}^n)} + Ce^{\beta_5\lambda}\int_{\Sigma_-(\omega)}\left|\partial_\nu u^{(j)}\right|^2 d\sigma dt\\
&\le C\,\|\chi\|^2_{H^5(\mathbb{R}^n)} + Ce^{\beta_5\lambda}\|\Lambda'_{q_1}(f^j_\lambda)-\Lambda'_{q_1}(f^j_\lambda)\|^2_{L^2(\Sigma_-(\omega))}.
\end{aligned}$$

Using again (3.9), we obtain

$$\int_{\Sigma_{+,\varepsilon/2}(\omega)}\left|\partial_\nu u^{(j)}\right|^2 e^{-2\lambda\phi_j(t,x;\omega)}dxdt \le C\left(\frac{1}{\lambda}+e^{\beta_6\lambda}\|\Lambda'_{q_1}-\Lambda'_{q_2}\|^2\right)\|\chi\|^2_{H^5(\mathbb{R}^n)}.$$

Hence it follows from (3.9) and (3.10) that

$$\left|\int_Q q(x)\chi_j^2(t,x)dxdt\right| \le \frac{C}{\sqrt{\lambda}}\,\|\chi\|^2_{H^5(\mathbb{R}^n)} + Ce^{\beta_7\lambda}\,\|\chi\|^2_{H^5(\mathbb{R}^n)}\,\|\Lambda'_{q_1}-\Lambda'_{q_2}\|.$$

Therefore we obtain

$$\begin{aligned}
\left|\int_{-T}^{T}\int_{\mathbb{R}^n}\chi^2(x)q(x+t\omega)dxdt\right| &\le \sum_{j=1}^{2}\left|\int_Q q(x)\chi_j^2(t,x)dxdt\right|\\
&\le C\left(\frac{1}{\sqrt{\lambda}}+e^{\beta\lambda}\|\Lambda'_{q_1}-\Lambda'_{q_2}\|\right)\|\chi\|^2_{H^5(\mathbb{R}^n)}.
\end{aligned}$$

This completes the proof of the lemma.

3.2 X-ray transform

The X-ray transform $\mathcal{P}$ maps a function in $\mathbb{R}^n$ into the set of its line integrals. More precisely, if $\omega\in\mathbb{S}^{n-1}$ and $x\in\mathbb{R}^n$,

$$\mathcal{P}(f)(\omega,x) := \int_{\mathbb{R}} f(x+s\omega)ds,$$

is the integral of f over the straight line through x with the direction ω. It is easy to see that $\mathcal{P}(f)(\omega,x)$ does not change if x is moved in the direction ω. Therefore we normally restrict x to $\omega^\perp = \{\theta\in\mathbb{R}^n;\ \theta\cdot\omega=0\}$, and we can consider $\mathcal{P}$ as a function on the tangent bundle $\mathcal{T}=\left\{(\omega,x):\ \omega\in\mathbb{S}^{n-1},\ x\in\omega^\perp\right\}$ (e.g., Natterer [22]).

Lemma 3.2 *There exist constants $C > 0$, $\mu > 0$, $\delta > 0$ and $\lambda_0 > 0$ such that for all $\omega \in \mathcal{V}_\varepsilon(\omega_0)$ we have*

$$|\mathcal{P}(q)(\omega, y)| \leq \frac{C}{\lambda^\delta} \|q\|_{L^\infty(\Omega)} + Ce^{\mu\lambda} \left\|\Lambda'_{q_1} - \Lambda'_{q_2}\right\|, \quad \text{a.e } y \in \mathbb{R}^n$$

for any $\lambda \geq \lambda_0$.

Proof . Let $\theta \in \mathcal{C}_0^\infty(\mathbb{R}^n)$ be a positive function which is supported in the unit ball and $\|\theta\|_{L^2(\mathbb{R}^n)} = 1$. Define

$$\chi_h(x) = h^{-n/2}\theta\left(\frac{x-y}{h}\right)$$

where $y \in \Omega_\varrho$ and $h > 0$ is sufficiently small.
Put

$$r(x, \omega) = \int_{-T}^{T} q(x - t\omega)dt.$$

Then we have

$$\begin{aligned}|r(y,\omega)| = \left|\int_{\mathbb{R}^n} \chi_h^2(x) r(y,\omega)dx\right| \quad &\leq \left|\int_{\mathbb{R}^n} \chi_h^2(x) r(x,\omega)dx\right| \\ &+ \left|\int_{\mathbb{R}^n} \chi_h^2(x)(r(y,\omega) - r(x,\omega))dx\right|.\end{aligned}$$

Since $H^\alpha(\Omega) \subset C^1(\overline{\Omega})$ by $\alpha > \frac{n}{2} + 1$ and $\|q\|_{H^\alpha(\Omega)} \leq M$, we have

$$|r(y,\omega) - r(x,\omega)| \leq C\,|x-y|\,.$$

Applying Lemma 3.1 with $\chi = \chi_h$, we obtain

$$|r(y,\omega)| \leq C\left(\frac{1}{\sqrt{\lambda}} + e^{\mu\lambda}\left\|\Lambda'_{q_1} - \Lambda'_{q_2}\right\|\right)\|\chi_h\|^2_{H^5(\mathbb{R}^n)} + C\int_{\mathbb{R}^n} |x-y|\,\chi_h^2(x)dx. \tag{3.11}$$

On the other hand, we have

$$\|\chi_h\|_{H^5(\mathbb{R}^n)} \leq Ch^{-5}, \quad \int_{\mathbb{R}^n} |x-y|\,\chi_h^2(x)dx \leq Ch.$$

Then by (3.1) and (3.11), we have for all $\omega \in \mathcal{V}_\varepsilon(\omega_0)$

$$\left|\int_{-T}^{T} q(y - t\omega)dt\right| \leq \frac{C}{\sqrt{\lambda}}h^{-10} + Ch^{-10}e^{\mu\lambda}\left\|\Lambda'_{q_1} - \Lambda'_{q_2}\right\| + Ch, \quad \text{a.e } y \in \Omega_\varrho.$$

We select h such that

$$h = \frac{1}{\sqrt{\lambda}}h^{-10}.$$

Then there exist constants $\delta > 0$ and $\beta > 0$ such that

$$\left|\int_{-T}^{T} q(y + t\omega)dt\right| \leq \frac{C}{\lambda^\delta} + Ce^{\beta\lambda}\left\|\Lambda'_{q_1} - \Lambda'_{q_2}\right\|, \quad \text{a.e } y \in \Omega_\varrho.$$

Since $T > \mathrm{Diam}\,\Omega$ and $q|_{\mathbb{R}^n \setminus (\overline{\Omega \cup \Omega_\varrho})} = 0$, we obtain for all $\omega \in \mathcal{V}_\varepsilon(\omega_0)$

$$|\mathcal{P}(q)(\omega, y)| = \left| \int_{\mathbb{R}} q(y + t\omega) dt \right| \le \frac{C}{\lambda^\delta} + Ce^{\beta\lambda} \left\| \Lambda'_{q_1} - \Lambda'_{q_2} \right\|, \quad \text{a.e } y \in \mathbb{R}^n,$$

so that the proof of the lemma is completed.

Let

$$\mathcal{K}_\varepsilon = \bigcup_{\omega \in \mathcal{V}(\omega_0)} \omega^\perp,$$

and let

$$\widehat{f}(\xi) = (\mathcal{F}f)(\xi) = (2\pi)^{-\frac{n-1}{2}} \int_{\omega^\perp} f(x) e^{-ix\cdot\xi} d\sigma_x$$

for $f \in L^1(\omega^\perp)$ where $d\sigma_x$ is the $(n-1)$-dimensional standard volume element on $x \in \omega^\perp$, while

$$\widehat{q}(\xi) = (2\pi)^{-\frac{n}{2}} \int_{\mathbb{R}^n} q(y) e^{-iy\cdot\xi} dy$$

for $q \in L^1(\mathbb{R}^n)$.

Lemma 3.3 *There exist constants $C > 0$, $\mu > 0$, $\delta > 0$ and $\lambda_0 > 0$ such that*

$$|\widehat{q}(\xi)| \le \frac{C}{\lambda^\delta} \|q\|_{L^\infty(\Omega)} + Ce^{\mu\lambda} \left\| \Lambda'_{q_1} - \Lambda'_{q_2} \right\|, \quad \xi \in \mathcal{K}_\varepsilon$$

for any $\lambda \ge \lambda_0$.

Proof . Let $q \in L^1(\mathbb{R}^n)$. By the change of variable $y = x + t\omega \in \omega^\perp \oplus \mathbb{R}\omega = \mathbb{R}^n$ with $dy = d\sigma dt$, noting that $\xi \in \omega^\perp$ implies $x \cdot \xi = x \cdot \xi + t\omega \cdot \xi = y \cdot \xi$, we have

$$\begin{aligned}
\mathcal{F}(\mathcal{P}q(\omega, \cdot))(\xi) &= (2\pi)^{-\frac{n-1}{2}} \int_{\omega^\perp} \int_{\mathbb{R}} q(x + t\omega) e^{-ix\cdot\xi} dt d\sigma \\
&= \sqrt{2\pi} (2\pi)^{-\frac{n}{2}} \int_{\mathbb{R}^n} q(y) e^{-iy\cdot\xi} dy = \sqrt{2\pi}\,\widehat{q}(\xi), \quad \xi \in \omega^\perp
\end{aligned}$$

(e.g., [22]). For $R > 0$ such that $\Omega \subset B(0, R)$, we obtain

$$\mathcal{F}(\mathcal{P}(q)(\omega, \cdot))(\xi) = (2\pi)^{-\frac{n-1}{2}} \int_{\omega^\perp \cap B(0,R)} \mathcal{P}(q)(\omega, x) e^{-ix\cdot\xi} dx = \sqrt{2\pi}\,\widehat{q}(\xi).$$

In terms of Lemma 3.2, the proof is completed.

3.3 Proof of the stability estimate

Let $B(0,\rho)=\{x\in\mathbb{R}^n;\ |x|<\rho\}$ and $|\gamma|=\gamma_1+\cdots+\gamma_n$ for $\gamma\in(\mathbb{N}\cup\{0\})^n$.

Lemma 3.4 *([31]) Let $\mathcal{W}$ be an open set of $B(0;1)$, and F an analytic function in $B(0;2)$ having the following property: there exist constants $M,\eta>0$ such that*

$$\|\partial^\gamma F\|_{L^\infty(B(0,2))}\le\frac{M|\gamma|!}{\eta^{|\gamma|}},\quad\forall\gamma\in(\mathbb{N}\cup\{0\})^n.$$

Then

$$\|F\|_{L^\infty(B(0,1))}\le(2M)^{1-\mu}\left(\|F\|_{L^\infty(\mathcal{W})}\right)^\mu,$$

where $\mu\in(0,1)$ depends on n, η and $|\mathcal{W}|$.

The lemma is conditional stability for the analytic continuation, and see Lavrent'ev, Romanov and Shishat·skiĭ[20] for classical results.

For fixed $\tau>0$ and $q\in L^1(\mathbb{R}^n)$, let us set $F_\tau(\xi)=\widehat{q}(\tau\xi)$ for $\xi\in\mathbb{R}^n$. Then it is easily seen that F is analytic and

$$|\partial^\gamma F_\tau(\xi)|\le(2\pi)^{-\frac{n}{2}}\|q\|_{L^1(\Omega)}\frac{\tau^{|\gamma|}}{((\text{Diam}\Omega)^{-1})^{|\gamma|}}\le C\frac{\tau^{|\gamma|}}{|\gamma|!(T^{-1})^{|\gamma|}}|\gamma|!\le C\frac{e^\tau}{(T^{-1})^{|\gamma|}}|\gamma|!.$$

Therefore, applying Lemma 3.4 in the set $\mathcal{W}=\mathcal{K}_\varepsilon\cap B(0,1)$ with $M=Ce^\tau$ and $\eta=T^{-1}$, we can take a constant $\mu\in(0,1)$ depending only on ε, n and T such that

$$|F_\tau(\xi)|\le Ce^{\tau(1-\mu)}\|F_\tau\|^\mu_{L^\infty(\mathcal{W})},\quad\forall\,\xi\in B(0,1).$$

Hence, by the fact that $\tau\mathcal{K}_\varepsilon=\{\tau\xi;\ \xi\in\mathcal{K}_\varepsilon\}=\mathcal{K}_\varepsilon$, we obtain

$$|\widehat{q}(\xi)|=\left|F_\tau(\tau^{-1}\xi)\right|\le Ce^{\tau(1-\mu)}\|F_\tau\|^\mu_{L^\infty(\mathcal{W})}=Ce^{\tau(1-\mu)}\|\widehat{q}\|^\mu_{L^\infty(\mathcal{K}_\varepsilon)},\quad\xi\in B(0,\tau).\tag{3.12}$$

We now estimate the $H^{-1}(\mathbb{R}^n)$ norm of q. For all $\tau>0$ we have

$$\begin{aligned}\|q\|^{2/\mu}_{H^{-1}(\mathbb{R}^n)}&=\left[\int_{|\xi|\le\tau}|\widehat{q}(\xi)|^2(1+|\xi|^2)^{-1}d\xi+\int_{|\xi|>\tau}|\widehat{q}(\xi)|^2(1+|\xi|^2)^{-1}d\xi\right]^{1/\mu}\\&\le C\left[\tau^n\|\widehat{q}\|^2_{L^\infty(B(0,\tau))}+\tau^{-2}\|q\|^2_{L^2(\Omega)}\right]^{1/\mu}.\end{aligned}$$

Substituting (3.12) and applying Lemma 3.3, we obtain

$$\|q\|^{2/\mu}_{H^{-1}(\mathbb{R}^n)}\le C\left[\tau^{n/\mu}e^{2\tau\left(\frac{1-\mu}{\mu}\right)}e^{C\lambda}\left\|\Lambda'_{q_1}-\Lambda'_{q_2}\right\|^2+\lambda^{-2\delta}\tau^{n/\mu}e^{2\tau\left(\frac{1-\mu}{\mu}\right)}+\tau^{-\frac{2}{\mu}}\right].\tag{3.13}$$

Let $\tau_0>0$ be sufficiently large and $\tau>\tau_0$. Set

$$\lambda=\tau^{\frac{n+2}{2\delta\mu}}e^{\tau\left(\frac{1-\mu}{\delta\mu}\right)}.$$

By $\tau > \tau_0$, we can assume that $\lambda > \lambda_0$. Then $\tau^{\frac{n}{\mu}} e^{2\tau\left(\frac{1-\mu}{\mu}\right)} \lambda^{-2\delta} = \tau^{-\frac{2}{\mu}}$ and (3.13) yields

$$\|q\|_{H^{-1}(\mathbb{R}^n)}^{2/\mu} \leq C\left[\tau^{\frac{n}{\mu}} e^{\psi(\tau)} \left\|\Lambda'_{q_1} - \Lambda'_{q_2}\right\|^2 + \tau^{-\frac{2}{\mu}}\right], \tag{3.14}$$

where ψ is defined by

$$\psi(\tau) = \left(2\tau\left(\frac{1-\mu}{\mu}\right) + C\tau^{\frac{n+2}{2\delta\mu}} e^{\tau\left(\frac{1-\mu}{\delta\mu}\right)}\right).$$

It is easily seen that

$$\tau^{\frac{n}{\mu}} e^{\psi(\tau)} \leq e^{e^{A\tau}}, \quad \tau > \tau_0$$

for some A depending only on Ω, ε, δ and μ. Substitute the above inequality into (3.14) and we obtain

$$\|q\|_{H^{-1}(\Omega)} \leq \|q\|_{H^{-1}(\mathbb{R}^n)} \leq C\left(e^{e^{A\tau}} \left\|\Lambda'_{q_1} - \Lambda'_{q_2}\right\|^2 + \tau^{-2/\mu}\right)^{\mu/2}.$$

Now, in order to minimize the right-hand side with respect to τ, we set

$$\tau = \frac{1}{A} \log\left|\log \|\Lambda'_{q_1} - \Lambda'_{q_2}\|\right| \tag{3.15}$$

and we obtain

$$\|q\|_{H^{-1}(\Omega)} \leq C\left[\left\|\Lambda'_{q_1} - \Lambda'_{q_2}\right\| + \left(\log\left|\log\left\|\Lambda'_{q_1} - \Lambda'_{q_2}\right\|\right|\right)^{-2/\mu}\right]^{\mu/2} \tag{3.16}$$

provided that the right-hand side of (3.15) $> \tau_0$. If the right-hand side $\leq \tau_0$ then there exists a constant $c_0 > 0$ such that

$$\left\|\Lambda'_{q_1} - \Lambda'_{q_2}\right\| \geq c_0.$$

Thus, we have

$$\|q\|_{H^{-1}(\Omega)} \leq C\|q\|_{H^{\alpha}(\Omega)} \leq \frac{2CM}{c_0^{\mu/2}} c_0^{\mu/2} \leq C' \left\|\Lambda'_{q_1} - \Lambda'_{q_2}\right\|^{\mu/2}.$$

Therefore, (3.16) holds in the both cases. The conclusion follows from the interpolation inequality between $H^{-1}(\Omega)$ and $H^{\alpha}(\Omega)$, and the Sobolev imbedding theorem.

References

[1] M. Belishev: *Boundary control in recostruction of manifolds and metrics (BC method)*, Inverse Problems 13 (1997), R1 - R45.

[2] M. Bellassoued: *Uniqueness and stability in determining the speed of propagation of second-order hyperbolic equation with variable coefficients*, Appl. Anal. 83 (2004), 983-1014.

[3] M.Bellassoued, D.Jellali and M.Yamamoto: *Lipschitz stability for a hyperbolic inverse problem by finite local boundary data,* Appl. Anal. 85 (2006), 1219-1243.

[4] A.L. Bukhgeim and M.V.Klibanov: *Uniqueness in the large of a class of multidimensional inverse problems.* Dokl. Akad. Nauk SSSR 260 (1981), 269–272.

[5] A.L.Bukhgeim and G.Uhlmann: *Recovering a potential from partial Cauchy data.* Comm. Partial Differential Equations 27 (2002), 653–668.

[6] A.P.Calderón: *On an inverse boundary value problem,* in Seminar on Numerical Analysis and its Applications to Continuum Physics, Rio de Janeiro, (1988), 65-73.

[7] F. Cardoso and R. Mendoza: *On the hyperbolic Dirichlet to Neumann functional,* Comm. Partial Differential Equations 21 (1996), 1235–1252.

[8] J. Cheng and G. Nakamura: *Stability for the inverse potential problem by finite measurements on the boundary,* Inverse Problems 17 (2001), 273-280.

[9] R. Cipolatti and Ivo F.Lopez: *Determination of coefficients for a dissipative wave equation via boundary measurements,* J. Math. Anal. Appl. 306 (2005) 317-329.

[10] H.Heck and J-N.Wang: *Stablity estimates for the inverse boundary value problem by partial Cauchy data,* Inverse Problems 22 (2006), 1787–1796.

[11] O.Yu.Imanuvilov and M.Yamamoto: *Global uniqueness and stability in determining coefficients of wave equations,* Comm. Partial Diff. Equations 26 (2001), 1409-1425.

[12] V.Isakov: *An inverse hyperbolic problem with many boundary measurements,* Comm. Partial Differential Equations 16 (1991), 1183-1195.

[13] V.Isakov: *Inverse Problems for Partial Differential Equations,* Springer-Verlag, Berlin, (1998).

[14] V. Isakov and Z. Sun: *Stability estimates for hyperbolic inverse problems with local boundary data,* Inverse Problems 8 (1992), 193-206.

[15] A. Katchalov, Y. Kurylev and M. Lassas: *Inverse Boundary Spectral Problems,* Chapman & Hall/CRC, Boca Raton, (2001).

[16] M.V. Klibanov: *Inverse problems and Carleman estimates,* Inverse Problems 8 (1992), 575-596.

[17] M.V. Klibanov and A. A. Timonov: *Carleman Estimates for Coefficient Inverse Problems and Numerical Applications,* VSP, Utrecht, (2004).

[18] Y.V. Kurylev and M. Lassas: *Hyperbolic inverse problem with data on a part of the boundary,* in ”Differential Equations and Mathematical Physics”, AMS/IP Stud. Adv. Math. 16, Amer. Math. Soc., Providence, (2000), 259-272.

[19] I. Lasiecka, J-L. Lions and R. Triggiani: *Non homogeneous boundary value problems for second order hyperbolic operators*, J.Math.Pure et Appl. 65 (1986), 149-192.

[20] M.M. Lavrent'ev, V.G. Romanov and S.P. Shishat· skiĭ: *Ill-posed Problems of Mathematical Physics and Analysis*, American Mathematical Society, Providence, Rhode Island, (1986).

[21] J-L. Lions and E. Magenes: *Non-homogenous Boundary Value Problems and Applications,* Volumes I and II, Springer-Verlag, Berlin, (1972).

[22] F. Natterer: *The Mathematics of Computarized Tomography*, John Wiley & Sons, Chichester, (1986).

[23] L. Rachele: *Uniqueness in inverse problems for elastic media with residual stress,* Comm. Partial Diff. Equations 28 (2003), 1787-1806.

[24] Rakesh: *Reconstruction for an inverse problem for the wave equation with constant velocity,* Inverse Problems 6 (1990), 91-98.

[25] Rakesh and W. Symes: *Uniqueness for an inverse problems for the wave equation*, Comm. Partial Diff. Equations 13 (1988), 87-96.

[26] A. Ramm and J. Sjöstrand: *An inverse problem of the wave equation*, Math. Z. 206 (1991), 119-130.

[27] V.G. Romanov: *Inverse Problems for Differential Equations,* in Russian, Novosibirsk State University, Novosibirsk, (1973).

[28] P. Stefanov and G. Uhlmann: *Stability estimates for the hyperbolic Dirichlet to Neumann map in anisotropic media*, J. Functional Anal. 154 (1998), 330-358.

[29] Z.Sun: *On continous dependence for an inverse initial boundary value problem for the wave equation*, J. Math. Anal. App. 150 (1990), 188-204.

[30] G. Uhlmann: *Inverse boundary value problems and applications,* Astéisque 20 (1992), 153-221.

[31] S.Vessela: *A continuous dependence result in the analytic continuation problem*, Forum Math.11 (1999), 695-703.

Preprint Series, Graduate School of Mathematical Sciences, The University of Tokyo

UTMS

2007–3 Takayuki Oda: *The standard* $(\mathfrak{g}, K)$*-modules of* $Sp(2, R)$ *I – The case of principal series –.*

2007–4 Masatoshi Iida and Takayuki Oda: *Harish-Chandra expansion of the matrix coefficients of* P_J *Principal series representation of* $Sp(2, \mathbb{R})$.

2007–5 Yutaka Matsui and Kiyoshi Takeuchi: *Microlocal study of Lefschetz fixed point formulas for higher-dimensional fixed point sets.*

2007–6 Shumin Li and Masahiro Yamamoto: *Lipschitz stability in an inverse hyperbolic problem with impulsive forces.*

2007–7 Tadashi Miyazaki: *The* $(\mathfrak{g}, K)$*-module structures of principal series representations of* $Sp(3, \mathbb{R})$.

2007–8 Wuqing Ning: *On stability of an inverse spectral problem for a nonsymmetric differential operator.*

2007–9 Tomohiro Yamazaki and Masahiro Yamamoto: *Inverse problems for vibrating systems of first order.*

2007–10 Yasufumi Osajima: *General asymptotics of Wiener functionals and application to mathematical finance.*

2007–11 Shuichi Iida: *Adiabatic limits of* η*-invariants and the meyer functions.*

2007–12 Ken-ichi Yoshikawa: $K3$ *surfaces with involution, equivariant analytic torsion, and automorphic forms on the moduli space II: a structure theorem.*

2007–13 M. Bellassoued, D. Jellali and M. Yamamoto: *Stability estimate for the hyperbolic inverse boundary value problem by local Dirichlet-to-Neumann map.*

The Graduate School of Mathematical Sciences was established in the University of Tokyo in April, 1992. Formerly there were two departments of mathematics in the University of Tokyo: one in the Faculty of Science and the other in the College of Arts and Sciences. All faculty members of these two departments have moved to the new graduate school, as well as several members of the Department of Pure and Applied Sciences in the College of Arts and Sciences. In January, 1993, the preprint series of the former two departments of mathematics were unified as the Preprint Series of the Graduate School of Mathematical Sciences, The University of Tokyo. For the information about the preprint series, please write to the preprint series office.

ADDRESS:
Graduate School of Mathematical Sciences, The University of Tokyo
3–8–1 Komaba Meguro-ku, Tokyo 153-8914, JAPAN
TEL +81-3-5465-7001 FAX +81-3-5465-7012