J. Math. Sci. Univ. Tokyo
6 (1999), 453–475.

A Remark on Spot Rate Models Induced by an Equilibrium Model

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Abstract. Cox-Ingersoll-Ross presented so-called CIR spot rate model, which is explained by their equilibrium model. We set an economy model with a slight modification of their model in terms of semimartingale and show the existence of equilibrium in our model. Furthermore, we discuss interest rate under equilibrium and show the general form of spot rate dynamics induced from our equilibrium model.

1. Introduction

Cox, Ingersoll and Ross (CIR abbr.) studied pricing various assets based on the equilibrium theory in their papers [2] [3]. In particular, It is well known that they derived an equilibrium spot rate model such as

$$dr_t = (m - br_t)dt + \sigma\sqrt{r_t}dB_t.$$

This model also has several good properties (mean-reverting, nonnegative, and so on) and is useful for computing bond prices and prices of interest-rate derivative securities.

Surveying real financial scene, besides CIR model, a lot of spot rate models are considered and used practically. Most of them, except CIR model, however, have not been sufficiently explained in the framework of equilibrium theory. From the view of economics, it is considered that interest rate should be determined by equilibrium relation between prices of two different time-point consumption good. Therefore one is left with the question that all the spot rate models should give us sort of explanation for its relation to the equilibrium theory. From another point of view, some equilibrium conditions may restrict the class of the spot rate models appropriate for an economic scheme. Such a question motivates us to do this research.

1991 *Mathematics Subject Classification.* 60G46, 60G60, 90A14.

In general, since the research of CIR, it has been understood that it is possible to form a spot rate model freely to a considerable degree, that is, equilibrium models like CIR do not limit the class of admissible spot rate models, though there have been only a few papers declaring this. One aim of this research is to mention this fact by analyzing an semimartingale-based equilibrium model, a slight refinement of original CIR model or some related ones.

There have been many researches on equilibrium models under uncertainty. Duffie and Duffie et al.([4] [5] [6] [7] [8]) discussed the dynamic equilibrium model from various interests and motivates us first. Karatzas, Lehoczky and Shreve([14] [15]) also researched the dynamic equilibrium in a pure exchange setting with strong concern about solving optimal problems. Both of them skillfully use stochastic methods. In most cases pure exchange models between agents are dealt with, though, the substance of the problem is not much different from that of the cases with production technology such as original CIR model. We begin with an equilibrium model similar to CIR setting and use stochastic methods for analysis. Speaking of semimartingale-based equilibrium model, Foldes ([10] [11]) considers a quite general case including jumps and shows the existence and uniqueness of equilibrium under some conditions on the model similar to the above ones, a little different from ours. Besides, Cvitanić and Karatzas([1]), Rogers([18], [19]), Jin and Grasserman([12]) mention that the existence of a nonnegative local martingale, regarded as pricing kernel or state price, is the key to equilibrium.

In this paper, we concentrate on discussion about an equilibrium model considering "production" and "consumption" close to the original CIR, in terms of continuous semimartingale. Although we only consider the case of single production technology case, the extension to multi-technology case or the case of technology with jumps is allowed to some extent after some preliminaries. Our model is not so different from others, but the firm and the representative agent are distinguished in point of the purpose of the subject's action. The existence of equilibrium is discussed and the sufficient conditions on utility function and technology for equilibrium existing are investigated. For example, utility functions such as $u(x,t) = e^{-\rho t}\dfrac{x^\alpha}{\alpha}, \quad \alpha \in (0,1), \quad \rho > 0$ are supported.

Furthermore, we discuss spot rate processes under equilibrium. As done in general, we take notice of a positive semimartingale called state price process or pricing kernel which plays an essential role. At last the author shows lots of general spot rate models are induced from an equilibrium scheme in a very simple setting (called LOG model). Ho-Lee model, Vasicek model, CIR model and so on are included as examples. The result shown there may not seem so important, but the significance is to make clear the connection between state variable model and spot rate process model within CIR scheme, which has not been stated in the literature. The author remarks that this illustrative result is close to that of Jin and Grasserman([12]) who obtain the result that every Heath-Jarrow-Morton model is supported by the CIR production economy.

This paper is organized as follows. In Section 2, we give the definition of production-consumption equilibrium and make it obvious that the existence of a nonnegative local martingale has an important role in our setting too. Section 3 gives the sufficient conditions on utility function and technology for equilibrium existing. In section 4, the relation between equilibrium model and spot rate models is discussed, especially for diffusion case.

The author thanks Prof. S. Kusuoka for a lot of useful advice and discussions, especially on the existence of equilibrium.

2. Production - Consumption Equilibrium Model

We first consider the mathematical representation for our economy. The interpretation for each element is given in Remark 2.4 later.

We consider time set as an interval $[0,T]$ with a finite time horizon T. We denote by L the space of progressively measurable processes on $[0,T]$ and by L_+ its subset of nonnegative processes.

Let $(\Omega, \mathcal{F}, (\mathcal{F}_t)_{t\in[0,T]}, P)$ be a filtered probability space satisfying the usual conditions and $\mathcal{F}_T = \mathcal{F}$.

First we give our fundamental definitions.

Definition 2.1. The economy is the collection $\mathcal{E} = \{X; (U^i, x^i)_{i=1,\cdots,m}\}$, where X is a continuous semimartingale, $U^i : L_+ \to \mathbf{R}$ is a strictly increasing function and $x^i > 0$ for each i $(i = 1, \cdots, m,$ m is a finite positive integer).

We set $x_0 = \sum_{i=1}^{m} x^i$ and $\mathcal{E}(X) = \exp(X - \frac{1}{2}\langle X, X\rangle)$, where $\langle X, X\rangle$ is the quadratic variation process of X.

DEFINITION 2.2. A production - consumption equilibrium for the economy $\mathcal{E}$ is a collection of processes $[(S^*, \pi^*), \delta^*, (c^{i*}, \theta^{i*})_{i=1,\cdots,m}]$, where π^* is a strictly positive semimartingale, satisfying the following conditions:

(1) δ^* solves the following problem:

maximize $E[\int_0^T \pi_t^* \delta_t dt]$ subject to $\delta \in L_+$ and $\int_0^T \mathcal{E}(X)_t^{-1} \delta_t dt \leq x_0$ a.s.

(2) $S_t^* = \frac{1}{\pi_t^*} E[\int_t^T \pi_s^* \delta_s^* ds | \mathcal{F}_t] \quad t \in [0, T]$

(3) For each agent i $(i = 1, \cdots, m)$, (c^{i*}, θ^{i*}) solves the next problem:

maximize $U^i(c)$ subject to (i) $c \in L_+, \theta \in L$ and

(ii) $\theta_t \pi_t^* S_t^* \leq \int_0^t \theta_{u-}(d(\pi_u^* S_u^*) + \pi_u^* \delta_u^* du) - \int_0^t \pi_u^* c_u du + \pi_0^* x^i \quad t \in [0, T]$ a.s.

(4) $\sum_{i=1}^{m} \theta^{i*} = 1$, $\sum_{i=1}^{m} c^{i*} = \delta^*$.

REMARK 2.3. If we consider a process K (allowed to be regarded as capital stock) which satisfies the following SDE

$$dK_t = K_t dX_t - \delta_t dt, \quad K_0 = x_0,$$

the constraint inequality $\int_0^T \mathcal{E}(X)_t^{-1} \delta_t dt \leq x_0$ in the above definition (1) coincides with $K_t \geq 0$.

REMARK 2.4. We may roughly interpret the above mathematical setting from an economic point of view. For more details of terminology, for example, refer to Duffie [4] [5].

It is supposed that the market consists of one firm with one production technology and m agents who play the role of consumer and investor. The firm is characterized by its technology and real output. Each agent is characterized by their utility function and their initial good. There is a single physical good which is produced by the firm and is used for consumption or investment. The firm pays the holders of the firm's share the product as dividend according to the ratio they hold. X is thought to represent the characteristics on the return of the technology. U^i is the i-th agent's utility function $(i = 1, \cdots, m)$ and $x^i > 0$ is the amount of good which i-th agent holds at the initial time.

As for Definition 2.2 of an equilibrium, the condition (1) means that δ^* is the technology's dividend-rate process that maximizes the initial share's price.

S^* is the firm's share price process. π^* is called a **state price process** owing to the condition (2). (See Duffie [5].)

Furthermore, (c^{i*}, θ^{i*}) stands for i-th agent's optimal consumption and portfolio process. The constraint inequality of (3) means that the cost of repurchasing the share at some time should be less than the total gain acquired from trading it by the time minus the cumulative consumption.

We suppose hereafter that $m = 1$ (one agent) and a utility function U is defined by the special form called the expected additive utility.

$$U(c) = E[\int_0^T u(c_t, t)dt] \tag{2.1}$$

where $u : (0, \infty) \times [0, T] \to \mathbf{R}$ is continuous and $u(\cdot, t) : (0, \infty) \to \mathbf{R}$ is concave, strictly increasing, continuously differentiable for each t in $[0, T]$.

The hypothesis of $m = 1$ may seem a bit strange. However, the concept of representative agent whose utility function, very roughly speaking, is represented such as a suitably weighted sum of all agents' utility functions) is often considered in economics. For that reason, the hypothesis is not necessarily unreasonable.

Theorem 2.5. *Set $\psi(x, t)$ be the inverse of $u_x(x, t)$ in x.*
If there exists a strictly positive local martingale N such that

(1) $\int_0^T \mathcal{E}(X)_t^{-1} \psi(N_t \mathcal{E}(X)_t^{-1}, t)dt = x_0$, *P-a.s.*

and

(2) $\{N_{\tau_n \wedge t}\}_{t\in[0,T]}$ *is a martingale, where*

$$\tau_n = \inf\{t > 0; \int_0^t \mathcal{E}(X)_s^{-1}\psi(N_s\mathcal{E}(X)_s^{-1}, s)ds > x_0(1 - \frac{1}{n})\}, \tag{2.2}$$

then

$$[(K^*, N\mathcal{E}(X)^{-1}); \delta^*; (c^*, 1)]$$

is a production-consumption equilibrium, where

$$\delta_t^* = c_t^* = \psi(N_t\mathcal{E}(X)_t^{-1}, t),$$

$$K_t^* = \mathcal{E}(X)_t\Big(x_0 - \int_0^t \mathcal{E}(X)_s^{-1}\delta_s^* ds\Big) \quad \text{for all } t \in [0, T].$$

Before the proof, we state the well-known result on concave functions as lemma.

LEMMA 2.6. *If* $\varphi : (a, b) \to \mathbf{R}$ *is continuously differentiable and concave, then*

$$(y - x)\varphi'(y) \le \varphi(y) - \varphi(x) \le (y - x)\varphi'(x), \quad a < x, y < b$$

PROOF OF THEOREM 2.5. First, we consider the condition (1) of Definition 2.2.

Let $\mathcal{C} = \{\delta \in L_+ | \int_0^T \mathcal{E}(X)_s^{-1}\delta_s ds \le x_0\}$. We see $\delta^* \in \mathcal{C}$ from the condition (1).

It is obvious that $\tau_1 < \tau_2 < \tau_3 < \cdots,\ \tau_n \uparrow T,\ n \to \infty$. It follows from the condition (2) that for all $\delta \in \mathcal{C}$

$$\begin{aligned}
&E[\int_0^T N_t\mathcal{E}(X)_t^{-1}\delta_t dt] \\
&= \lim_{n\to\infty} E[\int_0^{\tau_n} N_t\mathcal{E}(X)_t^{-1}\delta_t dt] \\
&= \lim_{n\to\infty} E[N_{\tau_n}\int_0^{\tau_n} \mathcal{E}(X)_t^{-1}\delta_t dt] \ \le\ \lim_{n\to\infty} E[N_{\tau_n}\int_0^{T} \mathcal{E}(X)_t^{-1}\delta_t dt] \\
&\le \lim_{n\to\infty} x_0 E[N_{\tau_n}] = x_0 E[N_0].
\end{aligned}$$

The second equality follows from a stochastic version of Fubini's theorem and the property of conditional expectation. The same method is used several times hereafter without notice.

On the other hand, for δ^*, from (2.2),

$$\begin{aligned}
&E[\int_0^T N_t \mathcal{E}(X)_t^{-1} c_t^* dt] \\
&= \lim_{n\to\infty} E[N_{\tau_n} \int_0^{\tau_n} \mathcal{E}(X)_t^{-1} \delta_t^* dt] \;=\; \lim_{n\to\infty} E[N_{\tau_n} x_0 \Big(1 - \frac{1}{n}\Big)] \\
&= \lim_{n\to\infty} x_0 \Big(1 - \frac{1}{n}\Big) E[N_{\tau_n}] = x_0 E[N_0].
\end{aligned}$$

This result implies that δ^* is an optimal solution of the first problem.

Second, we show $N\mathcal{E}(X)^{-1}$ is a state price process.

$$\begin{aligned}
&E[\int_t^T N_s \mathcal{E}(X)_s^{-1} \delta_s^* ds | \mathcal{F}_t] \\
&= \lim_{n\to\infty} E[\int_t^{\tau_n} N_s \mathcal{E}(X)_s^{-1} \delta_s^* ds | \mathcal{F}_t] \\
&= \lim_{n\to\infty} E[N_{\tau_n} \int_t^{\tau_n} \mathcal{E}(X)_s^{-1} \delta_s^* ds | \mathcal{F}_t] \\
&= \lim_{n\to\infty} E[N_{\tau_n} \Big\{ x_0 \Big(1 - \frac{1}{n}\Big) - \int_0^t \mathcal{E}(X)_s^{-1} \delta_s^* ds \Big\} | \mathcal{F}_t] \\
&= K_t^* \mathcal{E}(X)_t^{-1} N_t
\end{aligned}$$

Finally we show that c^* solves the agent's problem in (3) of Definition 2.2. Now the constraint inequality is reduced and equivalent to $c \in \mathcal{C}$.

By Lemma 2.6, for any $c \in \mathcal{C}$,

$$u(c_t, t) \le u(c_t^*, t) + u_x(c_t^*, t)(c_t - c_t^*) = u(c_t^*, t) + N_t \mathcal{E}(X)_t^{-1} (c_t - c_t^*).$$

So we have

$$\begin{aligned}
&E[\int_0^T u(c_t, t) dt] \\
&\le E[\int_0^T u(c_t^*, t) dt] + E[\int_0^T N_t \mathcal{E}(X)_t^{-1} (c_t - c_t^*) dt] \\
&= E[\int_0^T u(c_t^*, t) dt] + E[\int_0^T N_t \mathcal{E}(X)_t^{-1} c_t dt] - E[\int_0^T N_t \mathcal{E}(X)_t^{-1} c_t^* dt] \\
&\le E[\int_0^T u(c_t^*, t) dt]
\end{aligned}$$

The last inequality is from the result about δ and δ^* of the first step. This proves our assertion. $\square$

We consider the simple model called "LOG model" below. In this special case, we can show the existence of production - consumption equilibrium and get the components like a state price process explicitly. We will discuss more concrete results on LOG model in Section 4.

DEFINITION 2.7. We say the model is LOG model if the function U is given by

$$U(c) = E[\int_0^T e^{-\rho t} \log c_t dt],$$

where $\rho > 0$ is a constant.

COROLLARY 2.8. *For LOG model,*

$$[(K^*, \gamma^* \mathcal{E}(X)^{-1}), \delta^*, (c^*, 1)]$$

is a production-consumption equilibrium, where

$$\gamma^* = \frac{1 - e^{-\rho T}}{\rho x_0}, \quad \delta_t^* = c_t^* = \frac{e^{-\rho t}}{\gamma^*} \mathcal{E}(X)_t \quad \textit{for all } t \in [0, T],$$

$$K_t^* = \frac{\mathcal{E}(X)_t (e^{-\rho t} - e^{-\rho T})}{\rho} \quad \textit{for all } t \in [0, T].$$

PROOF. It is direct from Theorem 2.5 by taking $N = \gamma^*$. $\square$

We will discuss what conditions on u and X are sufficient to assure the existence of equilibrium in general in the following section.

3. Existence of Production-Consumption Equilibrium

We consider the additive utility (2.1) and suppose throughout this section that $u : [0, \infty) \times [0, T] \to \mathbf{R}$ is continuous and for each t in $[0, T]$, $u(\cdot, t) : (0, \infty) \to \mathbf{R}$ is continuously differentiable on $(0, \infty)$, concave and strictly increasing.

ASSUMPTION 3.1. *We assume the following conditions for u:*

(A-1) $\lim_{x\to\infty} u(x,t) = \infty$, *and* $\lim_{x\to\infty} \frac{u(x,t)}{x} = 0$.

(A-2) $u(0,t) = 0$, *and* $\lim_{x\downarrow 0} u_x(x,t) = \infty$, *where* u_x *is a partial derivative in* x.

(A-3) *There is some constant* $C_1 > 0$ *such that* $\max_{t\in[0,T]} u(x,t) \leq C_1 \min_{t\in[0,T]} u(x,t)$ *for all* $x \geq 0$.

(A-4) $\overline{\lim_{x\downarrow 0}}\ \overline{\lim_{y\to\infty}} \frac{u(xy,t)}{u(y,t)} = 0$.

(A-5) $\lim_{x\downarrow 0} \max_{t\in[0,T]} u_x(ax,t)x = 0$ *for all* $a > 0$ *and* $u_x(x,t)$ *is continuous in* t *for all* $x > 0$.

Besides, for X, *we suppose*

(A-6) $E[\max_{t\in[0,T]} \mathcal{E}(X)_t] < \infty$.

It is easily checked that $u(x,t) = e^{-\rho t}\frac{x^\alpha}{\alpha}$, $\alpha \in (0,1)$, $\rho > 0$ satisfies the above assumptions (A-1) - (A-5).

Theorem 3.2. *Under Assumption 3.1, there exists a local martingale* N *such that*

$$[(K^*, N\mathcal{E}(X)^{-1}); \delta^*; (c^*, 1)]$$

is a production-consumption equilibrium, where

$$\delta_t^* = c_t^* = \psi(N_t\mathcal{E}(X)_t^{-1}, t), \quad \psi(x,t) \text{ is the inverse of } u_x(x,t) \text{ in } x,$$

$$K^* = \mathcal{E}(X)_t\Big(x_0 - \int_0^t \mathcal{E}(X)_s^{-1}\delta_s^* ds\Big).$$

The following two propositions are essential to prove the above theorem. The first one states the existence of optimal consumption and its properties.

Proposition 3.3. *Under Assumption 3.1,* *(1)* $U_0 \equiv \sup_{c\in\mathcal{C}} U(c) < \infty$,

where $\mathcal{C} = \{c \in L_+ | \int_0^T \mathcal{E}(X)_t^{-1} c_t dt \leq x_0 \text{ a.s.}\}$

(2) There is a $c^* \in \mathcal{C}$ *such that* $U(c^*) = U_0$

(3) $c_t^* > 0$, *a.e.* t *a.s.*

(4) $\int_0^T \mathcal{E}(X)_t^{-1} c_t^* dt = x_0$ a.s.

PROOF. (1) By Lemma 2.6,

$$u(x,t) \leq (x-1)u_x(1,t) + u(1,t), \quad t \in [0,T], x \in \mathbf{R}_+.$$

So if $c \in \mathcal{C}$, we have

$$\begin{aligned} U(c) &\leq E[\int_0^T (c_t u_x(1,t) + u(1,t))dt] \\ &\leq \int_0^T u(1,t)dt + \max_{t\in[0,T]} u_x(1,t) E[\int_0^T \mathcal{E}(X)_t^{-1} c_t \mathcal{E}(X)_t dt] \\ &\leq \int_0^T u(1,t)dt + \max_{t\in[0,T]} u_x(1,t) E[\max_{t\in[0,T]} \mathcal{E}(X)_t \int_0^T \mathcal{E}(X)_t^{-1} c_t dt] \\ &\leq \int_0^T u(1,t)dt + \max_{t\in[0,T]} u_x(1,t) x_0 E[\max_{t\in[0,T]} \mathcal{E}(X)_t]. \end{aligned}$$

It follows from (A-6) that the last expression is finite and does not depend on c, hence so is $\sup_{c\in\mathcal{C}} U(c)$.

(2) Let $f(y,t) = u(\cdot,t)^{-1}(y), y \in [0,\infty)$. Then $f(y,t)$ is increasing, convex, continuously differentiable in $y \in (0,\infty)$, and it follows from (A-1) $\lim_{y\to\infty} \frac{f(y,t)}{y} = \infty$.

Let $\mathcal{K} = \{\xi \in L_+ | \int_0^T \mathcal{E}(X)_t^{-1} f(\xi_t,t)dt \leq x_0 \text{ a.s.}\}$. Then $\mathcal{K}$ is convex.

We also observe $\sup_{\xi\in\mathcal{K}} E[\int_0^T f(\xi_t,t)dt] < \infty$.

Unless it holds, we can take a sequence $\{\xi^{(n)}\}$ in $\mathcal{K}$ such that $E[\int_0^T f(\xi_t^{(n)},t)dt] > n$.

On the other hand, since $\xi^{(n)} \in \mathcal{K}$ for all n, we have

$$E[\int_0^T f(\xi_t^{(n)},t)dt] \quad = \quad E[\int_0^T \mathcal{E}(X)_t \mathcal{E}(X)_t^{-1} f(\xi_t^{(n)},t)dt]$$

$$
\begin{aligned}
&= E[(\max_{t\in[0,T]} \mathcal{E}(X)_t)\int_0^T \mathcal{E}(X)_t^{-1} f(\xi_t^{(n)},t)dt] \\
&= x_0 E[(\max_{t\in[0,T]} \mathcal{E}(X)_t)] < \infty.
\end{aligned}
$$

This contradicts the choice of the sequence.

Since for all $\alpha > 0$, there is a constant $c > 0$ such that $\dfrac{f(y,t)}{y} > \alpha$, for all $y > c$, we have

$$
\begin{aligned}
\sup_{\xi\in\mathcal{K}} E[\int_0^T 1_{\{\xi_t > c\}}\xi_t dt] &\le \frac{1}{\alpha}\sup_{\xi\in\mathcal{K}} E[\int_0^T 1_{\{\xi_t > c\}} f(\xi_t,t)dt] \\
&\le \frac{1}{\alpha}\sup_{\xi\in\mathcal{K}} E[\int_0^T f(\xi_t,t)dt] < \infty
\end{aligned}
$$

Therefore, $\mathcal{K}$ is uniformly integrable with respect to $dt\otimes dP$ on $[0,T]\times\Omega$.

Moreover, we have $U_0 = \sup_{\xi\in\mathcal{K}} E[\int_0^T \xi_t dt]$.

These results imply that $\mathcal{K}$ is a weakly sequential compact set in $L^1([0,T]\times\Omega)$ and that there is a $\xi^* \in \mathcal{K}$ such that $U_0 = E[\int_0^T \xi_t^* dt]$. Letting $c_t^* = f(\xi_t^*,t)$, we have the claim.

(3) Let $c_t^{(n)} = (1-\frac{1}{n})c_t^* + \frac{1}{nT}x_0\mathcal{E}(X)_t, \quad n \ge 1$. Then it is obvious that $c^{(n)} \in \mathcal{C}$.

Furthermore we have

$$
\begin{aligned}
& n(U(c^{(n)}) - U(c^*)) \\
= \ & E[\int_0^T n(u(c_t^{(n)},t) - u(c_t^*,t))dt] \\
\ge \ & E[\int_0^T u_x(c_t^{(n)},t)(\frac{1}{T}x_0\mathcal{E}(X)_t - c_t^*)dt] \\
= \ & \int_0^T E[u_x(c_t^{(n)},t)(\frac{1}{T}x_0\mathcal{E}(X)_t - c_t^*) \ ; \ \frac{1}{T}x_0\mathcal{E}(X)_t \ge c_t^*]dt \\
& + \int_0^T E[u_x(c_t^{(n)},t)(\frac{1}{T}x_0\mathcal{E}(X)_t - c_t^*) \ ; \ \frac{1}{T}x_0\mathcal{E}(X)_t < c_t^*]dt \\
\ge \ & \int_0^T E[u_x(c_t^{(n)},t)\frac{1}{T}x_0\mathcal{E}(X)_t \ ; \ c_t^* = 0]dt \\
& - \int_0^T E[u_x(\frac{1}{T}x_0\mathcal{E}(X)_t,t)c_t^* \ ; \ \frac{1}{T}x_0\mathcal{E}(X)_t < c_t^*]dt
\end{aligned}
$$

$$\geq \quad E[\int_0^T u_x(c_t^{(n)},t)\frac{1}{T}x_0\mathcal{E}(X)_t 1_{\{0\}}(c_t^*)dt] - E[\int_0^T u_x(\frac{1}{T}x_0\mathcal{E}(X)_t,t)c_t^* dt].$$

For every $a > 0$, (A-5) implies that for all $\varepsilon > 0$, there exists a $\delta > 0$ such that $\max_{t\in[0,T]} u_x(ax,t)x < \varepsilon$ for $0 < x < \delta$.

Thus, from (A-5) and (A-6) it follows that for all $a > 0$,

$$\begin{aligned} & E[\max_{t\in[0,T]} u_x(a\mathcal{E}(X)_t,t)\mathcal{E}(X)_t] \\ = \; & E[\max_{t\in[0,T]} u_x(a\mathcal{E}(X)_t,t)\mathcal{E}(X)_t; \mathcal{E}(X)_t < \delta] \\ & + E[\max_{t\in[0,T]} u_x(a\mathcal{E}(X)_t,t)\mathcal{E}(X)_t; \mathcal{E}(X)_t \geq \delta] \\ \leq \; & \varepsilon + (\max_{t\in[0,T]} u_x(a\delta,t))E[\max_{t\in[0,T]} \mathcal{E}(X)_t] < \infty \end{aligned}$$

Therefore, we note that

$$E[\int_0^T u_x(\frac{1}{T}x_0\mathcal{E}(X)_t,t)c_t^* dt] \leq x_0 E[\max_{t\in[0,T]} u_x(\frac{1}{T}x_0\mathcal{E}(X)_t,t)\mathcal{E}(X)_t] < \infty.$$

Therefore, if $P(|t \in [0,T]; c_t^* = 0| > 0) > 0$ (where $|A|$ means the Lebesgue measure of a measurable set $A \subset \mathbf{R}$), then, by using (A-2), we have

$$n(U(c^{(n)}) - U(c^*)) \to \infty, \quad n \to \infty$$

This contradicts the assumption for c^*.

(4) Let $\tilde{c}_t^{(n)} = \begin{cases} c_t^* & \text{if } t \in [0, T - \frac{1}{n}] \\ n\Big(x_0 - \int_0^{T-\frac{1}{n}} \mathcal{E}(X)_s^{-1} c_s^* ds\Big) & \text{if } t \in (T - \frac{1}{n}, T] \end{cases}$.

Then we have

$$n(U(\tilde{c}^{(n)}) - U(c^*)) = E[\int_{T-\frac{1}{n}}^T (u(\tilde{c}_t^{(n)},t) - u(c_t^*,t))dt].$$

Note that

$$u(y,t) \geq y u_x(y,t), \quad y > 0$$

and

$$u(y,t) \leq u(x,t) + u_x(x,t)(y-x) \leq u(x,t) + \frac{u(x,t)}{x}y, \quad x,y > 0.$$

Let $\varepsilon > 0$. Then we have

$$\begin{aligned}
& E[\int_{T-\frac{1}{n}}^{T} u(c_t^*, t))dt] \\
\leq \ & E[\int_{T-\frac{1}{n}}^{T} (u(n\varepsilon, t) + \frac{u(n\varepsilon, t)}{n\varepsilon} c_t^*)dt] \\
\leq \ & \int_{T-\frac{1}{n}}^{T} u(n\varepsilon, t)dt + \frac{1}{n\varepsilon}(\max_{t\in[0,T]} u(n\varepsilon, t))E[\int_{T-\frac{1}{n}}^{T} c_t^* dt] \\
\leq \ & \Big(1 + \frac{C_1}{\varepsilon} E[\int_{T-\frac{1}{n}}^{T} c_t^* dt]\Big) \int_{T-\frac{1}{n}}^{T} u(n\varepsilon, t)dt
\end{aligned}$$

The last inequality follows from (A-3).

Since $E[\int_0^T c_t^* dt] \leq x_0 E[\max_{t\in[0,T]} \mathcal{E}(X)_t] < \infty$, we observe that for any $\varepsilon > 0$, if n is sufficiently large, then

$$E[\int_{T-\frac{1}{n}}^{T} u(c_t^*, t)dt] \leq 2 \int_{T-\frac{1}{n}}^{T} u(n\varepsilon, t)dt.$$

Now we suppose that $P(\int_0^T \mathcal{E}(X)_t^{-1} c_t^* dt < x_0) > 0$. Then we see that there is a $\delta > 0$ such that

$$P(x_0 - \int_0^T \mathcal{E}(X)_t^{-1} c_t^* dt \geq \delta) \geq \delta.$$

Then we have for any $\varepsilon > 0$, if n is sufficiently large,

$$n(U(\tilde{c}^{(n)}) - U(c^*)) \geq \delta \int_{T-\frac{1}{n}}^{T} u(n\delta, t)dt - 2 \int_{T-\frac{1}{n}}^{T} u(n\varepsilon, t)dt.$$

The right-hand side, however, is positive because of the assumption (A-4) if ε is sufficiently small and n is sufficiently large. This is a contradiction to the assumption for c^*. □

Next we show that the optimal consumption satisfying the properties in the last proposition introduces a local martingale which meets the conditions stated in Theorem 2.5.

PROPOSITION 3.4. *Let* $c^* \in \mathcal{C}$ *satisfy* $U(c^*) = \sup_{c \in \mathcal{C}} U(c) < \infty$ *and* $\int_0^T \mathcal{E}(X)_t^{-1} c_t^* dt = x_0$ a.s..

Moreover, suppose (A-5) and (A-6) in Assumption 3.1.

Then there is a strictly positive local martingale $\{M_t\}_{t \in [0,T]}$ *satisfying the following:*

(1) $u_x(c_t^*, t)\mathcal{E}(X)_t = M_t$ *a.e.* t

(2) $\{M_{\tau_n \wedge t}\}_{t \in [0,T]}$ *is a martingale, where*

$$\tau_n = \inf\{t > 0; \int_0^t \mathcal{E}(X)_s^{-1} c_s^* ds > x_0(1 - \frac{1}{n})\}$$

PROOF. Let us take an arbitrary progressively measurable function $\xi : \Omega \times [0,T] \to \mathbf{R}$ with $|\xi_t| \leq 1$ and fix it. Let $\eta(t;s), t \in [0,T], s \in [0,T)$, be given by

$$\eta(t;s) = \begin{cases} c_t^* \xi_t & \text{if } t \in [0,s) \\ -(x_0 - \int_0^s \mathcal{E}(X)_u^{-1} c_u^* du)^{-1} (\int_0^s \mathcal{E}(X)_u^{-1} c_u^* \xi_u du) c_t^* & \text{if } t \in [s,T]. \end{cases}.$$

Then we have

$$\int_0^T \mathcal{E}(X)_t^{-1} \eta(t;s) dt = 0, \ \ s \in [0,T), \text{a.s.},$$

$$\int_0^T \mathcal{E}(X)_t^{-1} \eta(t;\tau_n) dt = 0,$$

and

$$|\eta(t;\tau_n)| \leq n c_t^*.$$

So we observe that

$$U(c^* + s\eta(\cdot;\tau_n)) \leq U(c^*)$$

for any $s \in (-1,1)$ with $(1+n)|s| < 1$. So we see that

$$\frac{d}{ds} U(c^* + s\eta(\cdot;\tau_n))|_{s=0} = 0.$$

Note $|u_x(c_t^* + s\eta(t;\tau_n), t)\eta(t;\tau_n)| \leq n u_x(\frac{1}{n+1} c_t^*, t) c_t^*$.

Since we have $E[\int_0^T u_x(\frac{1}{n+1}c_t^*, t)c_t^* dt] < \infty$ owing to (A-5) and (A-6) (analogous to the proof of (3) in the previous proposition), Lebesgue convergence theorem enables us to interchange between differentiation and integral.

So we have $E[\int_0^T u_x(c_t^*, t)\eta(t; \tau_n)dt] = 0$. Then

$$\begin{aligned} & E[\int_0^{\tau_n} u_x(c_t^*, t)c_t^* \xi_t dt] \\ = \; & E[(x_0 - \int_0^{\tau_n} \mathcal{E}(X)_u^{-1} c_u^* du)^{-1}(\int_0^{\tau_n} \mathcal{E}(X)_u^{-1} c_u^* \xi_u du)(\int_{\tau_n}^T u_x(c_t^*, t)c_t^* dt)]. \end{aligned}$$

Let $Y_n = (x_0 - \int_0^{\tau_n} \mathcal{E}(X)_u^{-1} c_u^* du)^{-1}(\int_{\tau_n}^T u_x(c_u^*, u)c_u^* du)$.

Then we have

$$E[\int_0^{\tau_n} c_t^* \{u_x(c_t^*, t) - E[Y_n|\mathcal{F}_t]\mathcal{E}(X)_t^{-1}\}\xi_t dt] = 0.$$

Since ξ is arbitrary, we have

$$(u_x(c_t^*, t)\mathcal{E}(X)_t - E[Y_n|\mathcal{F}_t])1_{[0,\tau_n)}(t) = 0, \quad dt \otimes dP\text{−a.e.}$$

Since $\tau_n \uparrow T$, we see that $\{u_x(c_t^*, t)\mathcal{E}(X)_t\}_{t\in[0,T]}$ has a right continuous version M. Then we observe that

$$M_{\tau_n \wedge t} = E[Y_{n+1}|\mathcal{F}_{\tau_n \wedge t}] \quad \text{a.s.}, \quad t \in [0, T),$$

so $M_{\tau_n \wedge t}$ is a martingale and M_t is a local martingale and strict positivity is apparent from the construction. □

Proof of Theorem 3.2. It is now clear since Proposition 3.3 and 3.4 guarantee the conditions in Theorem 2.5 are satisfied. □

4. Equilibrium Spot Rate Process in LOG Model

Definition 4.1. Let π be a state price process in an equilibrium and have the decomposition $\pi_t = \pi_0 + \pi_t^M + \pi_t^A$, where π^M is a local martingale with $\pi_0^M = 0$ and π^A is an adapted process of finite variation with $\pi_0^A = 0$.

An equilibrium cumulative spot rate process is a process R with $R_0 = 0$ satisfying

$$dR_t \;=\; -\frac{d\pi_t^A}{\pi_t} \tag{4.1}$$

If dR_t is absolutely continuous with respect to dt, we set $r_t = \frac{dR_t}{dt}$ and call r the equilibrium spot rate process.

We see a relation between an equilibrium spot rate and an equivalent martingale measure (EMM). Here, we define an EMM Q in the following sense.

Let (S^*, δ^*) be the pair of components of Definition 2.2 and $P(t,s)$ be a process defined by

$$P(t,s) = \frac{1}{\pi_t} E[\pi_s|\mathcal{F}_t] \text{ for } 0 \le \text{t} \le \text{s} \le \text{T}.$$

In most finance models, $P(t,s)$ is interpreted by s-maturity zero-coupon-bond price process with $P(t,t) = 1$ for all $t \in [0,T]$.

EMM Q is defined as a probability measure "equivalent" to the original measure P under which

$$\{\beta_t S_t^* + \int_0^t \beta_u \delta_u^* du\}_{0 \le t \le T}, \; \{\beta_t P(t,s)\}_{0 \le t \le s} \text{ for all } s \in [0,T]$$

are both martingale, where $\beta_t = \exp(-R_t)$, and R is a cumulative spot rate process.

We state below the relation between the density of EMM Q and a state price process π.

Hereafter we denote by E (resp. E^Q) the expectation with respect to P (resp. Q).

LEMMA 4.2. *Under the above notations, set* $\xi_t = e^{R_t}\frac{\pi_t}{\pi_0}$ *and define the density process of an equivalent probability measure* Q *as* $E[\frac{dQ}{dP}|\mathcal{F}_t] = \xi_t$.

Suppose $E[\pi_T] < \infty$.

If ξ *is a martingales under* P, Q *is an EMM.*

PROOF. We may suppose $\pi_0 = 1$.

Let $0 < u < t \leq T$. Note $E[\pi_T|\mathcal{F}_u] = E[\pi_t P(t,T)|\mathcal{F}_u]$.
Since ξ^{-1} is a martingake under Q, by the Bayes rule,

$$E[\pi_t P(t,T)|\mathcal{F}_u] = \frac{1}{\xi_u^{-1}} E^Q[\xi_t^{-1}\pi_t P(t,T)|\mathcal{F}_u] \quad \text{a.s.}.$$

This implies that $\{\beta_t P(t,T)\}_{0\leq t\leq T}$ is a Q-martingale,.

It follows from the same argument that $\{\beta_t S_t^* + \int_0^t \beta_u \delta_u^* du\}_{0\leq t\leq T}$ is a Q-martingale. □

Now we mention the procedure to induce the spot rate dynamics for LOG model.

PROPOSITION 4.3. *For LOG model, suppose that X admits the decomposition*

$$X_t = X_0 + M_t + A_t,$$

where M is a local martingale with $M_0 = 0$ and A is an adapted process of finite variation with $A_0 = 0$.

Then an equilibrium cumulative spot rate R satisfies the relation

$$R_t = A_t - \langle M, M\rangle_t.$$

PROOF. It immediately follows from $d\mathcal{E}(X)_t^{-1} = \mathcal{E}(X)_t^{-1}(-dM_t - dA_t + d\langle M, M\rangle_t)$ and (4.1). □

From now on, we only consider LOG model (see Definition 2.7 and Corollary 2.8). We also pay attention to general diffusion-type spot rate models (including multi-dimensional case) and see that they can be induced from our equilibrium setting (in LOG model).

For the purpose, we introduce an Ito process Y which connotes " state variables", that is, some economic parameters. We assume that the firm's production technology X is influenced by the state variable Y in the sense that the instantaneous change of X_t depends on the current value Y_t. We assume the followings:

ASSUMPTION 4.4.

(0) B is a standard Brownian motion on the probability space $(\Omega, \mathcal{F}, P)$ and we also take the filtration $(\mathcal{F}_t)_{t\in[0,T]}$ as Brownian filtration generated by B.

(1) Let Y be $\mathbf{R}$-valued Ito process defined by

$$dY_t = \mu(Y_t, t)dt + \sigma(Y_t, t)dB_t, \qquad Y_0 \in \mathbf{R}, \tag{4.2}$$

where $\mu, \sigma : \mathbf{R} \times [0, T] \to \mathbf{R}$, are functions which guarantee the existence of solution to this SDE.

(2) Let X satisfy

$$dX_t = [\varphi(Y_t, t) + \phi(Y_t, t)^2]dt + \phi(Y_t, t)dB_t, \tag{4.3}$$

where $\varphi : \mathbf{R} \times [0, T] \to \mathbf{R}$ is twice continuously partial differentiable in the first argument x and once in t and $\phi : \mathbf{R} \times [0, T] \to \mathbf{R}$ is continuous.

(3)

$$E[\exp\Big(2\int_0^T \phi(Y_s, s)^2 ds\Big)] < \infty \tag{4.4}$$

and

$$E[\exp\Big(-4\int_0^T \varphi(Y_s, s)ds\Big)] < \infty. \tag{4.5}$$

Now, we state our main result upon the dynamics of spot rate process under an equivalent martingale measure.

THEOREM 4.5. *Under Assumption 4.4, there exists a spot rate process r such that under an equivalent martingale measure Q and for some functions $\nu : \mathbf{R} \times [0, T] \to \mathbf{R}$, $\quad \rho : \mathbf{R} \times [0, T] \to \mathbf{R}$,*

$$dr_t = \nu(Y_t, t)dt + \rho(Y_t, t)d\hat{B}_t, \qquad r_0 = \varphi(Y_0, 0)$$

where Q has a density process given by

$$E[\frac{dQ}{dP}|\mathcal{F}_t] = \exp\Big(-\int_0^t \phi(Y_s, s)dB_s - \frac{1}{2}\int_0^t \phi(Y_s, s)^2 ds\Big),$$

and $\hat{B}$ is a standard Brownian motion under Q.

In particular, if φ is monotone in variable x, then we have

$$dr_t = \nu'(r_t, t)dt + \rho'(r_t, t)d\hat{B}_t$$

for some functions

$$\nu' : \mathbf{R} \times [0, T] \to \mathbf{R}, \quad \rho' : \mathbf{R} \times [0, T] \to \mathbf{R}.$$

Proof. It is sufficient to prove in the case of $X_0 = 0$. We have $r_t = \varphi(Y_t, t)$ from Proposition 4.3.

By using Ito's formula, we compute

$$\begin{aligned} dr_t &= \varphi_x(Y_t, t)dY_t + \varphi_t(Y_t, t)dt + \frac{1}{2}\varphi_{xx}(Y_t, t)d\langle Y, Y\rangle_t \\ &= \{\varphi_x(Y_t, t)\mu(Y_t, t) + \varphi_t(Y_t, t) + \frac{1}{2}\varphi_{xx}(Y_t, t)\sigma(Y_t, t)^2\}dt \\ &\quad +\varphi_x(Y_t, t)\sigma(Y_t, t)dB_t. \end{aligned}$$

We also have

$$\begin{aligned} \mathcal{E}(X)_t^{-1} &= \exp\Big(-X_t + \frac{1}{2}\langle X, X\rangle_t\Big) \\ &= \exp\Big(-\int_0^t [\varphi(Y_s, s) + \phi(Y_s, s)^2]ds \\ &\qquad -\int_0^t \phi(Y_s, s)dB_s + \frac{1}{2}\int_0^t \phi(Y_s, s)^2 ds\Big) \\ &= \exp\Big(-\int_0^t r_s ds\Big)\exp\Big(-\int_0^t \phi(Y_s, s)dB_s - \frac{1}{2}\int_0^t \phi(Y_s, s)^2 ds\Big). \end{aligned}$$

Set

$$\xi_t = \exp\Big(\int_0^t r_s ds\Big)\mathcal{E}(X)_t^{-1} = \exp\Big(-\int_0^t \phi(Y_s, s)dB_s - \frac{1}{2}\int_0^t \phi(Y_s, s)^2 ds\Big).$$

Since we get the Novikov's condition $E[\exp\Big(\frac{1}{2}\int_0^T \phi(Y_s, s)^2 ds\Big)] < \infty$ from (4.4), it follows that ξ is a P-martingale, and so $E[\xi_t] = 1$. We also see $\xi_T > 0$ a.s., that is, Q is equivalent to P.

Besides, we observe that $E[\mathcal{E}(X)_T^{-1}] < \infty$ immediately from Cauchy-Schwartz inequality and (4.5).

Hence Q is an EMM by Lemma 4.2.

Therefore the spot rate r satisfies the following SDE under Q:

$$\begin{aligned} dr_t &= \{\varphi_x(Y_t,t)[\mu(Y_t,t)-\phi(Y_t,t)\sigma(Y_t,t)] \\ &\qquad +\varphi_t(Y_t,t)+\frac{1}{2}\varphi_{xx}(Y_t,t)\sigma(Y_t,t)^2\}dt \\ &\quad +\varphi_x(Y_t,t)\sigma(Y_t,t)d\hat{B}_t \end{aligned}$$

Setting

$$\varphi_x(x,t)\sigma(x,t)=\rho(x,t)$$
$$\varphi_x(x,t)[\mu(x,t)-\phi(x,t)\sigma(x,t)]+\varphi_t(x,t)+\frac{1}{2}\varphi_{xx}(x,t)\sigma(x,t)^2=\nu(x,t),$$

completes the proof of the first part.

If φ is monotone in variable x, observing that $Y_t = \varphi^{-1}(r_t, t)$ implies that $\varphi_x(Y_t,t)$ is regarded as $\varphi_x(\varphi^{-1}(r_t,t),t)$ and so forth. So we can get an SDE representation for r without Y. Here φ^{-1} stands for the inverse function of φ in the first variable. □

COROLLARY 4.6. *Under Assumption 4.4, if $\varphi(x,t) = ax$ ($a \neq 0$ is some constant), then*

$$dr_t = a\{\mu(\frac{r_t}{a},t)-\phi(\frac{r_t}{a},t)\sigma(\frac{r_t}{a},t)\}dt + a\sigma(\frac{r_t}{a},t)d\hat{B}_t.$$

Next we want to consider primitive models which give some general-type spot rate models, that is, the concrete descriptions of SDE (4.2), (4.3) that the processes Y, X satisfy. For the purpose, we have to determine the proper functions σ, φ which lead to the intended model.

We give some examples along Collorary 4.6.

Example 4.7. (Ho - Lee model)

$$dY_t = (\frac{\sigma\mu}{\theta}+\beta\sigma)dt+\sigma dB_t, \qquad dX_t = \Big(\frac{\theta}{\sigma}Y_t+\beta^2\Big)dt+\beta dB_t$$

(where $\sigma, \mu, \theta, \beta$ are all positive constants). Then

$$dr_t = \mu dt + \theta d\hat{B}_t.$$

Example 4.8. (Vasicek model)

$$dY_t = \Big(\sigma[\frac{m}{\theta} + \beta] - bY_t\Big)dt + \sigma dB_t, \qquad dX_t = \Big(\frac{\theta}{\sigma}Y_t + \beta^2\Big)dt + \beta dB_t$$

(where $\sigma, m, b, \theta, \beta$ are all positive constants). Then

$$dr_t = (m - br_t)dt + \theta d\hat{B}_t.$$

Example 4.9. (CIR model and the extension)
Let $\frac{1}{2} \leq \alpha < 1$.

$$dY_t = \frac{1}{c}(m - bcY_t + \theta\beta c^{\alpha} Y_t^{\alpha})dt + \theta c^{\alpha-1} Y_t^{\alpha} dB_t,$$

$$dX_t = (cY_t + \beta^2)dt + \beta dB_t$$

(where m, b, θ, c, β are all positive constants). Then

$$dr_t = (m - br_t)dt + \theta r_t^{\alpha} d\hat{B}_t.$$

Example 4.10. (another example of CIR model)

$$dY_t = \frac{1}{c}(m - (bc - \theta\sigma\sqrt{c})Y_t)dt + \frac{\theta}{\sqrt{c}}\sqrt{Y_t}dB_t,$$

$$dX_t = (c + \sigma^2)Y_t dt + \sigma\sqrt{Y_t}dB_t$$

(m, b, θ, c, σ be all proper positive constants). Then

$$dr_t = (m - br_t)dt + \theta\sqrt{r_t}d\hat{B}_t$$

Remark 4.11. Example 4.10 is almost the same as the original model given in CIR [3]. They assume that the instantaneous expectation and variance of X are proportional to the current value of the state variable Y.

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(Received October 22, 1998)
(Revised February 24, 1999)

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