講義題目 Representation theory of the general linear groups after Riche and Williamson

授業の概要 This will be a survey of a recent monumental work by Simon Riche and Geordie Williamson [RW] on the representation theory of reductive algebraic groups over a field $\Bbbk$ of positive characteristic p. By the limitation of the lecture's capacity we will focus only on Parts 1 and 2 of [RW] dealing with the general linear groups.

So let $G = \mathrm{GL}_n(\Bbbk)$. We consider only algebraic representations over $\Bbbk$, called rational representations in Jantzen's book [J], of G. It is interesting not only in its own right but also for applications in the representation theory of the symmetric group $\mathfrak{S}_n$.

A central problem in the representation theory of G has been the determination of the irreducible characters of G. In the late 70's Lusztig formulated a conjecture for the irreducible characters of G in terms of Kazhdan-Lusztig polynomials associated to the affine 岩堀-Hecke algebra of G. It was long believed to be true for $p \geq 2n-3$ or even for $p > n$ until very recently Williamson had come up with counterexamples; it was proved by Andersen, Jantzen and Soegel for indefinitely large p resorting to the solution by Kazhdan and Lusztig of the corresponding conjecture for the quantum algebra at a p-th root of unity, which in turn relied on the solution by 柏原 and 谷崎 of the corresponding conjecture for the affine Lie algebra, see, e.g., [J08]. Subsequently, Fiebig gave an explit huge bound for p above which the conjecture holds.

The present work [RW] reformulates Lusztig's conjecture for character formulae for the indecomposable tilting modules in the principal block, a subcategory of all the representations of G, and replacing Kazhdan-Lusztig polynomials by p-Kazhdan-Lusztig polynomials, p-KL polynomials for short, and proves the formulae for $p > n$. The p-KL polynomials are associated to a diagrammatic category $\mathcal{D}$ introduced by Elias and Williamson, on which 阿部紀行 lectured last year. Their proof is given by defining an action of $\mathcal{D}$ on the principal block of G in terms of wall-crossing translation functors, and then establishing the correspondence of the indecomposable tilting modules in the principal block and the indecomposable objects in the antispherical quotient of $\mathcal{D}$. The action is a categorification of an action of the associated affine Lie algebra over $\mathbb{C}$ on the complexification of the Grothendieck group of the principal block. In order to change the base fields from $\mathbb{C}$ to $\Bbbk$, 2-categorification of the action by Khovanov and Lauda, Rouquier is exploited. Finally, the irreducible characters are deduced from the characters of the indecomposable tilting modules for $p \geq 2(n-1)$. The last bound is now improved by Sobaje to $p > n$.

Prerequisites: Basic results on the representation theory of G were covered by 阿部's lectures last year. For lots of category theory used in the work I found [中岡] very helpful. I will make a set of the class notes on the subject delivered during the last semester at Osaka City Univ. available.

[**J**] Jantzen, J. C., Representations of Algebraic Groups, Math. Surveys and Monographs **107**, 2003 AMS

[**J08**] Jantzen, J. C., *Character formulae from Hermann Weyl to the present*, In: Groups and Analysis, LNS **354**, LMS, Cambridge UP 2008, pp. 232-270

[**中岡**] 中岡宏行, 圏論の技法, 2015 日本評論社

[**RW**] Riche, S. and Williamson, G., Tilting Modules and the p-Canonical Basis, Astérisque **397**, 2018 SMF

成績評価方法 レポート