J. Math. Sci. Univ. Tokyo
5 (1998), 99–117.

Singularities of the Bergman Kernel for Certain Weakly Pseudoconvex Domains

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Abstract. Consider the Bergman kernel $K^B(z)$ of the domain $\mathcal{E}_m = \{z \in \mathbf{C}^n; \sum_{j=1}^n |z_j|^{2m_j} < 1\}$, where $m = (m_1, \ldots, m_n) \in \mathbf{N}^n$ and $m_n \neq 1$. Let $z^0 \in \partial\mathcal{E}_m$ be any weakly pseudoconvex point, $k \in \mathbf{N}$ the degenerate rank of the Levi form at z^0. An explicit formula for $K^B(z)$ modulo analytic functions is given in terms of the polar coordinates $(t_1, \ldots, t_k, r)$ around z^0. This formula provides detailed information about the singularities of $K^B(z)$, which improves the result of A. Bonami and N. Lohoué [4]. A similar result is established also for the Szegö kernel $K^S(z)$ of $\mathcal{E}_m$.

1. Introduction and Main Result

Let Ω be a bounded domain with smooth boundary in $\mathbf{C}^n$, $B(\Omega)$ the set of holomorphic L^2-functions on Ω. It is well-known that $B(\Omega)$ is a closed linear subspace of the Hilbert space $L^2(\Omega)$. The *Bergman kernel* $K^B(z)$ of the domain Ω is defined by

$$K^B(z) = \sum_j |\phi_j(z)|^2,$$

where $\{\phi_j\}$ is a complete orthonormal basis for $B(\Omega)$. The above series converges uniformly on any compact subset of Ω. It is very important to investigate the singularities of $K^B(z)$. This is mainly because they contain much information about the analytic and geometric invariants of the domain Ω.

First we consider the case where Ω is a strongly pseudoconvex domain. In this case C. Fefferman [13], L. Boutet de Monvel and J. Sjöstrand [5]

1991 *Mathematics Subject Classification*. 32A40, 32F15, 32H10.

Key words: Bergman kernel, Szegö kernel, weakly pseudoconvex domain of finite type, polar coordinates, irregular singular point, admissible approach region.

obtained the following asymptotic expansion for $K^B(z)$:

$$K^B(z) = \frac{\varphi^B(z)}{r(z)^{n+1}} + \psi^B(z) \log r(z), \tag{1.1}$$

where r is a defining function of Ω, i.e., $\Omega = \{z \in \mathbf{C}^n; r(z) > 0\}$ and $\mathrm{grad} r(z) \neq 0$ on $\partial\Omega$. The functions $\varphi^B(z)$ and $\psi^B(z)$ can be expressed as a power series of r. From the viewpoint of ordinary differential equations, this result may be interpreted that the Bergman kernel of a strongly pseudoconvex domain has the singularities of *regular singular type*.

Next we proceed to the case of weakly pseudoconvex domain of finite type (in the sense of J. J. Kohn [26] or J. P. D'Angelo [9]). In this case there is no such strong general result that is comparable with (1.1) in the strongly pseudoconvex case; yet there are many detailed results for the Bergman kernels of specific domains. We refer to [2],[8],[18],[10],[14],[19] for explicit computations, to [20],[36],[12],[6],[21],[22] for estimates of the size and to [3] for boundary limits on nontangential cone. Especially D. Catlin [6] and G. Herbort [22] gave precise estimates of $K^B(z)$ from above and below for certain class of domains whose degenerate rank of the Levi form equals one. In general, however, the singularities of $K^B(z)$ are so complicated that a unified treatment of them seems to be difficult.

In this paper, we pick up the specific domains

$$\mathcal{E}_m = \left\{ z = (z_1, \ldots, z_n) \in \mathbf{C}^n \,;\, \sum_{j=1}^{n} |z_j|^{2m_j} < 1 \right\}, \tag{1.2}$$

where $m = (m_1, \ldots, m_n) \in \mathbf{N}^n$ and $m_n \neq 1$, to clarify what is happening for the weakly pseudoconvex domains of finite type. Since $\mathcal{E}_m$ is a Reinhardt domain, the set of (normalized) monomials forms a complete orthonormal basis for $B(\mathcal{E}_m)$. Hence $K^B(z)$ can be represented by a convergent power series of $(|z_1|^2, \ldots, |z_n|^2)$, whose coefficients were explicitly computed in [23],[8],[4]. A. Bonami and N. Lohoué [4] gave an important integral representation for the Bergman kernel $K^B(z)$ of $\mathcal{E}_m$ (see (2.1)). From this representation they deduced a detailed information about the singularities of $K^B(z)$, though their result is yet to be improved.

From our point of view, we briefly review the result of [4]. Let $z^0 = (z_1^0, \ldots, z_n^0) \in \partial\mathcal{E}_m$ be any boundary point of $\mathcal{E}_m$, $k \in \mathbf{Z}_{\geq 0}$ the degenerate rank of the Levi form at z^0. We say that z^0 is a *strongly* (resp. *weakly*) *pseudoconvex point* if $k = 0$ (resp. if $k > 0$). Let I, P and Q be the subsets of $N = \{1, \dots, n\}$ defined by

$$(1.3) \qquad \begin{cases} I &= \{j \in N; m_j = 1\}, \\ P &= \{j \in N; z_j^0 = 0\} \setminus I, \\ Q &= \{j \in N; z_j^0 \neq 0\} \cup I. \end{cases}$$

Then the degenerate rank k equals the cardinality $|P|$ of P. One of the main results in [4] (p.181) states that the restriction of $K^B(z)$ to the subset $V = \{z \in \mathbf{C}^n; z_j = 0\,(j \in P)\}$ admits the following expression around z^0:

$$(1.4) \qquad K^B(z) = C_P^B \; \frac{\prod_{j\in Q} m_j^2 |z_j|^{2m_j-2}}{\{1 - \sum_{j\in Q} |z_j|^{2m_j}\}^{|Q|+|\frac{1}{m}|_P+1}} \;+\; O(1),$$

where C_P^B is a positive constant and $|\frac{1}{m}|_P = \sum_{j\in P} \frac{1}{m_j}$ ($|\frac{1}{m}|_\emptyset := 0$). The formula (1.4) is quite explicit, but still weak in the sense that it is valid only on the thin set V, and that the error term $O(1)$ is somewhat too loose.

Besides [4] there are some studies on the Bergman kernel (or Szegö kernel) of the domain $\mathcal{E}_m$ ([2],[23],[8],[16],[17]). In the case $m = (1, \dots, 1, m)$, explicit expressions for $K^B(z)$ are obtained ([2],[8],[4], see also Remark 2, §2.3), while there seems to be no explicit one for general m. The recent studies of Gebelt [16], Gong and Zheng [17] and Francsics and Hanges [15] are very interesting. N. W. Gebelt [16] generalized the method of producing the asymptotic expansion (1.1) due to Fefferman [13] to obtain the analogous results about the cretain domain which is approximated by $\mathcal{E}_m$ ($m = (1, \dots, 1, m)$). S. Gong and X. Zheng [17] gave a global estimate of $K^B(z)$ from above and below. G. Francsics and N. Hanges [15] expressed the Bergman kernel of $\mathcal{E}_m$ explicitly by using some generalized hypergeometric functions.

Now we state our main results. Our essential idea is to introduce the new variables (t, r), which we call the *polar coordinates* around z^0. Here $t = (t_j)_{j\in P}$ is defined by

$$t_j(z)^{2m_j} = \frac{|z_j|^{2m_j}}{1 - \sum_{k\in Q} |z_k|^{2m_k}} \qquad (j \in P),$$

and r is the defining function of $\mathcal{E}_m$, i.e.,

$$r(z) = 1 - \sum_{j=1}^{n} |z_j|^{2m_j}.$$

We call t the *angular variables* and r the *radial variable*, respectively. Then the map $F : z \mapsto (t, r)$ takes $\mathcal{E}_m$ onto the region:

$$D = \left\{ (t, r) \in \mathbf{R}^{|P|} \times (0, 1] \, ; \, t_j \geq 0 \ (j \in P), \sum_{j \in P} t_j^{2m_j} \leq 1 - r \right\}.$$

The accumulation points of $F(z)$ as $\mathcal{E}_m \ni z \to z^0$ are precisely those points which belong to the set $\overline{\Delta} \times \{0\}$, where $\overline{\Delta}$ is the closure of the *locally closed* simplex:

$$\Delta = \left\{ t = (t_j)_{j \in P} \, ; \, t_j \geq 0 \ (j \in P), \sum_{j \in P} t_j^{2m_j} < 1 \right\}.$$

Let $G = U \cap K$ be a locally closed subset of an Euclidean space, where U is open and K is closed, respectively. Then we say that $f \in C^\omega(G)$ if f is a real analytic function on some open neighborhood V of G in U, where V may depend on f.

The following theorem asserts that the asymptotic behavior of K^B as $\mathcal{E}_m \ni z \to z^0$ can be expressed most conveniently in terms of the polar coordinates (t, r).

THEOREM 1. *There is a function $\Phi^B(t) \in C^\omega(\Delta)$ such that*

$$(1.5) \quad K^B(z) \equiv \frac{n!}{\pi^n} \prod_{j \in Q} m_j^2 |z_j|^{2m_j - 2} \frac{\Phi^B(t(z))}{r(z)^{|Q| + |\frac{1}{m}|_P + 1}} \qquad \text{modulo} \ \ \mathrm{C}^\omega(\{z^0\}).$$

Here $\Phi^B(t)$ satisfies (i) *or* (ii).

(i) *If z^0 is a strongly pseudoconvex point* (*i.e.* $P = \emptyset$), $\Phi^B(t) = 1$ *identically.*

(ii) *If z^0 is a weakly pseudoconvex point* (*i.e.* $P \neq \emptyset$), *then $\Phi^B(t)$ is positive on Δ and is unbounded as $t \in \Delta$ approaches $\overline{\Delta} \setminus \Delta$.*

We remark that $\prod_{j \in Q} m_j^2 |z_j|^{2m_j - 2}$ does not contribute the singularities of $K^B(z)$ seriously since it is positive near z^0. Later we shall see that $\Phi^B(t)$ is essentially the Laplace transform of a certain auxiliary function expressible in terms of Mittag-Leffler's function (see (2.9)).

We mention a few implications of the formula (1.5) in order to compare it with the known results stated previously. First, if z^0 is a strongly pseudoconvex point, i.e. $P = \emptyset$, then the angular variables t do not appear and $\Phi^B(t) = 1$ identically, and therefore (1.5) reproduces the asymptotic expansion (1.1) due to C. Fefferman [13], L. Boutet de Monvel and J. Sjöstrand [5]. We remark that the logarithmic term in (1.1) does not appear in the present case. Secondly, the restriction of (1.5) to the subset V is just the substitution $t_j(z) = 0$ $(j \in P)$ into (1.5), which induces the formula (1.4) with the error term $O(1)$ replaced by a real analytic function. Thus the formula (1.5) improves that of Bonami and Lohoué [4] in the sense that it is valid in a wider domain and that the error term is more accurate.

From our theorem, we consider the behavior of $K^B(z)$ at a weakly pseudoconvex point from the following three angles: (a) *estimate*, (b) *boundary limit* and (c) *asymptotic formula.* We assume z^0 is a weakly pseudoconvex point and define the region $\mathcal{U}_\alpha(z^0)(=\mathcal{U}_\alpha) \subset \mathcal{E}_m$ by

$$\mathcal{U}_\alpha = \left\{ z \in \mathcal{E}_m; \sum_{j\in P} t_j(z) = \frac{\sum_{j\in P} |z_j|^{2m_j}}{1 - \sum_{j\in Q} |z_j|^{2m_j}} < \frac{1}{\alpha} \right\} \quad (\alpha > 1).$$

(a) By the boundedness of $\Phi^B(t)$ in (ii) we can precisely estimate the size of $K^B(z)$ on $\mathcal{U}_\alpha$. The region $\mathcal{U}_\alpha$ reminds us of the *admissible approach regions* considered in [35],[29],[30],[1] etc. (b) The boundary limit of $K^B(z) \cdot r(z)^{|Q|+|\frac{1}{m}|_P+1}$ as $z \to z^0$ on each $\mathcal{U}_\alpha$ is $n!\,\pi^{-n}\prod_{j\in Q} m_j^2 |z_j|^{2m_j-2}\Phi^B(t(z))$. Since this limit depends on the angular variables t, the value of the limit is changed by the curves on which we approach z^0. But the limit is uniquely determined on any nontangential cone. (c) In view of (1.5) the polar coordinates (t, r) is necessary to understand the asymptotic formula of $K^B(z)$ at z^0. This fact may be interpreted that the Bergman kernel has a singularity of irregular singular type at a weakly pseudoconvex point. The degeneration from the strong pseudoconvexity to the weak pseudoconvexity corresponds to the process of confluence from the regular singularity to the irregular singularity ([32],[31]). Recently the auther gave an asymptotic expansion of the Bergman kernel for certain class of weakly pseudoconvex domains by using a kind of polar coordinates similar to the above ([24]).

In more detail we investigate the structure of singularities of the Bergman kernel of $\mathcal{E}_m$. The singularities of $\Phi^B(t)$ at $\overline{\Delta} \setminus \Delta$ can also be expressed in a form similar to (1.5) by introducing new polar coordinates on the simplex Δ. Through the finite recursive procedure of this type we can completely understand the structure of the singularities of $K^B(z)$. This situation will be explained more precisely in Section 3.

This paper is organized as follows. In Section 2 we give the proof of Theorem 1. We divide the proof into two parts. In the first part we refine the error term $O(1)$ in (1.4). In the second part we introduce the polar coordinates and express the singularities of $K^B(z)$ explicitly. In Section 3 we completely investigate the structure of the singularities of $\Phi^B(t)$ through the finite recursive procedure described above. In Section 4 we give the proof of Lemma 2, which is necessary for the proof of Theorem 1. In Section 5 a similar result about the Szegö kernel of $\mathcal{E}_m$ is established.

Acknowledgment. I would like to express my deepest gratitude to Professor Katsunori Iwasaki for very useful conversations and his kind help during the preparation of this paper.

2. Proof of Theorem 1

In this section, we give the proof of Theorem 1. We write $A \gtrsim B$ to show that $|B/A|$ is bounded and when $A \gtrsim B$ and $A \lesssim B$, we write $A \approx B$. We set $|\frac{1}{m}|_R = \sum_{j\in R} \frac{1}{m_j}$ ($|\frac{1}{m}|_\emptyset := 0$) for $R \subseteq N := \{1, \dots, n\}$.

2.1. Integral representation of Bonami and Lohoué

Bonami and Lohoué [4] give an integral representation of the Szegö kernel of $\mathcal{E}_m$ (see (5.1) in §5). In the same fashion as in [4], we can obtain the following integral representation of the Bergman kernel of $\mathcal{E}_m$:

$$K^B(z) = \frac{1}{\pi^n} \int_0^\infty e^{-\tau} \prod_{j=1}^n F_{m_j}(|z_j|^2 \tau^{\frac{1}{m_j}}) \tau^{|\frac{1}{m}|_N} d\tau, \tag{2.1}$$

with

$$F_m(u) = m \sum_{\nu=0}^\infty \frac{u^\nu}{\Gamma(\frac{\nu}{m} + \frac{1}{m})},$$

where $m \in \mathbf{N}$. Here F_m is the derivative of Mittag-Leffler's function:

$$E_m(u) = \sum_{\nu=0}^\infty \frac{u^\nu}{\Gamma(\frac{\nu}{m} + 1)},$$

(i.e. $E'_m = F_m$).

We briefly explain the method of Bonami and Lohoué for obtaining the integral representation of $K^B(z)$. As mentioned in the Introduction, the following power series representation of $K^B(z)$ is given in [23],[8],[4]:

$$K^B(z) = c\sum_{\nu} \frac{\Gamma(\sum_{j=1}^n \frac{\nu_j}{m_j} + \sum_{j=1}^n \frac{1}{m_j} + 1)}{\prod_{j=1}^n \Gamma(\frac{\nu_j}{m_j} + \frac{1}{m_j})} \prod_{j=1}^n |z_j|^{2\nu_j}, \tag{2.2}$$

where $c = \frac{1}{2\pi^n}\prod_{j=1}^n m_j$. Next we represent the Gamma function in the numerator in terms of the integral expression and change the order of the integral and the sum. Finally we put in order the sum in the integral, then we can obtain (2.1).

2.2. Refinement of the error term $O(1)$

In this subsection, we investigate the error term $O(1)$ in (1.4) ([4], p.181) more precisely. The argument below is almost similar to that of Bonami and Lohoué [4].

Throughout this section, we investigate the Bergman kernel in a small neighborhood of the fixed boundary point $z^0 = (z_1^0, \ldots, z_n^0) \in \partial\mathcal{E}_m$. Let N, I, P and Q be defined by (1.3).

The following properties of Mittag-Leffler's function are necessary for the computation below.

Lemma 1 ([34],[38],[4]). *Regarding $F_m(u)$ as an entire function on the complex plane, $F_m(u)$ is expressed in the following form:*

$$F_m(u) = m^2 u^{m-1}\chi_{\mathcal{H}}(u)\, e^{u^m} + f_m(u), \tag{2.3}$$

where $\chi_{\mathcal{H}}(u) = \begin{cases} 1 & for \;\; u \in \mathcal{H} := \{|\arg u| < \frac{\pi}{2m}\} \\ 0 & otherwise, \end{cases}$ *and the function $f_m(u)$ has the following properties:* (i) *$f_m(u)$ is bounded in $\mathbf{C}$,* (ii) *$f_m(u)$ is holomorphic on $\mathcal{H}$, and* (iii) *there is a positive constant c such that $f_m(u) > c > 0$ for $u > 0$ and $\lim_{\substack{u\to 0\\ u\in\mathcal{H}}} f_m(u) = \frac{m}{\Gamma(\frac{1}{m})}$.*

Substituting (2.3) into the integral representation (2.1), we have

$$K^B(z) = \frac{n!}{\pi^n}\sum_{I\subseteq K\subseteq N} I_K(z),$$

where

$$(2.4)\quad I_K(z) = \frac{1}{n!}\prod_{j\in K} m_j^2|z_j|^{2m_j-2} \times \int_0^\infty e^{-[1-\sum_{j\in K}|z_j|^{2m_j}]\tau} \prod_{j\in N\setminus K} f_{m_j}(|z_j|^2\tau^{m_j})\tau^{|K|+|\frac{1}{m}|_{N\setminus K}}d\tau.$$

Applying Lemma 1 to (2.4), we obtain the following estimate for $I_K(z)$:

$$(2.5)\quad I_K(z) \approx \frac{\prod_{j\in K} m_j^2|z_j|^{2m_j-2}}{\left[1-\sum_{j\in K}|z_j|^{2m_j}\right]^{|K|+|\frac{1}{m}|_{N\setminus K}+1}} \quad \text{near } z^0 \text{ for } I\subseteq K\subseteq N.$$

By (2.5), we know that $I_K(z)$ $(I\subseteq K\subseteq N)$ is unbounded near z^0, if and only if $K\supseteq Q$. More precisely we have

Lemma 2.

$$\sum_{K\not\supseteq Q \text{ and } K\supseteq I} I_K(z) \in \mathrm{C}^\omega(\{z^0\}).$$

This lemma will be established in Section 4. It implies

$$(2.6)\qquad K^B(z) \equiv \frac{n!}{\pi^n}\sum_{K\supseteq Q} I_K(z) \quad \text{modulo } C^\omega(\{z^0\}).$$

Now, restricting $K^B(z)$ to the set $V=\{z\in\mathbf{C}^n; z_j=0\ (j\in P)\}$, we have

$$(2.7)\quad K^B(z)\equiv C_P^B\,\frac{\prod_{j\in Q} m_j^2|z_j|^{2m_j-2}}{\{1-\sum_{j\in Q}|z_j|^{2m_j}\}^{|Q|+|\frac{1}{m}|_P+1}} \quad \text{modulo } C^\omega(\{z^0\}).$$

In fact, $I_K(z)$ $(K\neq Q)$ vanishes identically on V and $f_m(0)=\frac{m}{\Gamma(\frac{1}{m})}$ by Lemma 1. The above formula is an improvement of (1.4) and if $z^0\in\partial\mathcal{E}_m$ is a strongly pseudoconvex point (i.e. $Q=N$), then we obtain (i) in the theorem.

Now we suppose that $z^0\in\partial\mathcal{E}_m$ is a weakly pseudoconvex point and investigate the behavior of $K^B(z)$ at z^0 without the above restriction. By (2.5) and (2.6), we obtain a precise estimate from above and below:

$$K^B(z)\approx \sum_{K\supseteq Q}\frac{\prod_{j\in K} m_j^2|z_j|^{2m_j-2}}{r(z)^{|K|+|\frac{1}{m}|_{N\setminus K}+1}} \qquad \text{near } z^0.$$

Note that the fact that $f_m(u) \approx 1$ for $u \geq 0$ plays an essential role in obtaining the above estimate. Furthermore we would like to investigate the asymptotic behavior of $K^B(z)$ at z^0. For this purpose, it is an important problem to obtain appropriate information about the function $F_m(u)$. Bonami and Lohoué ([4], pp.177–178) indicate that the asymptotic expansion of $K^B(z)$ can be obtained by using that of the function $F_m(u)$ at infinity. But the meaning of their expansion seems not to be clear. In this paper, we assert that Lemma 1 is sufficient information about $F_m(u)$ to obtain the asymptotic formula of $K^B(z)$. Instead of more detailed analysis of $F_m(u)$, we introduce another geometric idea, which will be mentioned in the next subsection.

2.3. New coordinates

In this subsection, we continue the argument of the previous subsection and complete the proof of the theorem.

From (2.3) and (2.4), we have

$$\begin{aligned}\sum_{K \supseteq Q} I_K(z) &= \frac{1}{n!} \prod_{j \in Q} m_j^2 |z_j|^{2m_j - 2} \int_0^\infty e^{-\tau[1 - \sum_{j \in Q} |z_j|^{2m_j}]} \\ &\quad \times \prod_{j \in P} \{ m_j^2 |z_j|^{2m_j - 2} \tau^{1 - \frac{1}{m_j}} e^{\tau |z_j|^{2m_j}} + f_{m_j}(|z_j|^2 \tau^{\frac{1}{m_j}}) \} \tau^{|Q| + |\frac{1}{m}|_P} d\tau \\ &= \frac{1}{n!} \prod_{j \in Q} m_j^2 |z_j|^{2m_j - 2} \\ &\quad \times \int_0^\infty e^{-\tau[1 - \sum_{j \in Q} |z_j|^{2m_j}]} \prod_{j \in P} F_{m_j}(|z_j|^2 \tau^{\frac{1}{m_j}}) \tau^{|Q| + |\frac{1}{m}|_P} d\tau.\end{aligned}$$

Now we introduce new variables $t = (t_j)_{j \in P}$, where

$$t_j^{2m_j} = \frac{|z_j|^{2m_j}}{1 - \sum_{k \in Q} |z_k|^{2m_k}} \quad (j \in P).$$

Then we have

$$\sum_{K \supseteq Q} I_K(z) = \prod_{j \in Q} m_j^2 |z_j|^{2m_j - 2} \frac{\Phi^B(t(z))}{r(z)^{|Q| + |\frac{1}{m}|_P + 1}}, \tag{2.8}$$

where

$$\Phi^B(t) = \frac{1}{n!} \Big[1 - \sum_{j \in P} t_j^{2m_j} \Big]^{|Q| + |\frac{1}{m}|_P + 1} \tag{2.9}$$

$$\times \int_0^\infty e^{-s} \prod_{j\in P} F_{m_j}(t_j^2 s^{\frac{1}{m_j}}) s^{|Q|+|\frac{1}{m}|_P} ds.$$

Substituting (2.8) into (2.6), we obtain (1.5) in Theorem 1. Since $\prod_{j\in Q} m_j^2 |z_j|^{2m_j-2}$ is positive near z^0, (2.8) implies that the singularities of $K^B(z)$ is essentially expressed in terms of the polar coordinates (t, r). We show the remaining assertions of Theorem 1.

First, we can obtain that $\Phi^B(t)$ is real analytic on the locally closed simplex

$$\Delta = \left\{ t = (t_j)_{j\in P} \, ; \, t_j \geq 0, \sum_{j\in P} t_j^{2m_j} < 1 \right\}, \tag{2.10}$$

in the same fashion as in the proof of Lemma 2.

Next we can obtain:

$$\Phi^B(t(z)) = \frac{1}{n!} \left[1 - \sum_{j\in P} t_j^{2m_j} \right]^{|Q|+|\frac{1}{m}|_P+1} \sum_{K\subseteq P} J_K(t(z)) \tag{2.11}$$

where

$$J_K(t) = \prod_{j\in K} m_j^2 t_j^{2m_j-2}$$
$$\times \int_0^\infty e^{-[1-\sum_{j\in K} t_j^{2m_j}]s} \prod_{j\in P\setminus K} f_{m_j}(t_j^2 s_j^{\frac{1}{m_j}}) s^{|Q|+|K|+|\frac{1}{m}|_{P\setminus K}} ds,$$

in the same fashion as in Subsection 2.2. Here each $J_K(t)$ has the following estimate:

$$J_K(t) \approx \frac{\prod_{j\in K} m_j^2 t_j^{2m_j-2}}{[1 - \sum_{j\in K} t_j^{2m_j}]^{|Q|+|K|+|\frac{1}{m}|_{P\setminus K}+1}}. \tag{2.12}$$

Now we claim (a) $\Phi^B(t)$ is positive on Δ and (b) $\Phi^B(t)$ is unbounded as $t \in \Delta$ approaches $\overline{\Delta} \setminus \Delta$.

(a) : Since $J_\emptyset(t) \gtrsim 1$ by (2.12), $\Phi^B(t)$ is positive on Δ by (2.11).

(b) : We consider the case where t approaches $t^0 = (t_j^0)_{j\in P} \in \overline{\Delta} \setminus \Delta$. Let $P_{[2]}, Q_{[2]}$ be the sets defined by $P_{[2]} = \{j \in P; t_j^0 = 0\}$, $Q_{[2]} = \{j \in P; t_j^0 \neq 0\}$ respectively. By (2.12), we have

$$\left[1 - \sum_{j\in P} t_j^{2m_j} \right]^{|Q|+|\frac{1}{m}|_P+1} J_{Q_{[2]}}(t) \approx \left[1 - \sum_{j\in Q_{[2]}} t_j^{2m_j} \right]^{-|Q_{[2]}|+|\frac{1}{m}|_{Q_{[2]}}}.$$

Since $Q_{[2]}$ is not empty, $[1-\sum_{j\in P} t_j^{2m_j}]^{|Q|+|\frac{1}{m}|_P+1} J_{Q_{[2]}}(t)$ is unbounded as $t \to t^0$. Hence we obtain (b) by (2.11).

This completes the proof of Theorem 1. $\square$

REMARKS. *1.* Since $|Q|+|K|+|\frac{1}{m}|_{P\setminus K}+1 \le |P|+|Q|+1 = n+1$ and $1-\sum_{j\in K} t_j^{2m_j} \ge 1-\sum_{j\in P} t_j^{2m_j}$, we have

$$J_K(t) \lesssim \left[1-\sum_{j\in P} t_j^{2m_j}\right]^{-|P|+|\frac{1}{m}|_P}$$

on Δ. By (2.11), we have

$$\Phi^B(t) \lesssim \left[1-\sum_{j\in P} t_j^{2m_j}\right]^{-|P|+|\frac{1}{m}|_P} \lesssim r(z)^{-|P|+|\frac{1}{m}|_P}. \tag{2.13}$$

The above estimate is optimal. In fact, we have $J_P(t) \approx \prod_{j\in P} t_j^{2m_j-2}[1-\sum_{j\in P} t_j^{2m_j}]^{-|P|+|\frac{1}{m}|_P}$. By (2.8), (2.13), we have

$$K^B(z) \lesssim \frac{1}{r(z)^{n+1}}.$$

2. In the case $m=(1,\dots,1,m)$, we can obtain the following closed expression of $K^B(z)$:

$$K^B(z) = \frac{n!}{\pi^n}\frac{\Phi^B(t)}{r^{n+\frac{1}{m}}}, \quad \text{with} \quad \Phi^B(t) = mT^{1-\frac{1}{m}}(1-T)^{n+\frac{1}{m}}\frac{d^n}{dT^n}\left(\frac{T^{n-1}}{1-T^{\frac{1}{m}}}\right),$$

where $T = t^{2m}$.

3. Recursive Formula

In this section, we investigate the structure of the singularities of the Bergman kernel of $\mathcal{E}_m$ in more detail. From the viewpoint of ordinary differential equations, the argument below reminds us of the process of step-by-step confluence from the regular singularity to the irregular singularity (see [25]). In this section, the results below can be justified in the same fashion in Section 2, so we omit the detailed proofs of them.

We remark that $\Phi^B(t)$ defined by (2.9) takes the same form as $K^B(z)$ in (2.1). So the argument in Section 2 applies to $\Phi^B(t)$ in place of $K^B(z)$, and the form of the singularities of $\Phi^B(t)$ can be written in the same fashion as in Theorem 1. Moreover we can completely understand the singularities of $\Phi^B(t)$ by finitely many recursive process of this kind.

We precisely explain this process. We suppose that $z^0 \in \partial\mathcal{E}_m$ is a weakly pseudoconvex point (i.e. $P \neq \emptyset$). We inductively define the sets $P_{[k]}$, $Q_{[k]} \subset N$, the variables $t_{[k]} = (t_{[k],j})_{j\in P_{[k]}}$, $r_{[k]}$, the simplex $\Delta_{[k]}$ and the function $\Phi_{[k]}(t_{[k]})$ on $\Delta_{[k]}$ in the following way.

First we set $P_{[1]} = P(\neq \emptyset)$, $Q_{[1]} = Q$, $t_{[1]} = (t_{[1],j})_{j\in P_{[1]}} = (t_j)_{j\in P}$, $r_{[1]} = r$, $\Delta_{[1]} = \Delta$ and $\Phi_{[1]}(t_{[1]}) = \Phi^B(t)$. Suppose that the sets $P_{[k-1]}(\neq \emptyset)$, $Q_{[k-1]} \subset P$ are settled, then the simplex $\Delta_{[k-1]}$ is defined by

$$\begin{aligned}\Delta_{[k-1]} &= \{t_{[k-1]} = (t_{[k-1],j})_{j\in P_{[k-1]}}; t_{[k-1],j} \geq 0, \sum_{j\in P_{[k-1]}} t_{[k-1],j}^{2m_j} < 1\} \\ &\subset \mathbf{R}^{|P_{[k-1]}|}.\end{aligned}$$

When we select a point $t_{[k-1]}^0 = (t_{[k-1],j}^0)_{j\in P_{[k]}} \in \overline{\Delta_{[k-1]}} \setminus \Delta_{[k-1]}$, the sets $P_{[k]}$, $Q_{[k]} \subset P_{[k-1]}$ are determined by

$$\begin{cases} P_{[k]} &= \{j \in P_{[k-1]}; t_{[k-1],j}^0 = 0\}, \\ Q_{[k]} &= \{j \in P_{[k-1]}; t_{[k-1],j}^0 \neq 0\}. \end{cases}$$

Furthermore the variables $t_{[k]} = (t_{[k],j})_{j\in P_{[k]}}$, $r_{[k]}$ are defined by

$$\begin{cases} t_{[k],j}^{2m_j} &= \frac{t_{[k-1],j}^{2m_j}}{1-\sum_{j\in Q_{[k]}} t_{[k-1],j}^{2m_j}} \quad (j \in P_{[k]}), \\ r_{[k]} &= 1 - \sum_{j\in Q_{[k]}} t_{[k-1],j}^{2m_j}. \end{cases}$$

Then we define the function $\Phi_{[k]}(t_{[k]})$ on the simplex $\Delta_{[k]}$ in the following. If $P_{[k]} = \emptyset$, then $\Phi_{[k]}(t_{[k]}) = 1$ identically. If $P_{[k]} \neq \emptyset$, then

$$\begin{aligned}\Phi_{[k]}(t_{[k]}) = \frac{1}{n!}&\left[1 - \sum_{j\in P_{[k]}} t_{[k],j}^{2m_j}\right]^{a_{[k]}+1} \\ &\times \int_0^\infty e^{-s} \prod_{j\in P_{[k]}} F_{m_j}(t_{[k],j}^2 s^{\frac{1}{m_j}}) s^{a_{[k]}} ds,\end{aligned} \tag{3.1}$$

where the constant $a_{[k]}$ is defined by $a_{[k]} = \sum_{j=1}^{k} |Q_{[j]}| + |\frac{1}{m}|_P - \sum_{j=2}^{k} |\frac{1}{m}|_{Q_{[j]}}$. In the above inductive process, we have

$$P = P_{[1]} \supsetneq P_{[2]} \supsetneq \cdots \supsetneq P_{[k-1]} \supsetneq P_{[k]}.$$

So there exists a positive integer $k^0 \leq |P|$ such that $P_{[k^0+1]} = \emptyset$. Thus we have defined $P_{[k]}, Q_{[k]}, t_{[k]}, r_{[k]}, \Delta_{[k]}, \Phi_{[k]}(t_{[k]})$ for $k = 1, 2, \ldots, k^0$ recursively. Moreover we set $Q_{[k^0+1]} = P_{[k^0]}$, $r_{[k^0+1]} = 1 - \sum_{j \in Q_{[k^0+1]}} t_{[k^0],j}^{2m_j}$ and $\Phi_{[k^0+1]}(t_{[k^0+1]}) \equiv 1$.

We remark that $\Phi_{[k]}(t_{[k]})$ in (3.1) takes the same form as $K^B(z)$ in (2.1). So we obtain the following proposition for $\Phi_{[k]}(t_{[k]})$ in the same manner as we have obtained Theorem 1 for $K^B(t)$.

PROPOSITION 1. *Suppose that* $1 \leq k \leq k^0$. *The function* $\Phi_{[k]}(t_{[k]})$ *is a positive and real analytic function on* $\Delta_{[k]}$ *and is unbounded as* $t_{[k]} \in \Delta_{[k]}$ *approaches* $t_{[k]}^0 \in \overline{\Delta_{[k]}} \setminus \Delta_{[k]}$. *Moreover we have the following recursive formula:*

$$\Phi_{[k]}(t_{[k]}) \equiv \prod_{j \in Q_{[k+1]}} m_j^2 t_{[k],j}^{2m_j-2} \frac{\Phi_{[k+1]}(t_{[k+1]})}{r_{[k+1]}^{|Q_{[k+1]}| - |\frac{1}{m}|_{Q_{[k+1]}}}} \text{ modulo } C^{\omega}(\{t_{[k]}^0\}). \tag{3.2}$$

We remark that the condition $k = k^0$ (resp. $1 \leq k \leq k^0 - 1$) corresponds to the strongly (resp. the weakly) pseudoconvex case in Theorem 1. The formula (3.2) recursively reduces $\Phi^B(t)$ to $\Phi_{[k^0+1]} \equiv 1$. Hence it may be interpreted that the above recursive process resolves the degeneration of the Levi form in the study of singularities of the Bergman kernel in the weakly pseudoconvex case.

4. Proof of Lemma 2

By (2.3), we obtain

$$\sum_{K \not\supseteq Q \text{ and } K \supseteq I} I_K(z) = \sum_{I \subseteq J \subset Q} \widetilde{I_J}(z),$$

where

$$\widetilde{I_J}(z) = \frac{1}{\pi^n} \prod_{j \in J} m_j^2 |z_j|^{2m_j-2} \int_0^{\infty} e^{-[1-\sum_{j \in J} |z_j|^{2m_j}]\tau}$$

$$\times \prod_{j\in Q\setminus J} f_{m_j}(|z_j|^2\tau^{\frac{1}{m_j}}) \prod_{j\in P} F_{m_j}(|z_j|^2\tau^{\frac{1}{m_j}})\tau^{|J|+|\frac{1}{m}|_{N\setminus J}} d\tau.$$

Thus it is sufficient to show that

$$\widetilde{I_J}(z) \in C^\omega(\{z^0\}), \tag{4.1}$$

for $I \subseteq J \subset Q$.

Let $\widehat{I_J}(u)$ be the function of complex variables $u = (u_1, \ldots, u_n) \in \mathbf{C}^n$ defined by

$$\begin{aligned}\widehat{I_J}(u) = \int_0^\infty & e^{-[1-\sum_{j\in J} u_j^{m_j}]\tau} \\ & \times \prod_{j\in Q\setminus J} f_{m_j}(u_j\tau^{\frac{1}{m_j}}) \prod_{j\in P} F_{m_j}(u_j\tau^{\frac{1}{m_j}})\tau^{|J|+|\frac{1}{m}|_{N\setminus J}} d\tau.\end{aligned} \tag{4.2}$$

In order to obtain (4.1), it is sufficient to show that there exists a neighborhood in $\mathbf{C}^n$ of $u^0 = (u_1^0, \ldots, u_n^0) := (|z_1^0|^2, \ldots, |z_n^0|^2)$ such that $\widehat{I_J}(u)$ is holomorphic there. Note that $\prod_{j\in J} m_j^2 |z_j|^{2m_j-2}$ is real analytic at z^0.

Now let $\mathcal{N}_J$ be the neighborhood of u^0 defined by

$$\begin{aligned}\mathcal{N}_J = \Big\{ & u \in \mathbf{C}^n; |u_j| < 1 \text{ for } j \in P, \\ & |u_j - u_j^0| < \frac{u_j^0}{2} \text{ for } j \in Q\setminus J \text{, and } 1 - \sum_{j\in J\cup P} |u_j|^{m_j} > \frac{\delta}{2}\Big\},\end{aligned}$$

where $\delta = 1 - \sum_{j\in J\cup P} u_j^0 > 0$. We show that $\widehat{I_J}(u)$ is holomorphic in $\mathcal{N}_J$. Since F_m is an entire function and f_m is holomorphic in the sector $\{u; |\arg u| < \frac{\pi}{2m}\}$ by Lemma 1 (ii), the integrand of (4.2) is holomorphic in $\mathcal{N}_J$ for $\tau > 0$. Each partial derivative of the integrand in (4.2) with respect to u_j is continuous on $\mathcal{N}_J \times (0, \infty)$. By Lemma 1, we have

$$|F_m(u)| \le c|u|^{m-1}e^{|u|^m} \text{ and } |f_m(u)| \le c,$$

on $\mathbf{C}$, where c is a positive constant. Thus we have

$$\begin{aligned}|\widehat{I_J}(u)| \le & \int_0^\infty e^{-[1-\sum_{j\in J}|u_j|^{m_j}]\tau} \\ & \times \prod_{j\in Q\setminus J} |f_{m_j}(u_j\tau^{\frac{1}{m_j}})| \prod_{j\in P} |F_{m_j}(u_j\tau^{\frac{1}{m_j}})| \, \tau^{|J|+|\frac{1}{m}|_{N\setminus J}} d\tau\end{aligned}$$

$$
\begin{aligned}
&\leq \quad c^{n+1-|J|}\int_0^\infty e^{-[1-\sum_{j\in J\cup P}|u_j|^{m_j}]\tau}\tau^{(m-1)|P|+|J|+|\frac{1}{m}|_{N\setminus J}}d\tau \\
&\leq \quad c^{n+1-|J|}\frac{\Gamma((m-1)|P|+|J|+|\frac{1}{m}|_{N\setminus J}+1)}{(\delta/2)^{(m-1)|P|+|J|+|\frac{1}{m}|_{N\setminus J}+1}}
\end{aligned}
$$

on $\mathcal{N}_J$. Hence we can see that $\widehat{I_J}(u)$ is holomorphic on $\mathcal{N}_J$ by the above inequalities. $\square$

Remark. Consider the smoothness of the Bergman kernel $K^B(z,w)$ off the diagonal (i.e. $\Delta := \{(z,w); z = w \in \partial\mathcal{E}_m\}$). Here $K^B(z,w) = \sum_j \phi_j(z)\overline{\phi_j(w)}$, where $\{\phi_j\}_j$ is as in the Introduction. We have

$$K^B(z,w) \in C^\omega(\overline{\mathcal{E}_m}\times\overline{\mathcal{E}_m}\setminus\Delta).$$

This can be obtained by putting together the proof of Lemma 2 and the argument in [4], pp.170–171.

5. The Szegö Kernel of $\mathcal{E}_m$

In this section, we establish a result similar to Theorem 1 for the Szegö kernel of $\mathcal{E}_m$. The result below is obtained in the same fashion as in the case of the Bergman kernel and we omit the proof.

Let Ω be a bounded domain in $\mathbf{C}^n$ with smooth boundary. Specify a surface element σ on the boundary $\partial\Omega$, and denote by $H^2_\sigma(\Omega)$ the set of holomorphic functions in Ω having L^2-boundary values with respect to σ. The *Szegö kernel* of Ω (with respect to σ) is defined by

$$K^S(z,w) = \sum_j |\tilde{\phi_j}(z)|^2$$

where $\{\tilde{\phi_j}\}$ is a complete orthonormal basis for $H^2_\sigma(\Omega)$.

We study the Szegö kernel of $\mathcal{E}_m$ with respect to the surface element which is introduced by Bonami and Lohoué in [4] (they denote the surface element by $d\mu_\alpha$).

An integral representation of $K^S(z)$ is given in [4] ((2.4), p.168):

$$K^S(z) = \frac{1}{2\pi^n}\int_0^\infty e^{-\tau}\prod_{j=1}^n F_{m_j}(|z_j|^2\tau^{\frac{1}{m_j}})\tau^{|\frac{1}{m}|_N-1}d\tau. \tag{5.1}$$

Since the difference between (2.1) and (5.1) does not give any essential influence on the argument in Section 2, we have a similar result about the singularities of $K^S(z)$.

THEOREM 2. *There is a function $\Phi^S(t) \in C^\omega(\Delta)$ such that*

$$K^S(z) \equiv \frac{(n-1)!}{2\pi^n} \prod_{j\in Q} m_j^2 |z_j|^{2m_j-2} \frac{\Phi^S(t(z))}{r(z)^{|Q|+|\frac{1}{m}|_P}} \qquad \text{modulo} \ \ \mathrm{C}^\omega(\{z^0\}).$$

Here $\Phi^S(t)$ also has the same properties as in Theorem 1 for $\Phi^B(t)$.

REMARKS. *1.* The precise expression of $\Phi^S(t)$ is the following:

$$\Phi^S(t) = \frac{1}{(n-1)!}\Big[1 - \sum_{j\in P} t_j^{2m_j}\Big]^{|Q|+|\frac{1}{m}|_P} \\ \times \int_0^\infty e^{-s} \prod_{j\in P} F_{m_j}(t_j^2 s^{\frac{1}{m_j}}) s^{|Q|+|\frac{1}{m}|_P - 1} ds.$$

In the case $m = (1, \ldots, 1, m)$, we have the following closed expression:

$$K^S(z) = \frac{(n-1)!}{2\pi^n} \frac{\Phi^S(t)}{r^{n-1+\frac{1}{m}}},$$

with

$$\Phi^S(t) = mT^{1-\frac{1}{m}}(1-T)^{n-1+\frac{1}{m}} \frac{d^{n-1}}{dT^{n-1}}\left(\frac{T^{n-2}}{1-T^{\frac{1}{m}}}\right),$$

where $T = t^{2m}$.

2. Consider the smoothness of the Szegö kernel $K^S(z,w)$ $(:= \sum_j \tilde{\phi}_j(z)\overline{\tilde{\phi}_j(w)})$ off the diagonal. We obtain

$$K^S(z,w) \in C^\omega(\overline{\mathcal{E}_m} \times \overline{\mathcal{E}_m} \setminus \Delta),$$

in the same fashion as in the case of the Bergman kernel. See Remark in Section 4.

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(Received December 10, 1996)

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