

## NAME

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Neal BEZ

## RESEARCH FIELD

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Functional Analysis, Real Analysis, Harmonic Analysis

## KEYWORDS

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Fourier restriction conjecture, Kakeya conjecture, geometric inequalities

## PRESENT RESEARCH

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At the heart of modern euclidean harmonic analysis is the Fourier restriction conjecture. This conjecture arose from key insight of Elias Stein in the late 1960s and says, roughly speaking, that for certain null sets  $S \subset \mathbb{R}^n$ ,  $n \geq 2$ , one can give meaning to the restriction of the Fourier transform of  $f \in L^p(\mathbb{R}^n)$  to  $S$  for a certain range of  $p$ .

The Fourier restriction conjecture enjoys deep connections to a number of other areas of mathematics. One example of this is the connection with the Kakeya conjecture from geometric measure theory. We say  $K \subset \mathbb{R}^n$  is a Kakeya set if it contains a unit line segment in every direction. Curiously, Abram Besicovitch showed in 1920 that one can construct planar Kakeya sets which are null. Despite this surprising phenomena, it is expected that Kakeya sets should be "big" in some sense and the Kakeya conjecture predicts that the fractal dimension of *any* Kakeya set in  $\mathbb{R}^n$  is equal to  $n$  (i.e. as big as it can be). Remarkably, it is known that if the Fourier restriction conjecture is true, then the Kakeya conjecture must also be true.

Certain multilinear versions of the conjectures have emerged and work of Bennett, Carbery and Tao (Acta Math. 2006) in particular has had a profound influence. Multilinear Kakeya estimates may be interpreted as a perturbation of the Brascamp–Lieb (BL) inequality. The BL inequality is a wide-reaching geometric inequality which can be seen as a generalization of Young's convolution inequality and the Loomis–Whitney inequality. In addition to contributing significantly to recent progress on the restriction/Kakeya conjectures, the BL inequality has rich connections with many other fields, such as convex geometry, information theory, optimization theory, and theoretical computer science.

My research has predominantly focused on topics which are connected at some level to the above. Most recently, various problems related to the BL inequality have particularly caught my attention.

## NOTICE FOR THE STUDENTS

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If you have enjoyed and become proficient in subjects like functional analysis, measure theory and Fourier analysis, then it is likely that you will enjoy working on research problems related to my work.