**Name:** Yukinobu Toda (IPMU)

**Research Field:** Algebraic Geometry

**Key words:** Derived category of coherent sheaves, Bridgeland stability condition, Calabi-Yau 3-fold, Donaldson-Thomas invariant, Gopakumar-Vafa invariant, Derived Algebraic Geometry

**Present Research:** My main research subjects are derived categories of coherent sheaves on algebraic varieties (especially on Calabi-Yau 3-folds), and Donaldson-Thomas (DT) invariants which count stable objects on them. The study of derived categories of coherent sheaves has been developed due to Kontsevich's Homological mirror symmetry, and it connects several mathematical (and physics) theories (birational geometry, representation theory, symplectic geometry, D-brane, etc). Some of representatives of my past works are as follows:

・The study of Bridgeland stability conditions on derived categories, especially construction problem of stability conditions on 3-folds and Bogomolov-Gieseker inequality conjecture.

・The constructions of moduli spaces of stable objects in the derived category, constructions of corresponding DT invariants, and a proof of Pandharipande-Thomas conjecture using wall-crossing formula of these invariants.

・A proposal of Gopakumar-Vafa invariants via vanishing cycle sheaves, and its relationship with Pandharipande-Thomas invariants which count some objects in the derived category.

Besides, I also studied non-commutative deformation theory and also DT theory on CalabiYau 4-folds. The recent goal of my research is to construct `categorical DT theory' which categorifies DT invariants, where I use derived algebraic geometry based on higher category theory. I have developed its basic theory in the case of local surfaces and several applications have been obtained, but for a general CY 3-fold even a definition is not yet given. By categorifying wall-crossing formulas of DT invariants which I proved so far, I expect to be able to develop new mathematics which connects wall-crossing of DT theory for CY 3-folds, birational geometry, derived categories of coherent sheaves and representation theory of Hall algebras.

**Notice for the students:** I want students to finish basic theory on algebraic geometry such as Hartshorne `Algebraic Geometry' before entering graduate course. It is important to understand theorem, but it is also useful to compute several examples in the research. Once learning basic theory of algebraic geometry, I recommend students to choose some interesting topics in the book `Derived category of coherent sheaves and Algebraic Geometry' and study them in details, by referring also related papers. It would be also good to learn differential geometry and representation theory of algebraic groups.

As there is no lecture course in IPMU, I recommend students to work mainly in Komaba during the master course, and we do seminars once in a week either in IPMU or by online. In the PhD course, I recommend students to work mainly in IPMU.