

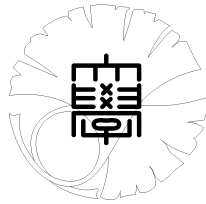
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**Twisted Alexander polynomial
and surjectivity of a group homomorphism**

by

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TWISTED ALEXANDER POLYNOMIAL AND SURJECTIVITY OF A GROUP HOMOMORPHISM

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ABSTRACT. If $\varphi : G \rightarrow G'$ is a surjective homomorphism, we prove that the twisted Alexander polynomial of G is divisible by the twisted Alexander polynomial of G' . As an application, we show non-existence of surjective homomorphism between certain knot groups.

1. INTRODUCTION

Suppose that G is a finitely presentable group with a surjective homomorphism to the free abelian group of rank l , e.g., abelianization. Let $\rho : G \rightarrow GL(n; R)$ be a linear representation. The twisted Alexander polynomial of G associated to ρ was introduced in [8] and is defined to be a rational expression of l indeterminates.

Let $\varphi : G \rightarrow G'$ be a surjective homomorphism. Each representation $\rho' : G' \rightarrow GL(n; R)$ naturally induces a representation of G , namely, $\rho = \rho' \circ \varphi$. In this paper we prove the following:

Main theorem . *The twisted Alexander polynomial of G associated to ρ is divisible by the twisted Alexander polynomial of G' associated to ρ' .*

The corresponding fact about the (classical) Alexander polynomial is well known [1].

We present two separate proofs of the main theorem. First we give a purely algebraic proof in §3. If G is a knot group, the twisted Alexander polynomial of G may be regarded as the Reidemeister torsion. In §4, we provide another proof of the main theorem in case when G and G' are knot groups, from the view point of Reidemeister torsion.

In the last section, we show non-existence of surjective homomorphism between certain knot groups, as an application of the main theorem.

2. TWISTED ALEXANDER POLYNOMIAL

In this section, we recall briefly the definition of the twisted Alexander polynomial.

Let G be a finitely presentable group. Choose and fix a presentation as follows:

$$G = \langle x_1, \dots, x_u \mid r_1, \dots, r_v \rangle.$$

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We denote by $\alpha : G \rightarrow \mathbb{Z}^l$ a surjective homomorphism to the free abelian group with generators t_1, \dots, t_l and $\rho : G \rightarrow GL(n; R)$ a linear representation, where R is a unique factorization domain. These maps naturally induce ring homomorphisms $\tilde{\rho}$ and $\tilde{\alpha}$ from $\mathbb{Z}[G]$ to $M(n; R)$ and $\mathbb{Z}[t_1^{\pm 1}, \dots, t_l^{\pm 1}]$ respectively, where $M(n; R)$ denotes the matrix algebra of degree n over R . Then $\tilde{\rho} \otimes \tilde{\alpha}$ defines a ring homomorphism

$$\mathbb{Z}[G] \rightarrow M(n; R[t_1^{\pm 1}, \dots, t_l^{\pm 1}]).$$

Let F_u be the free group on generators x_1, \dots, x_u and

$$\Phi : \mathbb{Z}[F_u] \rightarrow M(n; R[t_1^{\pm 1}, \dots, t_l^{\pm 1}])$$

the composite of the surjection $\mathbb{Z}[F_u] \rightarrow \mathbb{Z}[G]$ induced by the fixed presentation and the map $\tilde{\rho} \otimes \tilde{\alpha} : \mathbb{Z}[G] \rightarrow M(n; R[t_1^{\pm 1}, \dots, t_l^{\pm 1}])$.

We define the $v \times u$ matrix M whose (i, j) component is the $n \times n$ matrix

$$\Phi \left(\frac{\partial r_i}{\partial x_j} \right) \in M(n; R[t_1^{\pm 1}, \dots, t_l^{\pm 1}]),$$

where $\partial/\partial x$ denotes the Fox derivation. This matrix M is called the Alexander matrix of the presentation of G associated to the representation ρ .

It is easy to see that there is an integer $1 \leq j \leq u$ such that $\det \Phi(x_j - 1) \neq 0$. For such j , let us denote by M_j the $v \times (u-1)$ matrix obtained from M by removing the j -th column. We regard M_j as an $nv \times n(u-1)$ matrix with coefficients in $R[t_1^{\pm 1}, \dots, t_l^{\pm 1}]$. Moreover, for an $n(u-1)$ -tuple of indices

$$I = (i_1, i_2, \dots, i_{n(u-1)}), \quad (1 \leq i_1 < i_2 < \dots < i_{n(u-1)} \leq nv)$$

we denote by M_j^I the $n(u-1) \times n(u-1)$ square matrix consisting of the i_k -th row of the matrix M_j , where $k = 1, 2, \dots, n(u-1)$.

Then the twisted Alexander polynomial (see [8]) of a finitely presented group G for a representation $\rho : G \rightarrow GL(n; R)$ is defined to be a rational expression

$$\Delta_{G, \rho}(t_1, \dots, t_l) = \frac{\gcd_I(\det M_j^I)}{\det \Phi(x_j - 1)}$$

and moreover well-defined up to a factor $\epsilon t_1^{\epsilon_1} \dots t_l^{\epsilon_l}$, where $\epsilon \in R^\times, \epsilon_i \in \mathbb{Z}$. See [8], [6], [2] and [3] for more precise definition and applications.

3. MAIN THEOREM AND THE ALGEBRAIC PROOF

In this section, we prove the following main theorem of this paper.

Theorem 3.1. *Let G and G' be finitely presentable groups and α, α' surjective homomorphisms from G, G' to \mathbb{Z}^l respectively. Suppose that there exists a surjective homomorphism $\varphi : G \rightarrow G'$ such that $\alpha = \alpha' \circ \varphi$. Then $\Delta_{G, \rho}$ is divisible by $\Delta_{G', \rho'}$ for any representation $\rho' : G' \rightarrow GL(n; R)$, where $\rho = \rho' \circ \varphi$. That is to say, the quotient of $\Delta_{G, \rho}$ by $\Delta_{G', \rho'}$ is a genuine polynomial.*

Proof. Choose and fix a presentation

$$G = \langle x_1, x_2, \dots, x_u \mid r_1, r_2, \dots, r_v \rangle.$$

Since φ is surjective, then G' is generated by $\varphi(x_1), \dots, \varphi(x_u)$. Namely, G' can be presented as

$$G' = \langle \varphi(x_1), \varphi(x_2), \dots, \varphi(x_u) \mid s_1, s_2, \dots, s_{v'} \rangle.$$

For convenience, we also write x_i for $\varphi(x_i)$, that is, we consider that G' is generated by x_1, \dots, x_u . Here each relator r_i is written as

$$r_i = \prod_k u_k s_{l_{i_k}}^{\varepsilon_{i_k}} u_k^{-1}, \quad i = 1, 2, \dots, v, \quad 1 \leq l_{i_k} \leq v', \quad u_k \in F_u, \quad \varepsilon_{i_k} = \pm 1.$$

By applying the Fox derivation $\frac{\partial}{\partial x_j}$ and collecting terms of $\frac{\partial s_k}{\partial x_j}$, we get

$$(3.1) \quad \varphi \left(\frac{\partial r_i}{\partial x_j} \right) = \sum_{k=1}^{v'} A_{i,k} \frac{\partial s_k}{\partial x_j}.$$

Here $A_{i,k}$ ($1 \leq i \leq v$) is a sum of some $\varepsilon_{\bullet} \varphi(u_{\bullet})$, which does not depend on j . Let M_G and $M_{G'}$ be the Alexander matrices removed the u -th column:

$$M_G = \begin{pmatrix} \tilde{\rho} \otimes \tilde{\alpha} \left(\frac{\partial r_1}{\partial x_1} \right) & \cdots & \tilde{\rho} \otimes \tilde{\alpha} \left(\frac{\partial r_1}{\partial x_{u-1}} \right) \\ \vdots & \ddots & \vdots \\ \tilde{\rho} \otimes \tilde{\alpha} \left(\frac{\partial r_v}{\partial x_1} \right) & \cdots & \tilde{\rho} \otimes \tilde{\alpha} \left(\frac{\partial r_v}{\partial x_{u-1}} \right) \end{pmatrix}$$

$$M_{G'} = \begin{pmatrix} \tilde{\rho}' \otimes \tilde{\alpha}' \left(\frac{\partial s_1}{\partial x_1} \right) & \cdots & \tilde{\rho}' \otimes \tilde{\alpha}' \left(\frac{\partial s_1}{\partial x_{u-1}} \right) \\ \vdots & \ddots & \vdots \\ \tilde{\rho}' \otimes \tilde{\alpha}' \left(\frac{\partial s_{v'}}{\partial x_1} \right) & \cdots & \tilde{\rho}' \otimes \tilde{\alpha}' \left(\frac{\partial s_{v'}}{\partial x_{u-1}} \right) \end{pmatrix}.$$

By (3.1), we have

$$M_G = AM_{G'}$$

where $A = (\rho'(A_{i,k}))$ is a $nv \times nv'$ matrix. For $I = (i_1, i_2, \dots, i_{n(u-1)})$, as is easily shown,

$$\det M_G^I = \det (A^I M_{G'}) = \sum_K \pm (\det A_K^I) (\det M_{G'}^K)$$

where $K = (k_1, k_2, \dots, k_{n(u-1)})$ and A_K^I is the matrix consisting of the $k_1, k_2, \dots, k_{n(u-1)}$ -th columns of A^I . It follows that if $\det M_{G'}^I$ has a common divisor P for all I , then so does $\det M_G^I$. Moreover, the denominator of $\Delta_{G,\rho}$ is equal to that of $\Delta_{G',\rho'}$. This completes the proof. \square

The corresponding fact about the Alexander polynomial is well known. Let $G(K)$ be the knot group $\pi_1(S^3 - K)$ of a knot K in S^3 . For any knots K, K' , if there exists a surjective homomorphism from $G(K)$ to $G(K')$, then the Alexander polynomial of K' is divisible by that of K . Murasugi mentions that if there exists a surjective homomorphism from a knot group $G(K)$ to the trefoil knot group, then the twisted Alexander polynomial of $G(K)$ is divisible by that of the trefoil knot group. The main theorem is a generalization of the above.

We remark geometric settings of a surjective homomorphism between groups. Here we mention degree 1 maps. Let X and Y be d -dimensional compact manifolds. Suppose that $f : X \rightarrow Y$ is a degree 1 map. It is easy to see that its induced homomorphism $f_* : \pi_1(X) \rightarrow \pi_1(Y)$ is a surjective homomorphism.

In the knot group case, there exist the following situations except for degree 1 maps. First, there exists a surjective homomorphism from any knot group to the trivial knot group which is the infinite cyclic group. Secondly, if a knot K is a connected sum of K_1 and K_2 , then its knot group $G(K)$ is an amalgamated

product of $G(K_1)$ and $G(K_2)$. Then there exists a surjection from $G(K)$ to each factor group. Thirdly, if a knot K is a periodic knot of order n , then there exists a surjective homomorphism from $G(K)$ to $G(K_*)$ where K_* is its quotient knot of K .

4. ANOTHER PROOF FROM THE VIEW POINT OF REIDEMEISTER TORSION

In this section, we prove our theorem in the knot group case. It is done by using the Mayer-Vietoris argument of Reidemeister torsion.

Here let us consider a knot K in S^3 and its exterior $E(K)$. For the knot group $G(K) = \pi_1 E(K)$, we choose and fix a Wirtinger presentation

$$G(K) = \langle x_1, \dots, x_u \mid r_1, \dots, r_{u-1} \rangle.$$

The abelianization homomorphism

$$\alpha_K : G(K) \rightarrow H_1(E(K), \mathbb{Z}) \cong \mathbb{Z} = \langle t \rangle$$

is given by $\alpha_K(x_1) = \dots = \alpha_K(x_u) = t$. If we have no confusion, we write simply α for α_K as previous section. In this section, we take a unimodular representation $\rho : G(K) \rightarrow SL(n; \mathbb{F})$ over a field \mathbb{F} . As the definition of the twisted Alexander polynomial, we consider the tensor representation

$$\rho \otimes \alpha : G \rightarrow GL(n; \mathbb{F}[t, t^{-1}]) \subset GL(n; \mathbb{F}(t)).$$

Here $\mathbb{F}(t)$ denotes the rational function field over \mathbb{F} . If $\rho \otimes \alpha$ is an acyclic representation over $\mathbb{F}(t)$, that is, all homology groups over $\mathbb{F}(t)$ of $E(K)$ twisted by $\rho \otimes \alpha$ are vanishing, then Reidemeister torsion of $E(K)$ for $\rho \otimes \alpha$ can be defined. Furthermore the following equality holds. See [3, 5] for more details of definitions and proofs.

Theorem 4.1. *If $\rho \otimes \alpha$ is an acyclic representation, then we have*

$$\tau_{\rho \otimes \alpha}(E(K)) = \Delta_{G(K), \rho}(t)$$

up to a factor $\pm t^{nk}$ ($k \in \mathbb{Z}$) if n is odd and up to only t^{nk} if n is even.

From this theorem, we prove the main theorem as divisibility of Reidemeister torsion in the knot group case. Here we take a surjective homomorphism $\varphi : G(K) \rightarrow G(K')$. By changing an orientation of meridians if we need, we may assume that $\alpha_{K'} \circ \varphi = \alpha_K$. Let $\rho' : G(K') \rightarrow SL(n; \mathbb{F})$ be a representation. For simplicity, we write a representation ρ for its composition $\rho' \circ \varphi$.

Now we consider 2-dimensional CW-complexes $X(K)$ and $X(K')$ defined by their Wirtinger presentations. It is well-known that these complexes are simple homotopy equivalent to the knot exteriors. Then these Reidemeister torsions of $X(K)$ and $X(K')$ are equal to the twisted Alexander polynomials respectively. Here we consider twisted homologies of these complexes by using their CW-complex structure. The coefficient V is a $2n$ -dimensional vector space over a rational function field $\mathbb{F}(t)$. When V is regarded as a $G(K)$ -module by using ρ , it is denoted by V_ρ .

The homomorphism φ induces a chain map $\varphi_* : C_*(X(K), V_\rho) \rightarrow C_*(X(K'), V_{\rho'})$. We take a tensor representation $\rho \otimes \alpha_K : G(K) \rightarrow GL(n; \mathbb{F}(t))$. Assume that $\rho \otimes \alpha_K$ and $\rho' \otimes \alpha_{K'}$ are acyclic representations. Then we can prove the following.

Theorem 4.2. *$\tau(X(K); V_{\rho \otimes \alpha_K}) / \tau(X(K'); V_{\rho' \otimes \alpha_{K'}})$ is a polynomial in $\mathbb{F}[t, t^{-1}]$.*

We show the following proposition first.

Proposition 4.3. *The chain map $\varphi_* : C_*(X(K), V_{\rho \otimes \alpha_K}) \rightarrow C_*(X(K'), V_{\rho' \otimes \alpha_{K'}})$ is surjective.*

Proof. It is clear that φ induces an isomorphism on the 0-chains, and a surjection on the 1-chains. Then we only need to prove this proposition on the 2-chains.

We take a non-trivial 2-chain $z \in C_2(X(K'), V_{\rho' \otimes \alpha_{K'}})$. By the acyclicity of the chain complex $C_*(X(K'), V_{\rho' \otimes \alpha_{K'}})$, the boundary map $\partial : C_2(X(K'), V_{\rho' \otimes \alpha_{K'}}) \rightarrow C_1(X(K'), V_{\rho' \otimes \alpha_{K'}})$ is injective. Then the image ∂z is non-trivial in C_1 . On the other hand, by the surjectivity of

$$\varphi : C_1(X(K), V_{\rho \otimes \alpha_K}) \rightarrow C_1(X(K'), V_{\rho' \otimes \alpha_{K'}}),$$

there exists a 2-chain $w \in C_2(X(K), V_{\rho \otimes \alpha_K})$ such that $\varphi_*(w) = z$. By the commutativity of maps, in C_2

$$\varphi_*(\partial w) = \partial \varphi_*(w) = \partial z = 0.$$

Then we have $\partial w = 0$. Hence by the acyclicity, there exists $\tilde{w} \in C_*(X(K), V_{\rho \otimes \alpha_K})$ such that $\partial \tilde{w} = w$. Again by the commutativity, $\varphi(\tilde{w}) = z$. Therefore φ_* is surjective. \square

Proof of Theorem 4.2. From the above proposition, we can take the kernel D_* of this chain map φ_* and obtain a short exact sequence

$$0 \rightarrow D_* \rightarrow C_*(X(K), V_{\rho \otimes \alpha_K}) \rightarrow C_*(X(K'), V_{\rho' \otimes \alpha_{K'}}) \rightarrow 0.$$

Here we recall the following fact. For a short exact sequence $0 \rightarrow C'_* \rightarrow C_* \rightarrow C''_*$ of finite chain complexes, if two of them are acyclic complexes, then the third one is also acyclic. Furthermore, among the torsion of them, it holds that

$$\tau(C_*) = \tau(C'_*)\tau(C''_*)$$

up to some factor.

By applying the property of the product of torsion, we have

$$\tau(X(K); V_{\rho \otimes \alpha_K}) = \tau(X(K'); V_{\rho' \otimes \alpha_{K'}})\tau(D; V_{\rho \otimes \alpha_K}).$$

We only need to prove that $\tau(D; V_{\rho \otimes \alpha_K})$ is a polynomial. By the definition, 0-chains D_0 is vanishing. Because

$$\varphi_* : C_0(X(K), V_{\rho \otimes \alpha_K}) \rightarrow C_0(X(K'), V_{\rho' \otimes \alpha_{K'}})$$

is isomorphism. Hence by definition, its torsion is the determinant of $D_2 \rightarrow D_1$. Therefore it is a polynomial. \square

Remark 4.4. By the similar argument, we can prove that if $\varphi : G(K) \rightarrow G(K')$ is an injective homomorphism, then $\tau(X(K'); V_{\rho \otimes \alpha_{K'}})/\tau(X(K); V_{\rho \otimes \alpha_K})$ is a polynomial.

5. EXAMPLES

In this section, we show some examples of the twisted Alexander polynomials and an application for Theorem 3.1. We consider the problem: is there a surjective homomorphism from $G(K)$ to $G(K')$ for two given knots K, K' ? Here we study the problem in case when K' is the figure eight knot 4_1 . It is investigated by Murasugi when K' is the trefoil knot 3_1 . The numbering of the knots follows that of Rolfsen's book [7].

If the classical Alexander polynomial of K can not be divided by that of K' , we can determine whether or not there exists a surjective homomorphism from $G(K)$ to $G(K')$. In the knot table in [7], up to 9 crossings, the classical Alexander polynomial

of any knot is not divisible by that of $G(4_1)$ except for $8_{18}, 8_{21}, 9_{12}, 9_{24}, 9_{37}, 9_{39}$ and 9_{40} . That is to say, except for $8_{18}, 8_{21}, 9_{12}, 9_{24}, 9_{37}, 9_{39}$ and 9_{40} , there exists no surjective homomorphism from such a knot group to $G(4_1)$.

Next, we consider a representation $\rho : G(K) \rightarrow SL(2; \mathbb{Z}/p\mathbb{Z})$ and the twisted Alexander polynomial associated to ρ . Theorem 3.1 says that if the numerator of $\Delta_{G(K), \rho}$ for all representations $\rho : G(K) \rightarrow SL(2; \mathbb{Z}/p\mathbb{Z})$ fixed some prime p can not be divided by the numerator of $\Delta_{G(K'), \rho'}$ for a certain representation $\rho' : G(K') \rightarrow SL(2; \mathbb{Z}/p\mathbb{Z})$, then there exists no surjective homomorphism from $G(K)$ to $G(K')$.

Let us compute the twisted Alexander polynomials $\Delta_{G(4_1), \rho'}$ for a certain representation $\rho' : G(4_1) \rightarrow SL(2; \mathbb{Z}/7\mathbb{Z})$. The knot group $G(4_1)$ admits a presentation

$$G(4_1) = \langle x_1, x_2, x_3, x_4 \mid x_4 x_2 x_4^{-1} x_1^{-1}, x_1 x_2 x_1^{-1} x_3^{-1}, x_2 x_4 x_2^{-1} x_3^{-1} \rangle.$$

We can check easily that the following is a representation of $G(4_1)$:

$$\rho'(x_1) = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}, \rho'(x_2) = \begin{pmatrix} 1 & 0 \\ 3 & 1 \end{pmatrix}, \rho'(x_3) = \begin{pmatrix} 4 & 4 \\ 3 & 5 \end{pmatrix}, \rho'(x_4) = \begin{pmatrix} 2 & 4 \\ 5 & 0 \end{pmatrix}.$$

Then we obtain the Alexander matrix

$$M = \begin{pmatrix} 6 & 0 & 2t & 4t & 0 & 0 & 6t+1 & 6t \\ 0 & 6 & 5t & 0 & 0 & 0 & 0 & 6t+1 \\ 3t+1 & 3t & t & t & 6 & 0 & 0 & 0 \\ 4t & 2t+1 & 0 & t & 0 & 6 & 0 & 0 \\ 0 & 0 & 3t+1 & 3t & 6 & 0 & t & 0 \\ 0 & 0 & 4t & 2t+1 & 0 & 6 & 3t & t \end{pmatrix}.$$

The numerator P of the twisted Alexander polynomial $\Delta_{G(4_1), \rho'}$ is the determinant of M_4 obtained from M by removing the last two columns. Then we get

$$P = t^4 + t^3 + 3t^2 + t + 1.$$

Moreover, we calculate the numerator of the twisted Alexander polynomials of $G(8_{21})$ for all representations $G(8_{21}) \rightarrow SL(2; \mathbb{Z}/7\mathbb{Z})$ and get 24 polynomials. These calculations are made by author's computer program and the same results are obtained by Kodama Knot program [4]. None of them can be divided by P , then we conclude that there exists no surjective homomorphism from $G(8_{21})$ to $G(4_1)$. Similarly, if we deal with $p = 5, 7$, we get there exists no surjective homomorphism from $G(9_{12}), G(9_{24}), G(9_{39})$ to $G(4_1)$. On the other hand, we can give explicitly surjective homomorphisms from the rest knot groups $G(8_{18}), G(9_{37}), G(9_{40})$ to $G(4_1)$. Then we can determine whether or not there exists a surjective homomorphism from the group of all knot with up to 9 crossings to $G(4_1)$.

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