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Algebraic Property of Dilatation Constants of Piecewise Linear Structures of Anosov Foliations

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Abstract. Let ϕ_t be an orientable Anosov flow of a closed 3-manifold and $\mathcal{F}^s$, the stable foliation of the flow. If $\mathcal{F}^s$ has a λ-piecewise linear structure, then we show that it is equivalent to the one obtained by using a surface of section S of the flow. Then we prove, for a positive integer p, λ^p is equal to the dilatation of the first return mapping of S which is a pseudo-Anosov diffeomorphism of a compact surface with boundary. Therefore, λ is a zero of a monic reciprocal polynomial with integral coefficients. In particular, λ is not transcendental and this gives a negative answer to the question raised in [10]. We also comment on the Ghys inequality for the group $\mathrm{PL}_\lambda(S^1)$ of all orientation preserving piecewise linear homeomorphisms whose derivatives are integral powers of λ at each differentiable point.

§0. Introduction

Let Σ_g be a closed orientable surface of genus $g \geq 2$ equipped with a metric with the constant curvature -1. Since the universal covering of Σ_g is the Poincaré disk, there is a holonomy homomorphism

$$\pi_1(\Sigma_g) \to PSL(2, \mathbf{R}).$$

By identifying the circle at infinity of the Poincaré disk with $S^1 = \mathbf{R}/\mathbf{Z}$, it induces a homomorphism

$$\Phi_g : \pi_1(\Sigma_g) \to PSL(2, \mathbf{R}) \subset \mathrm{Diff}^\infty_+(S^1).$$

For any $\lambda > 1$, let $PL_\lambda(S^1)$ be the group of all orientation preserving piecewise linear homeomorphisms h of S^1 such that $h'(x) \in \{\lambda^n \mid n \in \mathbf{Z}\}$ at every differentiable point. If there exists $\lambda > 1$ and an orientation preserving homeomorphism h of S^1 such that, for any $\gamma \in \pi_1(\Sigma_g)$,

$$h \circ \Phi_g(\gamma) \circ h^{-1} \in PL_\lambda(S^1),$$

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then Φ_g is said to be λ-*piecewise linearizable*. For the following numbers $\lambda = \lambda_g, \nu_g$, Φ_g is λ-piecewise linearizable.
1)([10],[13]) λ_g is the largest eigen value of the matrix A_g, where

$$A_g = \begin{pmatrix} 2g^2-1 & 2g^2+2g \\ 2g^2-2g & 2g^2-1 \end{pmatrix}.$$

2)([4]) ν_g is the largest eigen value of the matrix B_g, where

$$B_g = \begin{pmatrix} 4g^2-2g-1 & 2g^2-2g \\ 8g^2-2 & 4g^2-2g-1 \end{pmatrix}.$$

All these examples are constructed by using a surface of section (also called a Birkhoff section) S of a geodesic flow on unit tangent bundles $T_1\Sigma_g$. Indeed, a surface of section S gives rise to a cross section $\hat{S}$ of the blow-up flow of the geodesic flow along a finite number of closed orbits which are contained in the boundary of the surface of section([8],[9]). Then the first return map f of $\hat{S}$ is a pseudo-Anosov diffeomorphism of compact surface $\hat{S}$ with boundary([7]). Then, for the dilatation λ of f, the stable foliation of the geodesic flow has a transversely λ-piecewise linear structure (see §1). If a λ-piecewise linearization of Φ_g is obtained in such a way, then the number λ must be algebraic. Then we have the following natural question which was first raised in [10].

QUESTION. Is there a transcendental number λ such that Φ_g is λ-piecewise linearizable for some integer $g \geq 2$?

In this paper, we prove the following theorem which gives the negative answer to Question. A polynomial $P(x)$ of degree n is called *reciprocal* if $x^n P(\frac{1}{x}) = P(x)$.

THEOREM 0.1. *Let Φ_g ($g \geq 2$) be as above. If Φ_g is λ-piecewise linearizable, then there exists a monic reciprocal polynomial $P(x)$ with integral coefficients such that λ is a zero of $P(x)$. In particular, for any transcendental or rational number λ, Φ_g is not λ-piecewise linearizable.*

This theorem is a special case of the following theorem, because the geodesic flow of a hyperbolic closed surface is an orientable transitive Anosov flow(see §1). An Anosov flow ϕ_t of a closed 3-manifold M is called *orientable* if M, the stable foliation $\mathcal{F}^s$, and the unstable foliation $\mathcal{F}^u$ are all orientable. The flow ϕ_t is *transitive* if every leaf of the stable foliation $\mathcal{F}^s$ is dense in M. It is known that every leaf of $\mathcal{F}^s$ is dense in M if it has a transversely piecewise linear structure (see [18]). Then the flow ϕ_t is transitive in that case.

THEOREM 0.2. *Let ϕ_t be an orientable Anosov flow of a closed 3-manifold with the stable foliation $\mathcal{F}^s$. If $\mathcal{F}^s$ has a transversely λ-piecewise linear structure, λ is a zero of a monic reciprocal polynomial $P(x)$ with integral coefficients.*

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§1. A Blowup Flow of an Anosov Flow

A smooth flow ϕ_t of a closed 3-manifold M is *Anosov* if there exists a continuous splitting $TM = E^0 \oplus E^s \oplus E^u$ of the tangent bundle TM of M into $D\phi_t$ invariant one dimensional subbundles with the following properties. Given a Riemannian metric, there are constants $a \geq 1$ and $b > 0$ such that
1) E^0 is tangent to the flow,
2) $\|D\phi_t(v)\| \leq ae^{-bt}\|v\|$ for any $v \in E^s$, $t \geq 0$, and
3) $\|D\phi_{-t}(v)\| \leq ae^{-bt}\|v\|$ for any $v \in E^u$, $t \geq 0$.
Both subbundles $E^0 \oplus E^s$ and $E^0 \oplus E^u$ are integrable, producing codimension one foliations $\mathcal{F}^s$ and $\mathcal{F}^u$ which are called a *stable foliation* and an *unstable foliation* of the flow ([1]). An Anosov flow is called *orientable* if M, E^0, E^s,and E^u are all orientable.

Let ϕ_t be a smooth orientable Anosov flow of a closed 3-manifold M and $\mathcal{F}^s$ (resp. $\mathcal{F}^u$) the stable (unstable) foliation of ϕ_t. Let γ be a closed orbit of ϕ_t. A new flow $\hat{\phi}_t : \hat{M} \to \hat{M}$ is defind as follows, which is called the *blowup flow* of ϕ_t along γ ([6]). A compact manifold $\hat{M}$ with boundary is obtained by replacing each point $x \in \gamma$ by the circle of normal directions

$$\{T_xM/T_x\gamma - \{0\}\}/\mathbf{R}_{>0} \cong S^1.$$

Then γ is replaced by a torus T_γ. The flow $\hat{\phi}_t$ is conjugate to ϕ_t on $M - \gamma$ and defined by $D\phi_t$ on T_γ. On T_γ, $\hat{\phi}_t$ has four closed orbits, 2 attracting and 2 repelling. A foliation $\hat{\mathcal{F}}^{\epsilon}$ $(\epsilon = s, u)$ is defined by $\hat{\mathcal{F}}^{\epsilon} = \mathcal{F}^{\epsilon}$ on $M - \gamma$ and T_{γ} is contained in a singular leaf of $\hat{\mathcal{F}}^{\epsilon}$. The flow $\hat{\phi}_t$ is an example of a pseudo-Anosov flow of a compact 3-manifold with boundary and has a Markov partition. When $\hat{\phi}_t$ has a cross section S, a first cohomology class u_S is defined as follows. Suppose an oriented loop c intersects S transversely. At any intersection point p, define $\delta_p = 1$ (resp. -1) if c passes through S at p in the same (resp. opposite) direction of the flow, and put $u_S(c) = \sum_{p \in c \cap S} \delta_p$. Then u_S determines a first cohomology class of $\hat{M}$ denoted by the same symbol. Then we have the following theorem due to D.Fried ([8]).

THEOREM 1.1. *For a cohomology class* $u \in H^1(\hat{M}, \boldsymbol{Z})$*, there is a cross section* S *of* $\hat{\phi}_t$ *such that* $u = u_S$*, if and only if* $u([c]) > 0$ *for any closed orbit* c*, where* $[c]$ *is the first homology class represented by the closed orbit* c *oriented in the flow direction.*

We note that this theorem is true for any blow-up flow $\hat{\phi}_t$ of any orientable transitive Anosov flow ϕ_t, because such flow ϕ_t has a surface of section ([9]).

When $\hat{\phi}_t$ has a cross section S, the first return mapping f of S is a pseudo-Anosov diffeomorphism of the compact surface with boundary ([7]). That is, there exist measured foliations (F^s, μ^s) (F^u, μ^u) with common prong singularities and a real number $\lambda > 1$ such that
1) F^{ϵ} is preserved by f for $\epsilon = s, u$,
2) F^s is transverse to F^u outside the singularities and the boundary of S,
3) $f_*\mu^s = \lambda^{-1}\mu^s$, $f_*\mu^u = \lambda\mu^u$.
Here, $f_*\mu^{\epsilon}(A) = \mu^{\epsilon}(f^{-1}(A))$ for any transverse arc A of F^{ϵ} $(\epsilon = s, u)$. In the case above, we can take $F^{\epsilon} = \hat{\mathcal{F}}^{\epsilon}|S$ $(\epsilon = s, u)$ by a standard argument constracting a Markov partition([5]). Then μ^{ϵ} gives rise to a transversely Euclidean structure of F^{ϵ}. By the condition 3) above, this structure can be extended to a transversely λ-affine structure of $\hat{\mathcal{F}}^{\epsilon}|(\hat{M} - \partial\hat{M})$ ([10]). Then we obtain a transversely λ-piecewise linear structure of $\mathcal{F}^{\epsilon}$ by recollapsing $\hat{M}$ to M. See §2 for the transverse structure of foliations of codimension one.

Note that all explained above is valid for a blowup flow of π_t along the union of a finite number of closed orbits.

§2. Transversely Piecewise Linear Foliations

An orientation preserving homeomorphism g of $\mathbf{R}$ is called *affine* if there are real numbers $a > 0$, b such that $g(x) = ax + b$ for any $x \in \mathbf{R}$ and called λ-*affine* when we can chose $a = \lambda^n$ for some $\lambda \geq 1$ and integer n. An orientation preserving homeomorphism g of $\mathbf{R}$ is called *piecewise linear* if there exists a sequence $\{x_i\}_{i=-\infty}^{\infty}$ of $\mathbf{R}$ such that
1) $x_i < x_{i+1}$ for any i,
2) $x_i \to \pm\infty$ if $i \to \pm\infty$,
3) $g|[x_i, x_{i+1}]$ is a restriction of an affine homeomorphism.
A piecewise linear homeomorphism g of $\mathbf{R}$ is called λ-*piecewise linear* if the derivative $g'(x)$ is an integral power of λ for any differentiable point x of g.

Recall that a codimension one foliation $\mathcal{F}$ on a 3-manifold M is a division of M by codimension one immersed submanifolds, called leaves which is defined by a family of local C^0 submersions $f_i : U_i \to \mathbf{R}$ such that $\{U_i\}$ is a open covering of M and that, for any $x \in U_i$, $f_i^{-1}(f_i(x))$ is contained in a leaf of $\mathcal{F}$, and a family of local homeomorphisms, called transition functions

$$g_{ij} : f_j(U_i \cap U_j) \to f_i(U_i \cap U_j)$$

such that $f_i = g_{ij} f_j$ on $U_i \cap U_j$.

A codimension one foliation is said to have a *transeversely affine* (resp. λ-*affine*, *piecewise linear*, λ-*piecewise linear*) *structure* if we can choose each transition function g_{ij} as the restrictions of an affine (resp. λ-affine, piecewise linear, λ-piecewise linear) homeomorphism of $\mathbf{R}$. A transversely 1-affine structure is usually called a *transversely Euclidean structure*. The next lemma is remarked in [18, remark 2.1].

Lemma 2.1. *Let $\mathcal{F}$ be a transversely piecewise linear foliation on a closed manifold M. Then there exists a compact subset K contained in the union of a finite number of leaves such that $\mathcal{F}|(M - K)$ is a transeversely affine foliation.*

The next theorem is the most important to prove Theorem 0.2.

Theorem 2.2. *Let ϕ_t be an Anosov flow with the weak stable foliation $\mathcal{F}^s$. If $\mathcal{F}^s$ has a transversely piecewise (resp. λ-piecewise) linear structure, then there exist a finite munber of closed orbits $\gamma_1, \cdots, \gamma_n$ such that the* *restriction* $\mathcal{F}^s|(M-\Gamma)$ ($\Gamma = \cup_{i=1}^n \gamma_i$) *has a transversely affine (resp.* λ-*affine) structure.*

PROOF. We prove in a piecewise linear case, because the same proof is valid for a λ- piecewise linear case. For a transversely piecewise linear structure of $\mathcal{F}$, take a compact set K as in Lemma 2.1 and fix it. Since we consider in the orientable category, then every leaf of $\mathcal{F}$ is homeomorphic to $\mathbf{R}^2$ or $S^1 \times \mathbf{R}$. Suppse $L_1, \cdots, L_n$ are the leaves such that $L_i \cap K \neq \emptyset$ for any i and $K \subset \cup_{i=1}^n L_i$. If L_i is homeomorphic to $\mathbf{R}^2$, then take an embedded compact 2-disk K_i in L such that $L_i \cap K \subset \mathrm{int} K_i$. If L_i is homeomorphic to $S^1 \times \mathbf{R}$, then take an embedded compact annulus K_i in L_i such that $L_i \cap K \subset \mathrm{int} K_i$ and that the unique closed orbit of ϕ_t in L_i is contained in $\mathrm{int} K_i$. Put $K' = \cup_{i=1}^n K_i$ and let $\{\gamma_1, \cdots, \gamma_n\}$ be the set of closed orbits of ϕ_t contained in K'. Then $\mathcal{F}^s|M-K'$ also has a transversely affine structure. That is, there is a C^0 devoloping submersion

$$\mathcal{D} : \widetilde{M-K'} \to \mathbf{R}$$

and a holonomy homomorphism

$$h : \pi_1(M-K') \to \mathrm{Aff}_+(\mathbf{R})$$

satisfying the equivariant condition

$$\mathcal{D}(\gamma x) = h(\gamma)(\mathcal{D}(x))$$

for any $\gamma \in \pi_1(M-K')$ and any $x \in \mathbf{R}$ ([14]). Here, the symbol tilder means a universal covering space and $\mathrm{Aff}_+(\mathbf{R})$ denotes the group of all affine homeomorphisms of $\mathbf{R}$. It is well known that to give a transversely affine structure is equivalent to to give such pair $(\mathcal{D}, h)$ as above. Since the inclusion map $\iota : M-K' \to M-\Gamma$ ($\Gamma = \cup_{i=1}^n \gamma_i$) induces an isomorphism $\iota_* : \pi_1(M-K') \to \pi_1(M-\Gamma)$, then $\widetilde{M-K'}$ is naturally included in $\widetilde{M-\Gamma}$. Then we easily see that there exists a natural extension

$$\mathcal{D}_\Gamma : \widetilde{M-\Gamma} \to \mathbf{R}$$

of $\mathcal{D}$ which is constant on each leaf of the lifted foliation of $\mathcal{F}^s|M-\Gamma$. Then it turns out that the pair $(\mathcal{D}_\Gamma, h \circ \iota_*^{-1})$ is equivariant. This complete the proof. □

§3. Proof of Theorem 0.2

Let ϕ_t be a smooth orientable Anosov flow on a closed 3-manifold M and $\mathcal{F}^s$ (resp. $\mathcal{F}^u$) the stable (unstable) foliation of the flow. By Theorem 2.2, there is a finite number of closed orbits $\gamma_1, \cdots, \gamma_n$ of ϕ_t such that $\mathcal{F}^s|(M-\Gamma)$ ($\Gamma = \cup_{i=1}^n \gamma_i$) has an transversely λ-affine structure. That is, there exist a developing map $\mathcal{D} : \widetilde{M-\Gamma} \to \mathbf{R}$ and a holonomy homomorphism h : $\pi_1(M-\Gamma) \to \mathrm{Aff}_\lambda(\mathbf{R})$ satisfying the equivariant condition, where $\mathrm{Aff}_\lambda(\mathbf{R})$ denotes the group of all λ-affine homeomorphisms of $\mathbf{R}$. By taking the blowup along Γ to the normal directions, we obtain a flow $\hat{\phi}_t : \hat{M} \to \hat{M}$, the stable foliation $\hat{\mathcal{F}}^s$, and the unstable foliation $\hat{\mathcal{F}}^u$ of $\hat{\phi}_t$. Here, we may assume each γ_i is replaced by a torus T_{γ_i}. Define a homomorphism $\hat{h}$: $\pi_1(\hat{M}) \to \mathrm{Aff}_\lambda(\mathbf{R})$ as to be the composition of the inverse of an isomorphism $\iota_* : \pi_1(M-\Gamma) \to \pi_1(\hat{M})$ and h, where $\iota : M-\Gamma \to \hat{M}$ is the inclusion map. Then define a homomorphism $u : \pi_1(\hat{M}) \to \mathbf{Z}$ by $u(\alpha) = \log_\lambda \hat{h}(\alpha)'(0)$ for any $\alpha \in \pi_1(\hat{M})$. For any closed orbit γ of $\hat{\phi}_t$ oriented by the flow direction, $u([\gamma])$ is positive, since the leaf holonomy of $\mathcal{F}^s$ is expanding along any closed orbit oriented in the flow direction, where $[\gamma]$ denotes the first integral homology class in $H_1(\hat{M}, \mathbf{Z})$ represented by the oriented loop γ. Then there exists a cross section $\hat{S}$ of $\hat{\phi}_t$ such that $u_{\hat{S}} = u$ by Theorem 1.1. The first return mapping $\hat{f}$ of $\hat{S}$ is a pseudo-Anosov homeomorphism of the compact surface with boundary whose invariant measured foliations are $\hat{\mathcal{F}}^s|\hat{S}$ and $\hat{\mathcal{F}}^u|\hat{S}$. Then there exist a transverse measure $\hat{\mu}^s$ of $\hat{\mathcal{F}}^s|\hat{S}$ and the dilatation $\lambda_0 > 1$ such that $\mu(\hat{f}(\tau)) = \lambda_0 \mu(\tau)$ for any transversal τ of $\hat{\mathcal{F}}^s|\hat{S}$. On one hand, since $u([c]) = 0$ for any oriented loop c in $\hat{S}$, then the transversely affine structure of $\hat{\mathcal{F}}^s$ induces a transversely Euclidean structure of $\hat{\mathcal{F}}^s|\hat{S}$. So, this transversely Euclidean structure gives us a transverse measure ν of it. Since $\hat{\mathcal{F}}^s|\hat{S}$ is uniquely ergodic([7]), then there exists a positive constant b such that $\mu = b\nu$. Now fix a small transverse arc A of $\mathcal{F}^s|(\hat{S} - \partial\hat{S})$. Then we have

$$\lambda_0 = \frac{\nu(\hat{f}(A))}{\nu(A)} = \frac{\mu(\hat{f}(A))}{\mu(A)} = \lambda^p$$

for some integer p, since ν is induced by the λ-affine structure of $\mathcal{F}^s|\hat{S}$. Now, since everything is orientable, then the dilatation of $\hat{f}$ is the maximal modulous root of the characteristic polynomial $P_{f_*}(x)$ of the induced homomorphism f_* on $H_1(S)$ ([20], [7]), where $f : S \to S$ is a pseudo-Anosov homeomorphism obtained from $\hat{f}$ by collapsing each boundary of $\hat{S}$ to a point. Then we have $P_{f_*}(\lambda^p) = P_{f_*}(\lambda_0) = 0$, and $P_{f_*}(x)$ is a monic reciprocal polynomial since the characteristic polynomial of a symplectic matrix is monic and reciprocal. Thus, $P_{f_*}(x^p)$ is the required polynomial and this completes the proof. □

§4. Ghys Inequality for $\mathbf{PL}_\lambda(S^1)$

Let Σ_g be a closed oriented hyperbolic surface of genus g. let $\mathrm{PL}_+(S^1)$ be the group of all orientation preserving piecewise linear homeomorphisms of $S^1 = \mathbf{R}/\mathbf{Z}$. Any homomorphism $\varphi : \pi_1(\Sigma_g) \to \mathrm{PL}_+(S^1)$ gives rise to an S^1-bundle E_φ over Σ_g which is often called the suspension of φ. The Euler number $eu(\varphi)$ of φ is defined to be the Euler class of the associated S^1-bundle E_φ evaluated on the fundamental class $[\Sigma_g]$. Any homomorphism satisfies the following Milnor-Wood inequality ([16], [21]):

$$|eu(\varphi)| \leq 2g - 2.$$

Conversely, for any integer m such that $|m| \leq 2g - 2$, there exists a homomorphism $\varphi : \pi_1(\Sigma_g) \to \mathrm{PL}_+(S^1)$ with $eu(\varphi) = m$ ([17]). Now let $\varphi : \pi_1(\Sigma_g) \to \mathrm{PL}_\lambda(S^1)$ with $|eu(\varphi)| = 2g - 2$ $(\lambda > 1)$. By changing the orientation of Σ_g, we may assume that $eu(\varphi) = 2 - 2g$. Any $\varphi(\pi_1(\Sigma_g))$-orbit on S^1 is dense in S^1 ([18]). Then φ is conjugate to the holonomy homomorphism Φ_g ([15]), that is, Φ_g is λ-piecewise linearizable. So, such a number λ is restricted by Theorem 0.1.

Theorem 4.1. *Suppose that $\lambda > 1$ can not be a zero of any monic reciprocal polynomial with integral coefficients. Then, for any integer $g \geq 2$ and any homomorphism $\varphi : \pi_1(\Sigma_g) \to PL_\lambda(S^1)$, the following inequality holds:*

$$|eu(\varphi)| < 2g - 2.$$

Let $\mathrm{PL}_{2,1}(S^1)$ be the group of all element f in $\mathrm{PL}_2(S^1)$ such that both $f([0])$ and all the non-differentiable points of f are containd in $\{p2^q \mid p, q \in \mathbf{Z}\}$ modulo $\mathbf{Z}$. The theorem was already proved for $\mathrm{PL}_{2,1}(S^1)$ essentially by the results in [11] and [12]. The theorem above is a large extension of it and the way of the proof is different from that of them.

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