受験番号＿＿＿＿＿＿＿＿

令和７（2025）年度　東京大学大学院

数理科学研究科　数理科学専攻　修士課程

英　語　（筆記試験）

令和６（2024）年８月２６日（月）
１０：２０　～　１１：４０

問題は全部で２題ある．２題とも解答すること．

（１）　解答しようとする各問ごとに解答用紙を１枚使用すること．
試験開始後，各解答用紙の所定欄に各自の**氏名**，**受験番号**と解答する**問題の番号**を記入すること．

（２）　試験開始後，この問題冊子の表紙の上部の受験番号欄に各自の**受験番号**を明記すること．ただし氏名を記入してはならない．

（３）　試験開始後，草稿用紙の上部に各自の**受験番号**を明記すること．ただし氏名を記入してはならない．

（４）　試験終了後に提出するものは，１題につき１枚，計**２枚の答案**，**１枚の草稿用紙**である．
着手した問題数が２題にみたない場合でも，氏名と受験番号のみを記入した白紙の答案を補い，２枚とすること．
指示に反したもの，**答案が２枚でないものは無効**とする．
問題冊子は回収する．

（５）　解答用紙の裏面を使用する場合は，表面右下に「裏面使用」と明記すること．

E 第 1 問

(1) 次の英文を和訳せよ．ただし，数学記号はそのまま訳文に用いても良い．

Definition. An integer greater than one whose only positive divisors are itself and one is called a prime number. An integer greater than one which is not a prime number is said to be composite.

All sorts of questions immediately spring to mind. How many primes are there? The answer is infinitely many. This means that there is no last prime. Or, alternatively, it can be thought of as meaning that there are more than 1 million primes, more than 1 billion primes, more than 1 trillion primes, in fact, more primes than any number that you care to name. This fact was known by Euclid over 2000 years ago, and his proof will be given shortly. What is the nth prime? For any given n, this question can always be answered in a finite amount of time. For example, the 664 999th prime is 10 006 721. But in the sense of giving a formula which yields the nth prime for all n, this has never been done. Is there a formula which at least gives only primes? No one has ever found one.

(中略)

Fermat (1601–1665) conjectured that the numbers

$$F_n = 2^{2^n} + 1$$

are primes for all integers $n \geq 0$. He checked this for $n = 0, 1, 2, 3, 4$ and found that the corresponding F_n's, 3, 5, 17, 257, and 65 537, are indeed primes. Since

$$F_5 = 4\,294\,967\,297,$$

Fermat did not attempt to verify his conjecture any further. Fermat undoubtedly had good reasons for believing his conjecture, nevertheless, he was wrong. Euler (1707–1783) found in 1732 that $641 | F_5$ and hence F_5 is composite.

[注] Euclid: ユークリッド (人名), Fermat: フェルマー（人名）， Euler: オイラー (人名)

[出典] Harold M. Stark, An Introduction to Number Theory, The MIT Press, Cambridge, Massachusetts, and London, England, 1978, p. 2 (一部改変).

(2) 次の英文の下線部を和訳せよ．

Also on the practical side, we will study results that belong to the domain of "Euclidean geometry" although they do not appear in Euclid's *Elements.* Some of these were discovered long ago, such as the fact that the three altitudes of a triangle meet in a point, which was known to Archimedes, while others were found more recently, such as the Euler line and the nine-point circle associated to a triangle. The technique of circular inversion, which became popular in the second quarter of the nineteenth century, provides an example of the modern transformational approach to geometry, and gives a convenient tool for the solution of classical problems such as the problem of Apollonius: to find a circle tangent to three given circles.

Finally, the investigation of the role of the parallel postulate has led to some of the most important developments arising out of Euclid's geometry. Already from the time of Euclid onward, commentators noted that this postulate was less elementary than the others, and they questioned whether it might not be a consequence of the other postulates. Two millennia of efforts to prove the parallel postulate by showing that its negation led to absurd (but not contradictory) results were considered failures until, in the mid-nineteenth century, a brilliant shift of perspective, with lasting consequences for the history of mathematics, admitted that these "absurd" conclusions were merely the first theorems in a new, strange, but otherwise consistent geometry. Thus were born the various non-Euclidean geometries that have been so valuable in the modern theory of topological manifolds, and in the development of Einstein's theory of relativity, to mention just two applications.

[注] parallel postulate: 平行線公準，Euclid: ユークリッド（人名），
Einstein: アインシュタイン（人名），theory of relativity: 相対性理論．

[出典] Robin Hartshorne, *Geometry: Euclid and beyond*, Undergraduate Texts in Mathematics, Springer-Verlag, New York. 2000, p. 5 (一部改変).

E 第 2 問

次の和文を英訳せよ．ただし，数学記号はそのまま訳文に用いても良い．

$\{f_n(z)\}$ を複素変数 z の関数 $f_1(z), f_2(z), \cdots, f_n(z), \cdots$ を項とする関数列とし，各関数 $f_n(z)$ の定義域はすべて同じであるとして，D をその定義域の部分集合とする．D に属するすべての点 z において複素数列 $\{f_n(z)\}$ が収束するとき関数列 $\{f_n(z)\}$ は D で収束するという．このとき $f(z) = \lim_{n\to\infty} f_n(z)$ はもちろん D で定義された関数で，$n \to \infty$ のとき $|f_n(z) - f(z)|$ は 0 に収束する．もしもこの収束が D で一様ならば関数列 $\{f_n(z)\}$ は D で一様に $f(z)$ に収束するという．すなわち:

定義　任意の正の実数 ε に対応して一つの自然数 $n_0(\varepsilon)$ が定まって，D に属するすべての点 z において

$$n > n_0(\varepsilon) \quad \text{ならば} \quad |f_n(z) - f(z)| < \varepsilon$$

となるとき，関数列 $\{f_n(z)\}$ は D で一様に $f(z)$ に収束するという．

[出典] 小平邦彦『複素解析 I』岩波講座 基礎数学，岩波書店 (1977)，p. 17 (一部改変).