Name: GIGA, Yoshikazu
ResearchField: Differential Equations, Functional Analysis, Real Analysis
Key words: Navier-Stokes equations, Curvature flow equations, Viscosity solutions, Nonlinear partial differential equations of diffusion type, Free boundary problem

Current research

Broadly speaking, my major field is (mathematical) analysis. Major topics of analysis concern limit and convergence of functions. For example, a typical problem in analysis is whether or not small perturbation of some quantity causes only small change of other related quantity. Such a problem is important not only in daily life but also in various fields of sciences and technology. As a result there is a huge variety of topics in analysis. In analysis my major field is mathematical analysis on nonlinear partial differential equations.

Various differential equations have been proposed with the purpose to model phenomena in nature such as motion of fluid, crystal growth and to handle problems in technology for example in image processing. My major research topic is nonlinear diffusion equations, which form an important class of partial differential equations (PDEs). (A PDE is an equation describing a function relation of a set of unknowns and their derivatives. Here unknowns depend in general on several independent variables as time and space. If the unknown depend only on one variable, the equation is called an ordinary differential equation.) My particular interest lies in solvability of PDEs under given conditions and in tracking behavior of solutions in a rigorous mathematical framework. Fortunately, my idea have led several important methods to analyze new phenomena. From various phenomena in natural science and technology including experiments there arises an inexhaustible set of mathematically new problems. Here, only two topics of my current interest will be explained.

(i) Free boundary problems: If one considers evolving two phases bounded for example by a crystal surface, the motion of phase boundary like a crystal surface is a priori unknown. If one has to solve motion of such a boundary as a part of problems, the problem is called a free boundary problem. Based on the theory of curvature flow equations developed by my colleague and myself, my major concern is whether it is possible to describe complicated shape like snow crystal by a mathematical model.

(ii) Navier-Stokes equations: The Navier-Stokes equations are fundamental equations modeling motion of incompressible viscous fluid. However, several fundamental questions remain open. For example, if one considers three-dimensional flow, it is a famous one-million dollar open problem proposed by Clay Institute whether or not the initial value problem admits a global-in-time smooth solution for non small initial data. This problem is of course one of my concern but I also intend to apply the theory of the Navier-Stokes equations to geofluid dynamics.

Besides these topics my fields include 'Blowup problems of semilinear heat equations', 'Discontinuous viscosity solutions', 'Higher order variational problems'. For example if you hit 'Aviles-Giga' in Web pages, you will see what kind of variational problem I have been involved. This is an interesting topic so if someone is interested, I am happy to guide him/her and pursue in this direction.

Prerequisites

(1) There is a huge variety of topics to be studied and there appears that necessary prerequisite is huge. However, it is better to study some particular branch of mathematics thoroughly (like measure theory, functional analysis, dynamical system, differential equations) than to learn many topics superficially. It is desirable not to avoid analytic calculation, for example, estimates of integrals, integration by parts although sometimes it is tedious.

(2) We welcome those who would like to enjoy mathematical conversation and to explain mathematics to other people.