令和６（2024）年度　東京大学大学院

数理科学研究科　数理科学専攻　修士課程

英　語　（筆記試験）

令和５（2023）年８月２８日（月）

１０：２０　～　１１：４０

問題は全部で２題ある．２題とも解答すること．

（１）　解答しようとする各問ごとに解答用紙を１枚使用すること．
試験開始後，各解答用紙の所定欄に各自の**氏名**，**受験番号**と解答する**問題の番号**を記入すること．

（２）　試験開始後，この用紙の下部に**受験番号**を明記すること．ただし氏名を記入してはならない．

（３）　試験開始後，草稿用紙の上部に各自の**受験番号**を明記すること．ただし氏名を記入してはならない．

（４）　試験終了後に提出するものは，１題につき１枚，計**２枚の答案**，**１枚の草稿用紙**である．
着手した問題数が２題にみたない場合でも，氏名と受験番号のみを記入した白紙の答案を補い，２枚とすること．
指示に反したもの，**答案が２枚でないものは無効**とする．
問題冊子は回収する．

（５）　解答用紙の裏面を使用する場合は，表面の右下に「裏面使用」と明記すること．

受験番号＿＿＿＿＿＿＿＿

E 第 1 問

(1) 次の英文の下線部を和訳せよ．ただし，数学記号および人名はそのまま訳文に用いても良い．

In mathematics, as in physics, in order to treat phenomena in a given (affine) space, one is naturally led to computation in the dual space. <u>One way, the most commonly used in analysis, is the Fourier transform. This transform, far from being of a local nature, is not easily adapted to calculus on manifolds. By contrast, Sato's method is perfectly suited to this case; you can complexify a real analytic manifold, and, instead of looking at the behavior at infinity of the Fourier transform, you look "where the boundary values come from." In technical terms, one regards the cotangent bundle (more precisely, $\sqrt{-1}$-times the cotangent bundle) as the conormal bundle to the real space in the complex space.</u> This was how Sato defined the analytic wave front set of a hyperfunction (in particular, of a distribution), a closed conic subset of the cotangent bundle, and he showed that if a hyperfunction u is a solution of the equation $Pu = 0$, then its wave front set is contained in the real part of the characteristic variety of the operator P. This was the starting point of microlocal analysis invented by Sato—a kind of revolution in analysis.

<u>Of course, at this time other mathematicians (especially L. Hörmander) and physicists (e.g., D. Iagolnitzer) had the intuition that the cotangent space was the natural space for analysis, and in fact this intuition had arisen much earlier. Indeed, pseudo-differential operators had existed before the wave front set appeared.</u> But Sato was the first to make the objects of analysis, such as distributions, live in the cotangent space, and for that purpose he constructed a key tool of sheaf theory, the microlocalization functor, that is, the "Fourier-Sato" transform of the specialization functor. This was also the origin of the microlocal theory of sheaves. In 1973 Sato and his two students, M. Kashiwara and T. Kawai, published a treatise on the microlocal analysis of PDE. Certainly this work had a considerable impact, although most analysts did not understand a single word. Hörmander and his school then adapted the classical Fourier transform to these new ideas, leading to the now popular theory of Fourier-integral operators.

[注] Fourier:（人名），Sato:（人名），conormal bundle: 余法束，Hörmander:（人名），Iagolnitzer:（人名），pseudo-differential operator: 擬微分作用素，wave front set: 波面集合．

[出典] Pierre Schapira, Mikio Sato, a Visionary of Mathematics. Notices of the American Mathematical Society 54 (2007), no. 2, p. 244（一部改変）．

(2) 次の英文を和訳せよ．ただし，数学記号および人名はそのまま訳文に用いても良い．

A *lattice* in $\mathbf{R}^n$ is an additive subgroup $L \subset \mathbf{R}^n$ which is additively generated by some basis $b_1, \ldots, b_n$ for the real vector space $\mathbf{R}^n$.

Choosing some basis $b_1, \ldots, b_n$ for L, we can form the fundamental domain P consisting of all $\xi_1 b_1 + \cdots + \xi_n b_n$ with $0 \leq \xi_i < 1$. Clearly every point of $\mathbf{R}^n$ is congruent modulo L to one and only one point of P. The volume (or Lebesgue measure)

$$\mathrm{vol}(P) = \int_P dx_1 \cdots dx_n$$

can be identified with the volume of the quotient torus $\mathbf{R}^n/L$. This volume is of course equal to the absolute value of the determinant of the matrix whose columns are $b_1, \ldots, b_n$. We write this briefly as

$$\mathrm{vol}(\mathbf{R}^n/L) = |\det(b_1, \ldots, b_n)|.$$

[注] Lebesgue:（人名）.

[出典] John Milnor and Dale Husemoller, Symmetric Bilinear Forms. Ergebnisse der Mathematik und ihrer Grenzgebiete, Band 73, Springer-Verlag, New York-Heidelberg, 1973, pp. 15–16（一部改変）.

E 第 2 問

次の和文を英訳せよ．ただし，数学記号はそのまま訳文に用いても良い．

定義　内積空間 V の 2 元 x, y が $\langle x, y\rangle = 0$ をみたすとき，x, y は互いに直交するという．

とくに $\mathbf{R}$ 上の内積空間の場合，$\langle x, y\rangle \in \mathbf{R}$ だから，$x \neq O,\, y \neq O$ に対して $-1 \leq \dfrac{\langle x, y\rangle}{||x||\,||y||} \leq 1$（Schwarz の不等式）であり，したがって $\cos\theta = \dfrac{\langle x, y\rangle}{||x||\,||y||}$ となる実数 $0 \leq \theta \leq \pi$ が定まる．この θ をベクトル x, y 間の角という．$x \neq O,\, y \neq O$ のとき，$\langle x, y\rangle = 0 \Longleftrightarrow \theta = \dfrac{\pi}{2}$.

命題　n 次元内積空間 V の有限個の O ではないベクトル $x_1, \ldots, x_k \in V$ が互いに直交する（$i \neq j$ のとき $\langle x_i, x_j\rangle = 0$）ならば，$x_1, \ldots, x_k$ は線形独立．とくに $k \leq n$ であり，$k = n$ ならば $x_1, \ldots, x_k$ は V の基底をなす．

[証明]　$\lambda_1 x_1 + \cdots + \lambda_k x_k = O\,(\lambda_i \in \mathbf{C})$ とする．各 x_j に対して $0 = \langle O, x_j\rangle = \lambda_1\langle x_1, x_j\rangle + \cdots + \lambda_k\langle x_k, x_j\rangle = \lambda_j ||x_j||^2$．仮定により $x_j \neq O$ だから $||x_j|| \neq 0$．したがって，$\lambda_j = 0$（$1 \leq j \leq k$）．ゆえに，$x_1, \ldots, x_k$ は線形独立である．　$\square$

[注]（一つの）内積空間 : (an) inner product space.

[出典] 岩堀長慶編『線形代数学』，裳華房，第 13 版（1996）, p. 191 （一部改変）.