Name: Asuka TAKATSU

Research field: Differential Geometry

Key words: Geometric analysis, Optimal transport theory, Convexity.

Present research:

My research objects are mainly Riemannian manifolds and metric measure spaces. I analyze how they are curved using an optimal transport theory. Here the optimal transport theory is a theory to find a minimum energy transport.

Let me explain this a bit more: there are two balls of equal radius on a metric measure space. We distribute matter uniformly on one ball and transport it to the other ball with minimum kinetic energy. If the radius is infinitesimally small, then the transport is almost a point-to-point transport hence a minimum energy transport is performed along a shortest path. Similarly, if we regard each ball as a collection of points, then a minimum energy transport between balls can be done along shortest paths. For example, in the case of Euclidean space, a minimum energy transport between balls is the parallel transport, where the shape of the ball is preserved. On the other hand, if the space is curved, then the shape that appears during a minimum energy transport is changed, where the difference can be estimated in terms of the lower *Ricci curvature* bound. This estimate is due to an inequality called the *Brunn–Minkowski inequality*, which expresses the convexity of the volume as a function.

It is also known that the contraction rate of the heat flow coincides with the lower Ricci curvature bound. This is related to the convexity of the *relative entropy*, which is a functional on the space of Borel probability measures.

Although these facts were proved in the 21st century, the optimal transport theory itself has a long history since the end of the 18th century and has recently been widely applied not only to geometric analysis but also to machine learning. Then I would like to do research on interesting things without getting too attached to the research area.

Notice for the students:

Needless to say, the more basic knowledge you have, the better. However it is impossible to know everything. Therefore I would like students who wish to work with me to have the courage to honestly say that they do not know when they come across something unknown, and to have the drive to know it.