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Calculation of the Hall Conductivity by Adiabatic Approximation

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Abstract. We consider the two dimensional electron system under a constant magnetic field perpendicular to the plane, and present a simple definition of the Hall conductivity which is the non-diagonal component of the conductivity tensor. To calculate the Hall conductivity, we adopt the adiabatic approximation and derive that it is equal to the Chern character modulo the order of any inverse power of the adiabatic parameter. Finite temperature correction is also considered and estimated to be exponentially small.

§1. Introduction

When the electric field E is applied to a very thin metallic plate subject to a perpendicular magnetic field, a current in the direction perpendicular to E is observed. The current is called the Hall current and the Hall conductivity is the ratio σ_H of the current to the electric potential. A remarkable fact first observed by von Klitzing and others[Kl] is that σ_H is quantized at very low temperature, viz. σ_H is an integral multiple of e^2/h. This is called the integer quantum Hall effect.

There is a large body of mathematical and physical literature on the integer quantum Hall effect. In these works the authors deduce the quantization of the Hall conductivity as follows: They take certain models and compute the Hall conductivity σ_H and show that σ_H is equal to a suitable topological invariant up to a constant; therefore, σ_H is quantized. There are, however, various possible models to be employed and various methods to compute the Hall conductivity.

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For example, in many physics literature (e.g., [TKNN]), the linear response theory is used to calculate the Hall conductivity. This is the standard method in theoretical physics and the product obtained is called "Kubo formula". And it is shown that the Hall conductivity is equal to the first Chern class over the torus. As the mathematical research, in [ASY, KS], the model is N-particle time-dependent Schrödinger operator on a 2-dimensional bounded domain. They used the method called "adiabatic switching" to calculate the Hall conductivity and showed that it is equal to the first Chern class over the torus. In [B2,N], the model is one-particle time-dependent Schrödinger operator with random kick rotor term on $\mathbf{R}^2$. They used the relaxation time approximation to calculate the Hall conductivity and showed that it is equal to the second Chern character first introduced by Connes in the non-commutative geometry[C].

In this paper, we present a simple approach to calculate the Hall conductivity. We adopt Bellissard's setting of one-particle ergodic Schrödinger operator, and consider a natural definition of the Hall conductivity. Then we calculate the Hall conductivity by using the adiabatic approximation and prove that it is equal to the Chern character which is the same as what appears in [B2]. The error of the adiabatic approximation is estimated to be the order of any inverse power of the adiabatic parameter.

Our model is described by the following time-dependent Schrödinger operator on $L^2(\mathbf{R}^2)$:

$$\begin{aligned} H_\omega(t) &:= (-i\nabla - A(x) - f(t)e_1)^2 + V_\omega(x), \\ t &\in [0,1], \quad x = (x_1, x_2) \in \mathbf{R}^2, \end{aligned} \tag{1.1}$$

where $A(x) = (-\frac{Bx_2}{2}, \frac{Bx_1}{2})$ corresponds to the constant magnetic field B: $dA_0 = Bdx_1 \wedge dx_2$. $e_1 := (1,0)$. $f(t) \in C^\infty(0,1)$ is a real valued function which satisfies $f(0) = 0$, $f(1) = 1$ and $f'(t) \in C_0^\infty(0,1)$. Due to the Faraday-Lenz law, the change of flux $f'(t)$ represents a time-dependent electric field perpendicular to x_1. By taking $f'(t)$ near 1, we regard the electric field is normalized. $V_\omega(x) \in L^\infty(\mathbf{R}^2)(\omega \in \Omega)$ is a real-valued random potential on a probability space Ω which can be considered to describe the disordered configuration. We assume that, for any $a \in \mathbf{R}^2$, there is a measure preserving map T^a on Ω such that $V_\omega(x)$ satisfies $V_\omega(x+a) = V_{T^a\omega}(x)$. In fact, Ω and $V_\omega(x)$ can be constructed from a potential $V(x) \in L^\infty(\mathbf{R}^2)$ by considering all of its translations and by identifying Ω with the weak$*$-closure of the set $\{V(\cdot - a) : a \in \mathbf{R}^2\}$(cf.[B2, B3]). Let $\mathbf{P}$ be a normalized, T-invariant and ergodic measure on Ω.

To define the Hall conductivity, we need some notations. We define the trace per area of an operator A on $L^2(\mathbf{R}^2)$ by:

$$\mathcal{T}(A) := \lim_{L\uparrow\infty} \frac{1}{|K_L|}\mathrm{Trace}(\chi_{K_L} A \chi_{K_L}), \tag{1.2}$$

whenever it exists, where $K_L := [-L, L] \times [-L, L] \subset \mathbf{R}^2$ and $|K_L| = 4L^2$ is its area. χ_{K_L} is the characteristic function of K_L. Let $\varepsilon_F \in \mathbf{R}$ be a constant and $P_\omega(t) := P_{(-\infty,\varepsilon_F)}(H_\omega(t))$ be the orthogonal projection below ε_F. This means we consider all electrons whose energy is below ε_F. In physics literature, ε_F is called the Fermi energy. To simplify the notation, we write H_ω and P_ω instead of $H_\omega(0)$ and $P_\omega(0)$ respectively. We assume throughout this paper (and is assumed also in many mathematical papers treating the integer quantum Hall effect) that

Assumption (A). *The Fermi energy ε_F lies in the gap of the spectrum of H_ω.*

When $V_\omega = 0$, the spectrum of H_ω is given explicitly by $\sigma(H_\omega) = \cup_{n=0}^{\infty}\{2(n + \frac{1}{2})B\}$. Since $V_\omega(x) \in L^\infty(\mathbf{R}^2)$, $\sigma(H_\omega)$ has gaps for sufficiently large B. It is true that this assumption is too strong and we wish to remove it eventually. However, the adiabatic approximation theorem to be used in what follows is not yet established without this Assumption (A). On the other hand, we should remark that, in [B2], it is proved that if the localization length is finite in the neighborhood of the Fermi energy, the Hall conductivity σ_ω to be computed in Corollary 1.3 below remains constant under the small variation of the Fermi energy.

Let $U_\omega(t), t \in [0,1], \omega \in \Omega$ be the unitary evolution of $H_\omega(t)$:

$$\begin{aligned} i\frac{\partial}{\partial t}U_\omega(t)\psi &= H_\omega(t)U_\omega(t)\psi, \\ U_\omega(0)\psi &= \psi, \quad \psi \in D. \end{aligned} \tag{1.3}$$

whose existence is guaranteed by a theorem of Kato and Yosida[RS2]. In (1.3), $D(\subset L^2(\mathbf{R}^2))$ is the domain of $H_\omega(t)$ which is independent of $t \in [0, 1]$.

We introduce the current operator perpendicular to x_2:

$$J_\omega(t) := U_\omega^*(t) i [H_\omega(t), x_2] U_\omega(t). \tag{1.4}$$

We define the charge transport in the time interval $[0,1]$ as the integration $\int_0^1 dt$ of the thermal average of $J_\omega(t)$ per area at temperature $T = 0$:

$$j_\omega := \int_0^1 dt\, \mathcal{T}(U_\omega^*(t) i [H_\omega(t), x_2] U_\omega(t) P_\omega). \tag{1.5}$$

Since the exchange of integration of t for trace per area in (1.5) is permitted for $\mathbf{P}$-a.e.ω by dominated convergence theorem,

$$j_\omega := \mathcal{T}((U_\omega^*(1) x_2 U_\omega(1) - x_2) P_\omega). \tag{1.6}$$

By (1.6), we can also regard j_ω as the thermal average of the operator of the displacement of particles. We suppose that, in general, j_ω can be written as $j_\omega = \sigma E + O(E^2)$ when the electric field E is small, and we may obtain the Hall conductivity by putting $\sigma := \lim_{E \downarrow 0} \frac{j_\omega}{E}$. Thus we consider the small electric field limit. To accomplish this situation, we let the flux increase very slowly by introducing the adiabatic parameter $\tau > 0$, putting $s := t/\tau$ and replacing t by s. The constant τ is taken so large that when s varies from 0 to 1, the real time t, which varies from 0 to τ, goes through long time, and electric field, which now becomes $f'(s)/\tau$, is small. We then replace $U_\omega(t)$ by the new time evolution $U_{\omega,\tau}(s)$:

$$\begin{aligned} i\frac{\partial}{\partial s} U_{\omega,\tau}(s)\psi &= \tau H_\omega(s) U_{\omega,\tau}(s)\psi, \\ U_{\omega,\tau}(0)\psi &= \psi, \quad \psi \in D. \end{aligned} \tag{1.7}$$

We define the charge transport in the time interval $[0,\tau]$ and under the electric field $f'(t)/\tau$ by

$$j_{\omega,\tau} := \mathcal{T}\left((U_{\omega,\tau}^*(1)\, x_2\, U_{\omega,\tau}(1) - x_2) P_\omega\right). \tag{1.8}$$

The well-definedness of (1.5) and (1.8) for $\mathbf{P}$-a.e.ω is proved in section 2. We define the Hall conductivity as the adiabatic limit of $j_{\omega,\tau}$:

$$\sigma_\omega := \lim_{\tau \uparrow \infty} j_{\omega,\tau}, \tag{1.9}$$

and one of the main purpose of this paper is to show that the limit (1.9) exists and that σ_ω is equal to the Chern character of P_ω. We remark that, instead of dividing $j_{\omega,\tau}$ by the electric field $\frac{1}{\tau}f'(t)$ in the unit time interval $t \in [0,1]$, we consider $j_{\omega,\tau}$ itself in the time interval $[0,\tau]$ in (1.9). However, this leads to the same result.

Following [Ka], we consider the adiabatic evolution $U^A_{\omega,\tau}(s)$ defined by:

$$\begin{aligned} i\frac{\partial}{\partial s}U^A_{\omega,\tau}(s)\psi &= \tau H^A_\omega(s)U^A_{\omega,\tau}(s)\psi, \\ U^A_{\omega,\tau}(0)\psi &= \psi, \quad \psi \in D. \end{aligned} \tag{1.10}$$

where $H^A_\omega(s) := H_\omega(s) + \frac{i}{\tau}[P'_\omega(s), P_\omega(s)]$. The operator $P'_\omega(s)$ is the derivative of $P_\omega(s)$ w.r.t.s in the operator norm. It is known that when τ is large, $U^A_{\omega,\tau}(s)$ approximates $U_{\omega,\tau}(s)$ in the operator norm up to the order of $O(\tau^{-1})$([Ka, ASY] and references therein). We define the adiabatically approximated charge transport:

$$j^A_{\omega,\tau} := \mathcal{T}\left((U^{A*}_{\omega,\tau}(1)\, x_2\, U^A_{\omega,\tau}(1) - x_2)P_\omega\right). \tag{1.11}$$

We estimate the error $j_{\omega,\tau} - j^A_{\omega,\tau}$ of this approximation in section 3. The result is

THEOREM 1.1. *Under Assumption (A), the error $j_{\omega,\tau} - j^A_{\omega,\tau}$ of the adiabatic approximation is estimated for $\mathbf{P} - a.e.\ \omega$ as follows*

$$j_{\omega,\tau} = j^A_{\omega,\tau} + O(\tau^{-\infty}), \quad \tau \to \infty. \tag{1.12}$$

We should remark that similar result is obtained in [KS] for the model of Avron, Seiler, Yaffe and Klein and our proof also uses some general results obtained in [KS] on the error estimate of the adiabatic approximation.

We have the following expression of $j^A_{\omega,\tau}$.

THEOREM 1.2. *Under Assumption (A), the adiabatically approximated charge transport $j^A_{\omega,\tau}$ can be written for $\mathbf{P} - a.e.\ \omega$ in the following form*

$$j^A_{\omega,\tau} = -i\mathcal{T}(P_\omega[\partial_{x_1}P_\omega, \partial_{x_2}P_\omega]P_\omega). \tag{1.13}$$

where $\partial_{x_j} P_\omega := i[P_\omega, x_j]$, $j = 1, 2$.

The proof of Theorem 1.2 is given in section 4. The RHS of (1.13) is the Chern character of P_ω[B2] and is independent of τ. Theorem 1.1 and Therem 1.2 imply that our purpose has been achieved.

COROLLARY 1.3. *The Hall conductivity defined in (1.9) exists and is equal to the Chern character for P-a.e.* ω*:*

$$\sigma_\omega = -i\mathcal{T}(P_\omega[\partial_{x_1} P_\omega, \partial_{x_2} P_\omega]P_\omega). \tag{1.14}$$

To prove the quantization of the Hall conductivity, we use the results of [ASS] but under the slightly different definitions remarked in section 5.

PROPOSITION 1.4. *The adiabatically approximated charge transport* $j^A_{\omega,\tau}$ *is written in terms of the Fredholm index on the Range of* P_ω *for* $\mathbf{P}$*-a.e.* ω*:*

$$j^A_{\omega,\tau} = \frac{1}{2\pi} Index(P_\omega \frac{z}{|z|} P_\omega), \tag{1.15}$$

where $z = x_1 + ix_2$, *and* $Index(P_\omega \frac{z}{|z|} P_\omega)$ *is the Fredholm Index of the multiplication operator* $z/|z|$ *on* $Range P_\omega$.

As the final topic, in section 6, we consider the zero temperature limit. We define the Hall charge transport at temperature $T(> 0)$ by

$$j_{\omega,\tau}(T) := \mathcal{T}\left((U^*_{\omega,\tau}(1)\, x_2\, U_{\omega,\tau}(1) - x_2) f_{T,\varepsilon_F}(H_\omega)\right), \tag{1.16}$$

where $f_{T,\mu}(\varepsilon) = (1 + \exp((\varepsilon - \mu)/kT))^{-1}$ is the Fermi-Dirac distribution function, k is the Boltzmann constant, and μ is the chemical potential. Adiabatically approximated charge transport $j^A_{\omega,\tau}(T)$ is defined similarly, that is, by the right hand side of (1.16) with $U^A_{\omega,\tau}(1)$ in place of $U_{\omega,\tau}(1)$.

PROPOSITION 1.5. *If the spectral gap where* ε_F *lives contains an interval* $(a, b) \subset \mathbf{R}$*, it holds that, for arbitrary* $\varepsilon > 0$ *and for* $\mathbf{P}$*-a.e.* ω*,*

$$j^A_{\omega,\tau}(T) = j^A_{\omega,\tau} + O(\exp(-(\Delta - \varepsilon)/kT)), \quad T \downarrow 0, \tag{1.17}$$

$$(1.18) \qquad j_{\omega,\tau}(T) = j_{\omega,\tau} + O(\exp(-(\Delta - \varepsilon)/kT)), \quad T \downarrow 0,$$

where $\Delta = \min(b - \varepsilon_F, \varepsilon_F - a)$.

At this point, we should remark about the difference of our work from [ASY, KS]. In our discussion, the flux is adiabatically switched to enforce the electric field. It is the same argument as in [ASY, KS]. But our model is different from theirs in several points: (1) Our definition of the conductivity is simpler. (2) The space on which our models work is $\mathbf{R}^2$, which is natural as the model of the bulk system in the grand canonical ensemble. (3) Our Hamiltonian is one-particle and based on Bellissard's setting of the ergodic Schrödinger operators. (4) Our topological invariant is Fredholm index of which we can compute the concrete value [ASS, B2].

In the following sections, we prove the theorems and propositions. In appendix, we study some relationship with the works of Avron, Seiler, Klein, and Yaffe[ASY,KS]. We show that σ_ω is equal to the trace per area of the adiabatic curvature. Moreover, in the model of Klein and Seiler, we calculate the Hall conductivity and obtain the same results as in [KS]. Next, we show that the Hall conductivity can be regarded as the thermal average of Berry's phase in a sense.

§2. Preparation

In this section, we introduce the following operator algebra $\mathcal{A}$ which is suitable for $H_\omega(t)$. The operator algebra is used by Bellissard [B1, 2, 3], Nakamura and Bellissard[NB]. Let $\mathcal{A}_0 := C_0(\Omega \times \mathbf{R}^2)$ be the set of continuous functions with compact support on $\Omega \times \mathbf{R}^2$. We define the $*$-algebra structure of $\mathcal{A}_0$ by

$$(2.1) \qquad AB(\omega, x) = \int_{\mathbf{R}^2} dy\, A(\omega, y)B(T^{-y}\omega, x - y)e^{\frac{iB}{2}x\wedge y},$$

$$(2.2) \qquad A^*(\omega, x) = \bar{A}(T^{-x}\omega, -x),$$

for $A, B \in \mathcal{A}_0, \omega \in \Omega, x \in \mathbf{R}^2$. For each $\omega \in \Omega$, this $*$-algebra has a representation π_ω on $L^2(\mathbf{R}^2)$:

$$(2.3) \qquad (\pi_\omega(A)\psi)(x) = \int_{\mathbf{R}^2} dy\, A(T^{-x}\omega, y - x)e^{\frac{iB}{2}y\wedge x}\psi(y),$$

for $A \in \mathcal{A}_0, \psi \in L^2(\mathbf{R}^2)$. It is easy to check the following properties.

(1) $\pi_\omega(AB) = \pi_\omega(A)\pi_\omega(B)$, $\pi_\omega(A)^* = \pi_\omega(A^*)$.

(2) $\pi_\omega(A)$ is bounded and $\|\pi_\omega(A)\|_{op} \le \|A\|_{\infty,1}$, where $\|\cdot\|_{op}$ is the operator norm and

$$\|A\|_{\infty,1} = \max\left\{\sup_{\omega\in\Omega}\int_{\mathbf{R}^2} dy\,|A(\omega,y)|,\ \sup_{\omega\in\Omega}\int_{\mathbf{R}^2} dy\,|A^*(\omega,y)|\right\}$$

This follows from mimicking the proof of Young's inequality.

(3) $\pi_\omega(A)$ satisfies the covariance relation:

$$U(a)\pi_\omega(A)U(a)^* = \pi_{T^a\omega}(A), \tag{2.4}$$

where $U(a)$ is the magnetic translation:

$$(U(a)\psi)(x) = e^{\frac{iB}{2}x\wedge a}\psi(x-a). \tag{2.5}$$

We define a C^*-norm:

$$\|A\| := \sup_{\omega\in\Omega}\|\pi_\omega(A)\|_{op}, A \in \mathcal{A}_0,$$

on $\mathcal{A}_0$. Let $\mathcal{A} = C^*(\Omega \times \mathbf{R}^2)$ be the completion of $\mathcal{A}_0$ with respect to this norm.

Our Hamiltonian is naturally described by $\mathcal{A}$. The following proposition appears in [B3, NB]. Write $\rho(H_\omega(t))$ for the resolvent set of $H_\omega(t)$.

PROPOSITION 2.1. *For $z \in \rho(H_\omega(t))$, $(H_\omega(t) - z)^{-1}$ is represented by an element of $\mathcal{A}$.*

A trace on $\mathcal{A}_0$ is defined by

$$\mathcal{T}_{\mathbf{P}}(A) := \int d\mathbf{P}(\omega)A(\omega,0), \quad \text{for } A \in \mathcal{A}_0, \tag{2.6}$$

whenever it exists. By Birkhoff's ergodic theorem, $\mathcal{T}_{\mathbf{P}}(A) = \mathcal{T}(\pi_\omega(A))$ for $\mathbf{P}$-a.e. ω. Hence $\mathcal{T}(\pi_\omega(A))$ is constant almost surely.

For $p \geq 1$, we define $L^p(\mathcal{A}, \mathcal{T})$ as the completion of $\mathcal{A}_0$ under the norm

$$\|A\|_p := \Big(\mathcal{T}((A^*A)^{p/2})\Big)^{2/p}, \quad \text{for } A \in \mathcal{A}_0. \tag{2.7}$$

It is easy to see

$$\mathcal{T}(AB) = \mathcal{T}(BA), \quad A \in \mathcal{A}, B \in \mathcal{L}^1(\mathcal{A}, \mathcal{T}) \tag{2.8}$$

for $\mathbf{P}$-a.e. ω by (2.1) and (2.6).

We define the differential structure on $\mathcal{A}_0$ by

$$\partial_{x_j} A(x, \omega) := i x_j A(\omega, x), \quad j = 1, 2, \ A \in \mathcal{A}_0. \tag{2.9}$$

This differential corresponds to the commutator with x_j on the representation, $\pi_\omega(A)$:

$$\pi_\omega(\partial_{x_j} A) = i[\pi_\omega(A), x_j], \quad j = 1, 2, \tag{2.10}$$

where the commutator $[A, B]$ of a pair of operators A, B is defined as a form on $D(A) \cap D(B)$ and extended to an operator. Using this differential structure, we define the Sobolev space $\mathcal{H}^1$ as the completion of $\mathcal{A}_0$ under the inner product

$$< A, B >_{\mathcal{H}^1} := \mathcal{T}(A^* B) + \mathcal{T}(\nabla A^* \cdot \nabla B). \tag{2.11}$$

From Assumption (A) and Proposition 2.1, P_ω is represented by an element of $\mathcal{A}$, and this was the reason why we abused the notation, $\partial_{x_j} P_\omega = i[P_\omega, x_j]$, $j = 1, 2$ in (1.13). Moreover, P_ω belongs to $\mathcal{H}^1$ due to the exponential decay property of its integral kernel[ASS]. Thus it is easy to see the RHS of (1.13) is finite for $\mathbf{P}$-a.e. ω.

Before ending this section, we shall confirm that physical quantities defined in introduction are all well-defined.

PROPOSITION 2.2. *The quantities j_ω, $j_{\omega,\tau}$, $j^A_{\omega,\tau}$, $j_{\omega,\tau}(T)$, and $j^A_{\omega,\tau}(T)$ are all well-defined for $\mathbf{P}$-a.e. ω.*

Proof. We show Proposition 2.2 for j_ω only. The proof for others is similar. We write j_ω as

$$j_\omega = \int_0^1 dt\, \mathcal{T}(U_\omega^*(t) i[H_\omega(t), x_2] U_\omega(t)(i + H_\omega)^{-1}(i + H_\omega)P_\omega)$$

$(i+H_\omega)^{-1}$ maps $L^2(\mathbf{R}^2)$ onto D and $U_\omega(t)$ maps D to D. And $[H_\omega(t), x_2] = -2(-i\nabla - A(x) - f(t)e_1)$. Hence the operator $U_\omega^*(t)i[H_\omega(t), x_2]U_\omega(t)(i + H_\omega)^{-1}$ is bounded on $L^2(\mathbf{R}^2)$. On the other hand, the integral kernel of the operator $(i + H_\omega)P_\omega$ has the exponential decay property[ASS]. Since the bounded operator on $L^2(\mathbf{R}^2)$ is an ideal of $\mathcal{L}^1(\mathcal{A}, \mathcal{T})$, j_ω is proved to be well-defined. To prove the well-definedness of $j_{\omega,\tau}(T)$ and $j^A_{\omega,\tau}(T)$, we use the rapid decay property of the integral kernel of $f_{T,\varepsilon_F}(H_\omega)$[Y]. □

§3. Adiabatic limit

In this section, we prove Theorem 1.1 that ensures $\sigma^A_{\omega,\tau}$ is very good approximation to $\sigma_{\omega,\tau}$. In what follows, we omit the ω-dependence for simplicity but we may not forget that some arguments hold only for $P-a.e.\ \omega$ in view of Birkhoff's ergodic theorem. We first recall some known results about the adiabatic evolution from [KS]. We define

$$\Omega(s) := U_\tau^{A*}(s) U_\tau(s), \quad s \in [0, 1], \tag{3.1}$$

to compare $U_\tau(s)$ with $U^A_\tau(s)$. It satisfies the Volterra integral equation

$$\Omega(s) = 1 - \int_0^s dt\, K_\tau(t, P)\Omega(t), \quad s \in [0, 1], \tag{3.2}$$

where $K_\tau(t, P) = U_\tau^{A*}(t)[P'(t), P(t)]U^A_\tau(t)$. To solve this, we set the following operator sequence

$$\Omega_0(s) = 1,\ \Omega_{j+1}(s) = -\int_0^s dt\, K_\tau(t, P)\Omega_j(t), \quad j \geq 1. \tag{3.3}$$

We use the following lemma due to Klein and Seiler.

Lemma 3.1 ([KS]). *$\Omega(s)$ has the asymptotic expansion in τ*

$$\Omega(s) \sim \sum_{j=0}^{\infty} \Omega_j(s), \quad s \in [0,1],\ \tau \to \infty, \tag{3.4}$$

in the sense that,

$$\|\Omega_{2j-1}(s)\|_{op} + \|\Omega_{2j}(s)\|_{op} = O(\tau^{-j}), \quad s \in [0,1],\ j \geq 1, \tag{3.5}$$

and

$$\|R_{2j}(s)\|_{op} + \|R_{2j+1}(s)\|_{op} = O(\tau^{-j-1}), \quad s \in [0,1],\ j \geq 0, \tag{3.6}$$

where $R_N(s) = \Omega(s) - \sum_{j=0}^{N} \Omega_j(s)$. The above estimates hold uniformly in $s \in [0,1]$.

Proof of Theorem 1.1. We insert the definition (3.1) of $\Omega(s)$ into (1.8):

$$\begin{aligned} j_{\omega,\tau} :=& \mathcal{T}\left((U_\tau^*(1)\, x_2\, U_\tau(1) - x_2)P\right) \\ =& \int_0^1 dt\, \mathcal{T}\left(U_\tau^*(t) i\tau [H, x_2] U_\tau(t) P\right) \\ =& \int_0^1 dt\, \mathcal{T}\left(\Omega^*(t) U_\tau^{A*}(t) i\tau [H_\omega, x_2] U_\tau^A(t) \Omega(t) P\right). \end{aligned}$$

We use Lemma 3.1 to expand $\Omega(1)$. Hence it is sufficient to show, for arbitrary $n \geq 1$,

$$\sum_{i+j=n} \mathcal{T}(\Omega_i^*(t) U_\tau^{A*}(t)\, i\tau[H, x_2]\, U_\tau^A(t) \Omega_j(t) P) = O(\tau^{-\infty}), \tag{3.7}$$

as τ tends to infinity. We claim that

$$\text{the LHS of (3.7)} = \sum_{i+j=n} \mathcal{T}(U_\tau^{A*}(t)\, i\tau[H, x_2]\, U_\tau^A(t) \Omega_j(t) \Omega_i^*(t) P) + O(\tau^{-\infty}). \tag{3.8}$$

The equation (3.8) is proved in the same way as the proof of Theorem A.4 in [KS] and this is the place where the assumption $f'(t) \in C_0^\infty(\mathbf{R})$ is necessary. We also used the cyclicity of the trace per area(2.8). By virtue of (3.8), we have only to show that, for arbitrary $n \geq 1$,

$$\sum_{i+j=n} \mathcal{T}(U_\tau^{A*}(t)\, i\tau[H, x_2]\, U_\tau^A(t)\Omega_j(t)\Omega_i^*(t)P) = 0. \tag{3.9}$$

We calculate $\Omega_j(t)$ explicitly

$$\Omega_j(t) = (-1)^j \int_0^t ds_1 \int_0^{s_1} ds_2 \cdots \int_0^{s_{j-1}} ds_j\, K(s_1)K(s_2)\cdots K(s_j), \tag{3.10}$$

$$\Omega_j^*(t) = \int_0^t ds_1 \int_{s_1}^t ds_2 \cdots \int_{s_{j-1}}^t ds_j\, K(s_1)K(s_2)\cdots K(s_j). \tag{3.11}$$

By using (3.10) and (3.11), it is elementary to prove (3.9). In fact, when $n = 1$, (3.9) follows since $K(s) + K^*(s) = 0$. When $n = 2k(k \in \mathbf{N})$, we compute as follows:

$$\begin{aligned}\Omega_{2k}(t) + \Omega_{2k-1}(t)\Omega_1^*(t) + \cdots + \Omega_{k+1}(t)\Omega_{k-1}^*(t) = \\ (-1)^{k-1} \int_{0 \leq s_{k+1} \leq s_k \leq \cdots \leq s_1 \leq t} ds_1 ds_2 \cdots ds_{k+1} \\ \int_{s_{k+1} \leq s_{k+2} \leq \cdots \leq s_{2k} \leq t} ds_{k+2} ds_{k+3} \cdots ds_{2k} \\ K(s_1)K(s_2)\cdots K(s_{2k}).\end{aligned} \tag{3.12}$$

$$\begin{aligned}\Omega_{k-1}(t)\Omega_{k+1}^*(t) + \Omega_{k-2}(t)\Omega_{k+2}^*(t) + \cdots + \Omega_1(t)\Omega_{2k-1}^*(t) + \Omega_{2k}^*(t) = \\ (-1)^{k-1} \int_{0 \leq s_k \leq s_{k-1} \leq \cdots \leq s_1 \leq t} ds_1 ds_2 \cdots ds_k \\ \int_{s_k \leq s_{k+1} \leq \cdots \leq s_{2k} \leq t} ds_{k+1} ds_{k+2} \cdots ds_{2k} \\ K(s_1)K(s_2)\cdots K(s_{2k}).\end{aligned} \tag{3.13}$$

$$\begin{aligned}\Omega_k(t)\Omega_k^*(t) = \\ (-1)^k \int_{0 \leq s_k \leq s_{k-1} \leq \cdots \leq s_1 \leq t} ds_1 ds_2 \cdots ds_k \int_0^t ds_{k+1} \int_{s_{k+1} \leq s_{k+2} \leq \cdots \leq s_{2k} \leq t} ds_{k+2} \\ ds_{k+3} \cdots ds_{2k} K(s_1)K(s_2)\cdots K(s_{2k}).\end{aligned} \tag{3.14}$$

From (3.12), (3.13), and (3.14), we have (3.9). When $n = 2k+1 (k \in \mathbf{N})$, we compute similarly as follows:

$$\begin{aligned}(3.15)\quad & \Omega_{2k+1}(t) + \Omega^*_{2k}(t)\Omega^*_1(t) + \cdots + \Omega_{k+1}(t)\Omega^*_k(t) = \\ & (-1)^{k+1} \int_{0 \le s_{k+1} \le s_k \le \cdots \le s_1 \le t} ds_1 ds_2 \cdots ds_{k+1} \\ & \int_{s_{k+1} \le s_{k+2} \le \cdots \le s_{2k} \le t} ds_{k+2} ds_{k+3} \cdots ds_{2k} \\ & \qquad K(s_1)K(s_2)\cdots K(s_{2k}).\end{aligned}$$

$$\begin{aligned}(3.16)\quad & \Omega_k(t)\Omega^*_{k+1}(t) + \Omega_{k+1}(t)\Omega^*_k(t) + \cdots + \Omega^*_{2k+1}(t) = \\ & (-1)^{k} \int_{0 \le s_{k+1} \le s_k \le \cdots \le s_1 \le t} ds_1 ds_2 \cdots ds_{k+1} \\ & \int_{s_{k+1} \le s_{k+2} \le \cdots \le s_{2k} \le t} ds_{k+2} ds_{k+3} \cdots ds_{2k} \\ & \qquad K(s_1)K(s_2)\cdots K(s_{2k}).\end{aligned}$$

From (3.15) and (3.16), (3.9) follows. This completes the proof of Theorem 1.1. □

§4. Derivation of the Chern character

In this section, we prove Theorem 1.2: we derive the Chern character from the definition of $j^A_{\omega,\tau}$(1.11).

PROOF OF THEOREM 1.2. We decompose $j^A_{\omega,\tau}$ into two parts:

$$\begin{aligned}(4.1)\qquad \sigma^A_\tau = & \int_0^1 ds \mathcal{T}(U^{A*}_\tau(s) i\tau [H^A(s), x_2] U^A_\tau(s)P) \\ =: & I + II,\end{aligned}$$

where

$$(4.2)\qquad I = \int_0^1 ds\ \mathcal{T}(U^{A*}_\tau(s) i\tau [H(s), x_2] U^A_\tau(s)P),$$

$$II = \int_0^1 ds\ \mathcal{T}(U_\tau^{A*}(s) i\tau \left[\frac{i}{\tau}[P'(s), P(s)], x_2\right] U_\tau^A(s)P). \tag{4.3}$$

In the first equality in (4.1), we used (1.10) and exchanged the integration $\int_0^1 dt$ for trace per area $\mathcal{T}$. This exchange is permitted for **P**-a.e. ω by dominated convergence theorem. We show $I = 0$ and $II = -i\mathcal{T}(P_\omega[\partial_{x_1} P_\omega, \partial_{x_2} P_\omega]P_\omega)$ below, and this completes Theorem 1.2. At this point, we notice that $U_\tau^A(s)$ has the intertwining property [ASY]:

$$U_\tau^A(s)P = P(s)U_\tau^A(s), \quad s \in [0,1], \tag{4.4}$$

And from the definition, $H(s)$, $P(s)$ and $P'(s)$ satisfy the following relations:

$$H(s) = e^{if(s)x_1} H e^{-if(s)x_1}, \tag{4.5}$$

$$P(s) = e^{if(s)x_1} P e^{-if(s)x_1}, \tag{4.6}$$

$$P'(s) = if'(s) e^{if(s)x_1} [\, x_1, P\,] e^{-if(s)x_1}. \tag{4.7}$$

By using (4.4)-(4.7) and the cyclicity of the trace per area (2.8), we obtain

$$I = i\tau\mathcal{T}(P(\partial_{x_2} H)P), \tag{4.8}$$

$$II = -i \int_0^1 ds\ f'(s)\mathcal{T}(P[\,\partial_{x_1} P, \partial_{x_2} P\,]P). \tag{4.9}$$

The RHS of (4.8) vanishes due to Proposition 3 of [B2]. From the definition of $f(s)$, we complete Theorem 1.2. □

The proof of Theorem 1.2 implies the following:

PROPOSION 4.1. *Let $U_\omega^A(s)$ be the unitary operator which satisfies*

$$\begin{aligned} \left\{ \frac{\partial}{\partial s} - [P'(s), P(s)] \right\} U_\omega^A(s)\psi &= 0, \\ U_\omega^A(0)\psi &= \psi, \quad \psi \in L^2(\mathbf{R}^2). \end{aligned} \tag{4.10}$$

Then,

(1) j^A_ω is represented by the adiabatic transport $U^A_\omega(s)$, i.e., for $\mathbf{P}$*-a.e. ω,*

$$j^A_\omega = \mathcal{T}((U^{A*}_\omega(1)x_2U^A_\omega(1) - x_2)P_\omega). \tag{4.11}$$

(2)

$$j^A_\omega = i\mathcal{T}(U^{A*}_\omega(1)\partial_{x_2}U^A_\omega(1)P_\omega). \tag{4.12}$$

Proposition 4.1 follows from straightfoward computation and we omit the proof. The RHS of (4.12) can be regarded as the extended version of Berry's phase (see Proposition A.3 in appendix).

§5. Proof of Proposition 1.4

Proposition 1.4 may be proved following the argument of the proof of Theorem 6.8 of [ASS] by changing the following two definitions. The proof goes without any other modifications.

The first, in [ASS], the adiabatic curvature is defined as

$$w_{12} = -i\left([P, \Lambda_1]Q[P, \Lambda_2] - (1 \leftrightarrow 2)\right), \tag{5.1}$$

where Λ_1, Λ_2 are switching functions and $Q = I - P$. Alternatively, we define

$$\tilde{w}_{12} = -i\left([P, x_1]Q[P, x_2] - (1 \leftrightarrow 2)\right). \tag{5.2}$$

The exponential decay property of the integral kernel of P[ASS] implies that $[P, x_j](j = 1, 2)$ is bounded.

The second, we replace the definition of the Hall charge transport in [ASS]:

$$\frac{Q}{2\pi} = -\lim_{L\uparrow\infty} \mathrm{Trace}(\chi_{K_L} w_{12} \chi_{K_L}), \tag{5.3}$$

by trace per area: $\mathcal{T}(\tilde{w}_{12})$ which is the Chern character of P up to a constant in Theorem 1.2.

§6. Zero temperature limit

In this section, we prove Proposition 1.5. which comes from the exponential property of $f_{T,\varepsilon_F}(\varepsilon)$.

Proof of Proposition 1.5. We prove (1.17) only, since (1.18) may be proved similarly. We write

$$\sigma_\tau^A(T) = \sigma_\tau^A + \mathcal{T}((U_\tau^{A*}(1)\, x_2\, U_\tau^A(1) - x_2)(f_{T,\varepsilon_F}(H) - P)).$$

The second term is written

$$\mathcal{T}((U_\tau^{A*}(1)\, x_2\, U_\tau^A(1) - x_2)(H+i)^{-2}(H+i)^2(f_{T,\varepsilon_F}(H) - P)).$$

We can show that the operator $(U_\tau^{A*}(1)\, x_2\, U_\tau^A(1) - x_2)(H+i)^{-2}$ is bounded by the argument in the proof of Proposision 2.2. And $\mathcal{T}((H+i)^2(f_{T,\varepsilon_F}(H) - P)) = \int(\varepsilon+1)^2(f_{T,\varepsilon_F}(\varepsilon) - \chi_{(-\infty,\varepsilon_F)}(\varepsilon))d\mathcal{N}(\varepsilon)$ where $\mathcal{N}(\varepsilon)$ is the integrated density of states:

$$\mathcal{N}(\varepsilon) = \lim_{L\uparrow\infty} \frac{1}{|K_L|} \sharp\{\text{eigenvalues of } \chi_{K_L} H \chi_{K_L} \le \varepsilon\}.$$

Thus the result follows from the dominated convergence theorem. □

Appendix

In appendix, we show that our definition of the Hall conductivity gives the results similar to the works of Avron, Seiler, Klein, and Yaffe [ASY][KS]. The Hamiltonian is essentially the same as what appears in [ASS]:

$$H_\omega(t,s) := (-i\nabla - A(x) - f(t)e_1 - se_2)^2 + V_\omega(x), \tag{A.1}$$

where $x \in \mathbf{R}^2$, $(t,s) \in R := [0,1]\times[0,1] \subset \mathbf{R}^2$, $\omega \in \Omega$. In (A.1), $e_1 = (1,0)$, $e_2 = (0,1)$, and $A(x)$, $f(t)$, and $V_\omega(x)$ are the same as defined in section 1. Let $P_\omega(t,s) := \chi_{(-\infty,\varepsilon_F]}(H_\omega(t,s))$. Following [ASSS], we regard $RangeP_\omega(t,s)$ as the infinite-dimensional bundle on R, and consider a covariant derivative on $RangeP_\omega(t,s)$:

$$\nabla := d + A, \tag{A.2}$$

where $A := -[dP_\omega, P_\omega]$ is called the adiabatic connection. The corresponding curvature is defined by

$$\Omega := \nabla^2 = P_\omega(dP_\omega \wedge dP_\omega)P_\omega, \tag{A.3}$$

In (A.3), we regard dP_ω as the operator-valued 1-form on R. We shall calculate the Hall conductivity in the same way as in section 1. We define the scaled unitary evolution $U_{\omega,\tau}(t,s)$ and the adiabatic evolution $U^A_{\omega,\tau}(t,s)$ of $H_\omega(t,s)$ similarly as (1.7) and (1.10). And we define the Hall conductivity using $H_\omega(t,s)$ as follows:

$$j_{\omega,\tau} := \mathcal{T}((U^*_{\omega,\tau}(1,0)x_2U_{\omega,\tau}(1,0) - x_2)P_\omega(0,0)), \tag{A.4}$$

$$\sigma_\omega := \lim_{\tau\uparrow\infty} j_{\omega,\tau}, \tag{A.5}$$

$$j^A_{\omega,\tau} := \mathcal{T}((U^{A*}_{\omega,\tau}(1,0)x_2U^A_{\omega,\tau}(1,0) - x_2)P_\omega(0,0)). \tag{A.6}$$

In reality, (A.4)-(A.6) are just the rewrites of (1.8), (1.9), and (1.11) respectively since $H_\omega(t,0) = H_\omega(t)$ so that σ_ω defined in (A.5) is already given in Corollary 1.3. However, σ_ω has another expressions given below.

Proposition A.1. *The Hall conductivity σ_ω is equal to the integration on R of the thermal average of the adiabatic curvature for* **P**- *a.e. ω, i.e.,*

$$\sigma_\omega = -i\int_R \mathcal{T}(\Omega) \tag{A.7}$$

The proof of Proposition A.1 is similar to that of Theorem 1.2 and we only show the sketch.

Sketch of proof. From the definition of $P_\omega(t,s)$, it follows that

$$P_\omega(t,s) = e^{if(t)x_1+isx_2}P_\omega(0,0)e^{-if(t)x_1-isx_2}. \tag{A.8}$$

By (A.8), $P_\omega(t,s)$ satisfies the relation

$$\frac{\partial P_\omega}{\partial t}(t,s) = -f'(t)\partial_{x_1}P_\omega(t,s), \quad \frac{\partial P_\omega}{\partial s}(t,s) = -\partial_{x_2}P_\omega(t,s). \tag{A.9}$$

Then, the direct computation of the RHS of (A.7) gives the result.

Next, we shall consider the N-body version of Proposition A.1. Following [KS], the Hamiltonian is defined by

$$H_N(t,s) := \sum_{j=1}^{N}(-i\nabla_j - A(x^j) - f(t)e_1 - se_2)^2 + \sum_{j=1}^{N} V(x^j) + \sum_{k<l} |x^k - x^l|^{-1}, \tag{A.10}$$

on $\Lambda^N L^2(K_L)$, $L > 0$, with periodic boundary conditions, where $x = (x^1, \cdots, x^N)$, $x^j \in K_L (1 \le j \le N)$, and $V(x) \in L^\infty(K_L)$. The space $\Lambda^N L^2(K_L)$ is the antisymmetric subspace of $\bigotimes_{j=1}^N L^2(K_L)$. We assume that there is a non-degenerate ground state $\Psi \in \Lambda^N L^2(K_L)$ and let P_Ψ be the corresponding orthogonal projection. We define the Hall conductivity σ in the spirit of section 1:

$$\sigma := \lim_{\tau \uparrow \infty} \sum_{j=1}^{N} Trace_{K_L}((U_\tau^*(1) x_2^j U_\tau(1) - x_2^j) P_\Psi), \tag{A.11}$$

where $U_\tau(t)(t \in [0,1], \tau > 0)$ is the time scaled unitary evolution of $H_N(t)$. We should note that the RHS of (A.11) is not a statistical quantity. By the similar computations in the proof of Proposition A.1, we obtain the same result as in [KS].

PROPOSITION A.2.

$$\sigma = -i \int_R Trace_{K_L}(P_\Psi(dP_\Psi \wedge dP_\Psi)P_\Psi). \tag{A.12}$$

In the end of appendix, we shall return to the Hamiltonian (A.1). And we prove that the Hall conductivity σ_ω has different representation using a parallel transport w.r.t. the adiabatic connection.

PROPOSITION A.3. *Let $U_\omega^A(t,s)$ be the adiabatic parallel transport w.r.t. the covariant derivative ∇, i.e., $U_\omega^A(t,s)$ is unitary which satisfies*

$$\nabla_{(\frac{\partial}{\partial t},0)} U_\omega^A(t,s)\psi = \left(\frac{\partial}{\partial t} - [\frac{\partial P_\omega}{\partial t}(t,s), P_\omega(t,s)]\right) U_\omega^A(t,s)\psi = 0,$$

$$U_\omega^A(0,s)\psi = \psi, \quad s \in [0,1], \quad \psi \in L^2(\mathbf{R}^2). \tag{A.13}$$

Then, the Hall conductivity σ_ω defined in (A.5) has the following form for $\mathbf{P}$*-a.e.* ω*.*

$$\sigma_\omega = -i \int_{\partial R} \mathcal{T}(U_\omega^{A*}(t,s) dU_\omega^A(t,s) P_\omega(0,s)). \tag{A.14}$$

If the dimension of $Range P_\omega(t,s)$ was equal to one, $Range P_\omega(t,s)$ would be a $U(1)$-bundle and the RHS of (A.14) would be equal to Berry's phase. Thus, we can say that, the Hall conductivity σ_ω is equal to the thermal average of Berry's phase in wide sense.

PROOF. This proof is based on the proof of Proposition 3.1 in [KS]. The Hall conductivity σ_ω may be written in the following:

$$\sigma_\omega = -i\mathcal{T}(\partial_{x_2} A_\omega(0,0) P_\omega(0,0)), \tag{A.15}$$

where $A_\omega(t,s) = -[\frac{\partial P_\omega}{\partial t}(t,s), P_\omega(t,s)]$. The equation (A.15) follows easily from Proposition 4.1.(1). We use the relations (A.8),(A.9), and the intertwining property of $U_\omega^A(t,s)$ in (A.15). The product obtained is

$$\sigma_\omega = i \int_R dtds\ \mathcal{T}(U_\omega^{A*}(t,s)(\frac{\partial}{\partial s} A_\omega(t,s)) U_\omega^A(t,s) P_\omega(0,s)). \tag{A.16}$$

We notice that $(\frac{\partial}{\partial s} A_\omega(t,s)) U_\omega^A(t,s) = \frac{\partial}{\partial s}\left(A_\omega(t,s) U_\omega^A(t,s)\right) - A_\omega(t,s)(\frac{\partial}{\partial s} U_\omega^A(t,s))$. Then, a short computation yields

$$\sigma_\omega = -i \int_0^1 ds\ \mathcal{T}(U_\omega^{A*}(1,s)(\frac{\partial}{\partial s} U_\omega^A(1,s)) P_\omega(0,s)). \tag{A.17}$$

On the other hand,

$$\mathcal{T}(U_\omega^{A*}(0,s)(\frac{\partial}{\partial s}U_\omega^A(0,s))P_\omega(0,s)) = 0, \tag{A.18}$$

which follows from (A.13), and for $t \in [0,1]$,

$$\begin{aligned}\mathcal{T}(U_\omega^{A*}(t,s)(\frac{\partial}{\partial t}U_\omega^A(t,s))P_\omega(0,s)) &= \mathcal{T}(P_\omega(t,s)A_\omega(t,s)P_\omega(t,s)) \\ &= 0.\end{aligned} \tag{A.19}$$

In the first equality in (A.19), we used (A.13), the intertwining property of $U_\omega^A(t,s)$, and the cyclicity of trace per area (2.8). In the second equality in (A.19), we used the fact that $P_\omega(t,s)(\frac{\partial}{\partial t}P_\omega(t,s))P_\omega(t,s) = 0$ which follows easily from the equation $\frac{\partial}{\partial t}(P_\omega(t,s))^2 = \frac{\partial}{\partial t}P_\omega(t,s)$. By combing (A.17), (A.18), and (A.19), we complete the proof. □

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