J. Math. Sci. Univ. Tokyo
19 (2012), 309–312.

Corrigendum to "Tame-blind Extension of Morphisms of Truncated Barsotti-Tate Group Schemes"

By Yuichiro Hoshi

The author discovered an error in the discussion of Step 6 in the proof of [1], Lemma 3.3, that is applied in the proof of the main results of [1]. (The error in question is as follows: In the discussion of Step 6, the author stated that it follows from Step 4, (4-iii), that the $\mathrm{Ker}(f_G^*: M_X \to M_G)$-part of ω_1 is equal to 0. However, in general, Step 4, (4-iii), does *not* imply it.) Therefore, the author would like to *replace* [1], Theorem 3.4 [hence also [1], Theorem 0.1] (respectively, [1], Corollary 3.6) by Theorem A below, which is a *tame-blind criterion* for a homomorphism between the generic fibers of finite flat commutative group schemes to extend to a homomorphism between the original group schemes (respectively, Theorem B below). Moreover, the author would like to *withdraw* [1], Corollary 3.5 [hence also [1], Corollary 0.2]; [1], Remark 3.7; [1], Corollary 3.8 [hence also [1], Corollary 0.3].

In the remainder of the present paper, let p be a prime number, R a complete discrete valuation ring, K the field of fractions of R, $\overline{K}$ an algebraic closure of K, and $K^{\mathrm{tm}} \subseteq \overline{K}$ the maximal tamely ramified extension of K. Suppose that K is of characteristic 0, and that the residue field of R is of characteristic p. Write v_p for the p-adic valuation of K such that $v_p(p) = 1$, e_K for the absolute ramification index of K, and $\epsilon_K^{\mathrm{Fon}} \stackrel{\mathrm{def}}{=} 2 + v_p(e_K)$ (cf. [1], Definition 2.4).

Theorem A (Tame-blind criterion for a homomorphism between the generic fibers to extend to a homomorphism between the original group schemes). *Let G be a truncated p-Barsotti-Tate group scheme over R (cf. [1], Definition 2.12), H a finite flat commutative group scheme over R, and $f_K: G_K \stackrel{\mathrm{def}}{=} G \otimes_R K \to H_K \stackrel{\mathrm{def}}{=} H \otimes_R K$ a homomorphism of group schemes over K. Write $X \subseteq G \times_R H$ for the scheme-theoretic image of the composite*

$$G_K \xrightarrow{(\mathrm{id}, f_K)} G_K \times_K H_K \xrightarrow{\subseteq} G \times_R H$$

2010 *Mathematics Subject Classification.* Primary 14L15; Secondary 11S15, 14L05.

(*thus, one verifies easily that the structure of group scheme of* $G \times_R H$ *determines a natural structure of* [*necessarily finite flat commutative*] *group scheme of* X) *and* X^D *for the Cartier dual of* X *over* R (*cf. the discussion entitled "Group schemes" in* [1], §0). *Suppose that* G *is of level* (*cf.* [1], *Definition 2.1,* (ii)*;* [1]*, Remark 2.13,* (i)) $\geq 3\epsilon_K^{\mathrm{Fon}}$. *Then the following conditions are equivalent:*

(i) *The homomorphism* f_K *uniquely extends to a homomorphism of group schemes* $G \to H$ *over* R.

(ii) *The* R*-valued cotangent space* $t^*_{X^D}(R)$ *of* X^D (*cf. the discussion entitled "Group schemes" in* [1], §0) *has no* $\epsilon_K^{\mathrm{Fon}}$*-primitive element* (*cf.* [1]*, Definition 2.8,* (ii))*, i.e., for any* $\omega \in t^*_{X^D}(R)$*, if* $p^{\epsilon_K^{\mathrm{Fon}}}\omega = 0$*, then* $\omega \in p \cdot t^*_{X^D}(R)$.

Proof. One verifies easily that condition (i) is equvalent to condition (i′): *The composite* $X \hookrightarrow G \times_R H \stackrel{\mathrm{pr}_1}{\to} G$ *is an isomorphism.* Moreover, since G is of level $\geq 3\epsilon_K^{\mathrm{Fon}}$, the implication (i′) ⇒ (ii) follows immediately from [1], Lemma 2.15, together with [1], Remark 2.13, (ii). Thus, it remains to verify the implication (ii) ⇒ (i′). To this end, suppose that condition (ii) is satisfied. Let R' be a complete discrete valuation ring which is *faithfully flat* over R such that its residue field is *perfect*, and, moreover, its absolute ramification index is equal to e_K (cf. the second paragraph of the proof of [1], Theorem 3.4). Then one verifies easily that the scheme-theoretic image of the base-change of the displayed composite in the statement of Theorem A by $R \hookrightarrow R'$ is naturally isomorphic to $X \otimes_R R'$; moreover, the composite $X \hookrightarrow G \times_R H \stackrel{\mathrm{pr}_1}{\to} G$ is an *isomorphism* if and only if the composite $X \otimes_R R' \hookrightarrow (G \times_R H) \otimes_R R' \stackrel{\mathrm{pr}_1}{\to} G \otimes_R R'$ is an *isomorphism.* Now let us observe that since there exists a natural isomorphism of R'-modules $t^*_{X^D}(R) \otimes_R R' \stackrel{\sim}{\to} t^*_{X^D \otimes_R R'}(R')$, it follows from condition (ii) that $t^*_{X^D \otimes_R R'}(R')$ has no $\epsilon_K^{\mathrm{Fon}}$-primitive element. Thus, since there exists a natural isomorphism $(X \otimes_R R')^D \stackrel{\sim}{\to} X^D \otimes_R R'$ over R', to verify the implication (ii) ⇒ (i′), by replacing R by R', we may assume without loss of generality that the residue field of R is *perfect.* Then since $t^*_{X^D}(R)$ has no $\epsilon_K^{\mathrm{Fon}}$-primitive element (cf. condition (ii)), and G, hence also G^D (cf. [1], Remark 2.13, (ii)), is *truncated p-Barsotti-Tate*, it follows immediately from [1], Lemma 2.11, together with [1], Lemma 2.15, that the Cartier dual $G^D \to X^D$ of the composite of condition (i′), hence also the composite of condition (i′) itself, is an *isomorphism*. This completes the proof of the implication (ii) ⇒ (i′), hence also of Theorem A. □

Theorem B (Points of truncated Barsotti-Tate group schemes). *Let G be a truncated p-Barsotti-Tate group scheme over R (cf.* [1]*, Definition 2.12). Suppose that G is of level (cf.* [1]*, Definition 2.1,* (ii)*;* [1]*, Remark 2.13,* (i)) $\geq 3\epsilon_K^{\mathrm{Fon}}$*. Then G is étale over R if and only if $G(K^{\mathrm{tm}}) = G(\overline{K})$.*

Proof. *Necessity* is immediate. Thus, it remains to verify *sufficiency*. Now let us observe that it follows from a similar argument to the argument used in the proof of Theorem A concerning "R'" that, to verify *sufficiency*, we may assume without loss of generality that the residue field of R is *perfect*. Moreover, it follows immediately from the definition of "$\epsilon_K^{\mathrm{Fon}}$" that, to verify *sufficiency*, by replacing R by the normalization of R in a suitably *tamely ramified* finite extension of K, we may assume without loss of generality that $G(K) = G(\overline{K})$. Then since $G(K) = G(\overline{K})$, and G is *finite* over R, one verifies easily that there exist a finite *étale* commutative group scheme H over R and a homomorphism of group schemes $H \to G$ over R which induces an *isomorphism* between their generic fibers. On the other hand, since H is *étale* over R, one verifies easily that $t_H^*(R) = \{0\}$ (cf. the discussion entitled "Group schemes" in [1], §0), hence also that $d_H^\circ = 0$ (cf. [1], Definition 2.8, (i)). Thus, it follows from [1], Lemma 2.10, (ii), together with our assumption that G is of level $\geq 3\epsilon_K^{\mathrm{Fon}}$, that the existence of such a homomorphism $H \to G$ implies that $d_G^\circ = 0$. In particular, since G is *truncated p-Barsotti-Tate* and of level $\geq 3\epsilon_K^{\mathrm{Fon}}$, it follows immediately from [1], Lemma 2.15, that $t_G^*(R) = \{0\}$, i.e., G is *étale* over R. This completes the proof of *sufficiency*, hence also of Theorem B. □

References

[1] Hoshi, Y., Tame-blind extension of morphisms of truncated Barsotti-Tate group schemes, J. Math. Sci. Univ. Tokyo **16** (2009), no. 1, 23–54.

(Received January 26, 2012)
(Revised June 7, 2012)

Research Institute for Mathematical Sciences
Kyoto University
Kyoto 606-8502, JAPAN
E-mail: yuichiro@kurims.kyoto-u.ac.jp