Name： SAKAI Hidetaka
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Present research

Elliptic functions and hypergeometric functions were the driving forces behind 19th century mathematics. In addition to function theory, algebraic geometry developed from elliptic curves, and the study of Riemann surfaces led to topology. Number theory also has a lot to offer from elliptic functions and hypergeometric functions.

It may seem simple, but my interest is in constructing the next generation of special function theory from functions that satisfy the Painlevé equations. The Painlevé equations are nonlinear, non-autonomous ordinary differential equations. In 2012, a research group from Ukraine gave an expression for the solution. This expression is also related to two-dimensional conformal field theory and four-dimensional gauge theory, and has attracted attention.

I used rational surface theory to organize the theory of the Painlevé equation, and as a result, a world emerged that naturally extended to the discrete Painlevé equation. Furthermore, I considered the generalization of the Painlevé equation to higher dimensions using the deformation theory of linear differential equations. From there, together with my collaborators, we classified systems of equations with four-dimensional phase space into 40 types.

Many systems of equations and their solutions appear, each with its own characteristics, making it fascinating, almost like natural history.

Notice for the students

As a study guide, the following subjects are relevant to this field of study:

Things that will definitely be useful to study: Complex Functions, Differential Equations in Complex Domains.

Depending on your research direction and ideas, other subjects that may be useful include: Algebraic Geometry, Algebraic Analysis, and Representation Theory of Infinite-dimensional Lie Algebras.