Name ; Hisayosi MATUMOTO
Research field : Lie groups, Lie algebras, and representation theory
Key words : Semisimple Lie group, unitary representation, generalized Verma module
Present research
(1) Degenerate principal series
In a joint work with Peter E. Trapa, we stidied degenerate principal series of $G = Sp(p,q)$ and $SO^*(2n)$ with an infinitesimal character appearing as a weight of some finite-dimensional G-representation. We show at a most singular parameter each irreducible constituent is weakly unipotent and unitarizable. We cosider the case of $SO^*(2n)$ here. We write the Levi part of a maximal parabolic subgroup as $GL(k,H) \times SO^*(2(n-2k))$. If $3k \leqslant n$, I_P is irreducible and isomorphic to a derived functor module. If $3k > n$, we conjecture there are $k-1$ irreducible constituents in I_P other than derived functor modules of the maximal Gelfand-Kirillov dimension. However, it remains open at this point.
(2) Homomorphisms between generalized Verma modules
Let $\mathfrak{g}$ be a complex semisimple Lie algebra and let $\mathfrak{p}$ be its parabolic subalgebra. The induced module of one-dimensional representation of $\mathfrak{p}$ is called a (scalar) generalized Verma module. If $\mathfrak{p}$ is a Borel subalgebra, it is called a Verma module. Around 1970, the existence condition of homomorphisms between Verma modules is found by Verma and Bernstein-Gelfand-Gelfand. In 1970s, Lepowsky studied homomorphisms between generalized Verma modules and obtained some fundamental result. However, the classification of the homomorphisms is known only for the case of the commutative niradical (Boe 1985) and a rank one parabolic associated witha symmetric pair. I classified the homomorphisms between scalar generalized Verma modules associated to maximal parabolic sualgebras and I explained how to use the operators constructed in the maximal case to get some operators in general. I conjectures that all the homomorphisms arise in this way; this statement generalizes the result of Bernstein-Gelfand-Gelfand.
Notice for students
The elementary theory of semisimple Lie algebras, such as root systems, Weyl groups, and the highest weight theory , is an essencial prelimanary to the representation theory of Lie algebras and Lie groups. So, I strongly encourage students to complete reading a standard textbook of Lie algebras such as Humphreys' Introduction to Lie algebras and representation theory", before entering the graduate school,