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Current Research Overview: My research field is algebraic geometry, with two main themes: enumerative geometry and derived categories of coherent sheaves. Enumerative geometry studies how many geometric objects, such as points and curves, exist on algebraic varieties. In particular, curve counting is an important subject that is deeply related to superstring theory in theoretical physics. Since there are in general infinitely many curves, it is not possible to count them directly. Instead, one defines counting invariants that assign a "virtual number" of curves and studies their properties.. The other main theme, the derived category of coherent sheaves, may be viewed as a framework for capturing geometric information on algebraic varieties in categorical terms. It has become clear that derived categories are deeply connected not only with algebraic geometry, but also with many other areas such as symplectic geometry, representation theory, and noncommutative algebra. In particular, the space of Bridgeland stability conditions constructed from derived categories is an important object of study closely related to mirror symmetry and birational geometry. So far, I have applied the theory of derived categories and stability conditions to enumerative geometry, especially Donaldson–Thomas theory, and have clarified qualitative properties of DT invariants, such as the DT/PT correspondence and rationality conjectures, from a categorical viewpoint. I also proposed the BMT (Bayer–Macrì–Toda) conjecture on stability conditions on three-dimensional algebraic varieties, and in joint work with Maulik gave a mathematical definition of Gopakumar–Vafa invariants. One of the interesting aspects of this research is the connection between the concrete theory of enumerative geometry and the abstract theory of derived categories. In recent years, I have been working on "categorical DT theory," which aims to lift counting invariants to categorical objects. Through this, I hope to develop a new form of categorical geometry that connects enumerative geometry with various other areas of mathematics. For example, in joint work with Tudor Pădurariu, we have given a new formulation of the Dolbeault geometric Langlands conjecture from the viewpoint of categorical DT theory, and are approaching this conjecture from the perspective of enumerative geometry.

What I Expect from Students: It is desirable first to acquire the basic knowledge of algebraic geometry through books such as Hartshorne's *Algebraic Geometry* and Vakil's *The Rising Sea*. Understanding theorems is important, but becoming familiar with concrete examples and calculations is also very useful for research. After mastering the basics of algebraic geometry, it is a good idea to proceed to literature connected to either enumerative geometry or the study of derived categories of coherent sheaves. For example, I recommend books such as Fulton's *Intersection Theory* and Huybrechts's *Fourier–Mukai Transforms in Algebraic Geometry*. After that, one may choose a topic of interest from my writings, such as *Recent Progress on the Donaldson–Thomas Theory* or *Derived Categories of Coherent Sheaves and Algebraic Geometry*, and study it in depth together with related research papers. It is also useful to study differential geometry and the representation theory of algebraic groups in addition to algebraic geometry. On the other hand, it is not necessary to study theoretical physics, such as superstring theory, in depth in advance. Since Kavli IPMU, where I am based, does not offer regular lecture courses for students, master's students are expected to be based primarily at Komaba, and we will hold a seminar about once a week either at Kavli IPMU or at Komaba. From the doctoral stage onward, I recommend making Kavli IPMU your primary base.