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On the Pseudo-Cyclicity of Some Iwasawa Modules Associated to Abelian Fields

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Abstract. Let p be an odd prime number, and $K/\mathbf{Q}$ a totally imaginary finite abelian extension of the first kind, with the Galois group Δ. Let $\mathcal{U}_\infty$ (resp. $\mathcal{E}_\infty$) denote the projective limit of the semi-local units (resp. the global units) of the fields in the cyclotomic $\mathbf{Z}_p$-extension of K. We will show that $(\mathcal{U}_\infty/\mathcal{E}_\infty)^+$ contains a cyclic $\Lambda[\Delta]$-submodule of finite index.

Introduction

Let p be a fixed prime number, and $\mathbf{Z}_p$ be the ring of p-adic integers. We denote by $\mathbf{Q}_\infty$ the cyclotomic $\mathbf{Z}_p$-extension of the rational number field $\mathbf{Q}$. Let K be a finite abelian extension of $\mathbf{Q}$, satisfying $K \cap \mathbf{Q}_\infty = \mathbf{Q}$. Let K_∞ be the cyclotomic $\mathbf{Z}_p$-extension of K, i.e. $K_\infty = K\mathbf{Q}_\infty$; and for each $n \geq 0$, let K_n be the intermediate field of K_∞/K such that K_n is a cyclic extension of degree p^n over K. Put $\Delta = \mathrm{Gal}(K/\mathbf{Q})$ and $\Gamma = \mathrm{Gal}(K_\infty/K)$.

Let L_∞ be the maximal unramified abelian p-extension over K_∞, and let $X = \mathrm{Gal}(L_\infty/K_\infty)$. Then X is a module over the completed group ring $\mathbf{Z}_p[[\mathrm{Gal}(K_\infty/\mathbf{Q})]]$ in a natural way. Identifying Δ with $\mathrm{Gal}(K_\infty/\mathbf{Q}_\infty)$ and $\mathbf{Z}_p[[\Gamma]]$ with the formal power series ring $\Lambda = \mathbf{Z}_p[[T]]$, X becomes a $\Lambda[\Delta]$-module, and it is known that X is finitely generated torsion over Λ. Under this condition, one can see that various Iwasawa modules which are defined with respect to K_∞/K also have the structure of $\Lambda[\Delta]$-modules. Let $J \in \Delta$ denote the complex conjugation. For a $\Lambda[\Delta]$-module M, we will put

$$M^+ = \{m \in M | J(m) = m\}, \quad M^- = \{m \in M | J(m) = -m\}.$$

Assume that K contains a primitive p-th root ζ_p of unity. Supposing that X^+ is a finite module, Greenberg has proved that X^- contains a cyclic $\Lambda[\Delta]$-submodule of finite index (Greenberg[Gr2] Theorem 5). In the following,

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we will call such a $\Lambda[\Delta]$-module a *pseudo-cyclic* $\Lambda[\Delta]$-module. Let M_∞ be the maximal abelian p-extension over K_∞ unramified outside p, and let $\mathcal{X} = \mathrm{Gal}(M_\infty/K_\infty)$. Then it is known that X^- is pseudo-isomorphic to $\mathcal{X}^+$, that is, isomorphic up to finite, and denoted by $X^- \sim \mathcal{X}^+$ (see §3). Therefore, the pseudo-cyclicity of X^- is equivalent to that of $\mathcal{X}^+$. For each prime divisor v of K_n lying above p, let $U_{n,v}$ be the group of local units in the v-completion $K_{n,v}$ which are congruent to 1 modulo the maximal ideal, and let $\mathcal{U}_n = \prod_{v|p} U_{n,v}$. Let E_n be the image of the group of all units in K_n by the embedding $K_n \hookrightarrow \prod_{v|p} K_{n,v}$. Let $\mathcal{E}_n$ be the closure of $E_n \cap \mathcal{U}_n$ in $\mathcal{U}_n$. We will denote by $\mathcal{U}_\infty$ and $\mathcal{E}_\infty$ the projective limits of $\mathcal{U}_n$ and $\mathcal{E}_n$ respectively, being taken with respect to the norm maps. Then it is known that $(\mathcal{U}_\infty/\mathcal{E}_\infty)^+$ is isomorphic to a $\Lambda[\Delta]$-submodule of $\mathcal{X}^+$ (cf.Washington[W] Corollary13.6). Therefore if X^- is a pseudo-cyclic $\Lambda[\Delta]$-module, then $(\mathcal{U}_\infty/\mathcal{E}_\infty)^+$ is also a pseudo-cyclic $\Lambda[\Delta]$-module (see Lemma 4).

The purpose of the paper is to prove the pseudo-cyclicity of $(\mathcal{U}_\infty/\mathcal{E}_\infty)^+$ directly, without supposing the finiteness of X^+, that is, our main result is the following:

Theorem. *Let p be an odd prime number and let K be a totally imaginary finite abelian extention of $\mathbf{Q}$. Suppose that K is of the first kind, i.e. its conductor is not divisible by p^2. Then $(\mathcal{U}_\infty/\mathcal{E}_\infty)^+$ contains a cyclic $\Lambda[\Delta]$-submodule of finite index.*

If we suppose the finiteness of X^+, then we have that $(\mathcal{U}_\infty/\mathcal{E}_\infty)^+$ is of finite index in $\mathcal{X}^+$. Therefore we have $(\mathcal{U}_\infty/\mathcal{E}_\infty)^+ \sim \mathcal{X}^+$. Hence our theorem can be used to show the pseudo-cyclicity of X^- (Greenberg loc.cit.), which we shall state as a corollary at the end of this paper.

An outline of the paper is the followings: in §1, for one prime divisor v, we consider the structure of the $\Lambda[\mathrm{Gal}(K_v/\mathbf{Q}_p)]$-module $U_{\infty,v} = \varprojlim U_{n,v}$. That is, we study the structure of some modules which are defined with respect to a local $\mathbf{Z}_p$-extension. In §2, we assume that $K/\mathbf{Q}$ is of the first kind, and study the structure of the $\Lambda[\Delta]$-module $\mathcal{U}_\infty$. First, we show that the consideration of $\mathcal{U}_\infty$ is reduced to that of $U_{\infty,v}$, and then, using our result in §1, we give the structure of the $\Lambda[\Delta]$-module $\mathcal{U}_\infty$. We note that, when $K = \mathbf{Q}(\zeta_p)$, the structure of the $\Lambda[\Delta]$-module $\mathcal{U}_\infty$ was known by

Iwasawa[Iw1]; furthermore when K is a finite abelian field with degree relatively prime to p, it was known by Gillard[Gi]. In §3, we study the Kummer duality, and then the structure of adjoint modules as $\Lambda[\Delta]$-modules, which were previously known as Λ-modules. Finally, after we prepare an algebraic lemma in §4, using our result concerning $\mathcal{U}_\infty$ in §2 and the results in §§3-4, we prove the main theorem and mention its corollary.

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1. Local Theory

1.1. Let p be a fixed prime number. We fix an algebraic closure Ω_p of the p-adic number field $\mathbf{Q}_p$, and always consider algebraic extensions of $\mathbf{Q}_p$ to be contained in Ω_p.

Let k be a finite extension of $\mathbf{Q}_p$ (in Ω_p) with

$$[k : \mathbf{Q}_p] = d.$$

We denote by k_{ab} the maximal abelian p-extension over k. By local class field theory, there is a canonical isomorphism

$$\mathrm{Gal}(k_{ab}/k) \xrightarrow{\sim} A_k,$$

where A_k denotes the p-adic completion of the multiplicative group $k^\times$: $A_k = \varprojlim k^\times/(k^\times)^{p^n}$. Then, we can write

$$A_k = \pi^{\mathbf{Z}_p} \times U_k,$$

where π is a uniformizing parameter of k, and U_k is the principal units of k, that is, the units congruent to 1 modulo the maximal ideal.

We denote by k_{ur} the maximal unramified abelian p-extension over k. Since the inertia group of $\mathrm{Gal}(k_{ab}/k)$ is isomorphic to U_k, k_{ur}/k is a $\mathbf{Z}_p$-extention:

$$\mathrm{Gal}(k_{ur}/k) \cong \mathbf{Z}_p.$$

Let W_k be the group of all p-th power roots of unity in k. Then W_k is a subgroup of U_k and U_k/W_k is a free $\mathbf{Z}_p$-module of rank d. Therefore we obtain

$$\mathrm{Gal}(k_{ab}/k) \cong W_k \oplus {\mathbf{Z}_p}^{d+1}.$$

From the above, it follows that there are $d+1$ independent $\mathbf{Z}_p$-extensions over k. In particular there exist $\mathbf{Z}_p$-extensions over k different from k_{ur}/k.

Let k_∞ be a $\mathbf{Z}_p$-extention over k; and for each $n \geq 0$, let k_n denote the intermediate field of k_∞/k such that k_n is a cyclic extension of degree p^n over k. If k_∞ is a $\mathbf{Z}_p$-extention over k different from k_{ur}/k, then there exists an $n_0 \geq 0$ such that $k_\infty \bigcap k_{ur} = k_{n_0}$. Hence k_∞/k_n is a totally ramified extension for $n \geq n_0$.

1.2. We fix a $\mathbf{Z}_p$-extension $\mathbf{Q}_{p,\infty}$ of $\mathbf{Q}_p$ different from $\mathbf{Q}_{p,ur}/\mathbf{Q}_p$. Let k be a finite abelian extension of $\mathbf{Q}_p$ such that $k \bigcap \mathbf{Q}_{p,\infty} = \mathbf{Q}_p$, and let $k_\infty = k\mathbf{Q}_{p,\infty}$. Then we obtain a $\mathbf{Z}_p$-extention k_∞/k different from k_{ur}/k. Put $\Gamma = \mathrm{Gal}(k_\infty/k)$ and $D = \mathrm{Gal}(k/\mathbf{Q}_p)$.

Let $M_n,\ 0 \leq n \leq \infty$, be the maximal abelian p-extension over k_n and let

$$X = \mathrm{Gal}(M_\infty/k_\infty).$$

Then Γ acts on X by conjugation. Fix a topological generator γ_0 of Γ, and identify the completed group ring $\mathbf{Z}_p[[\Gamma]]$ with the formal power series ring $\mathbf{Z}_p[[T]]$ by $\gamma_0 = 1+T$. Then we can make X into a $\mathbf{Z}_p[[T]]$-module. Furthermore, identifying D with $\mathrm{Gal}(k_\infty/\mathbf{Q}_{p,\infty})$, we can also make X into a $\mathbf{Z}_p[D][[T]]$-module. Here we will consider the structure of the $\mathbf{Z}_p[D][[T]]$-module X. In the following, we write $\Lambda = \mathbf{Z}_p[[T]]$ and $\Lambda[D] = \mathbf{Z}_p[D][[T]]$.

For each $n \geq 0$, we define the element $\omega_n \in \Lambda = \mathbf{Z}_p[[T]]$ by

$$\omega_n = (1+T)^{p^n} - 1.$$

Then we have

$$\omega_n X = \mathrm{Gal}(M_\infty/M_n),\ X/\omega_n X = \mathrm{Gal}(M_n/k_\infty).$$

We have already seen in §1.1 that $X/\omega_0 X = X/TX = \mathrm{Gal}(M_0/k_\infty) = \mathrm{Gal}(k_{ab}/k_\infty)$ is finitely generated over $\mathbf{Z}_p$. Hence by Nakayama's lemma, X is finitely generated over Λ (cf.Washington [W],Lemma 13.16).

Since $M_n = k_{n,ab}$, $\mathrm{Gal}(M_n/k_n)$ and hence $\mathrm{Gal}(M_n/k_\infty)$ are both finitely generated $\mathbf{Z}_p$-modules. Let X_n be the submodule of X containing $\omega_n X$ such that $X_n/\omega_n X$ is the torsion $\mathbf{Z}_p$-submodule of $X/\omega_n X = \mathrm{Gal}(M_n/k_\infty)$. Clearly X_n is a $\Lambda[D]$-module, and

$$Y = \bigcap_{n=0}^{\infty} X_n,\ X' = X/Y$$

are, also, $\Lambda[D]$-modules.

We denote by W the group of all p-th power roots of unity in Ω_p, and let

$$W_n = W_{k_n} = W \bigcap {k_n}^{\times}, \qquad 0 \leq n \leq \infty.$$

We obviously have

$$W_0 \subseteq W_1 \subseteq \cdots \subseteq W_n \subseteq \cdots \subseteq W_\infty \subseteq W, \quad W_\infty = \bigcup_{n=0}^{\infty} W_n.$$

Hence either W_∞ is finite and $W_\infty = W_n$ for sufficiently large $n \geq 0$, or $W_\infty = W$ and $k_\infty = k(W)$. First, we consider the case where $k_\infty = k(W)$. Let $\kappa : \Gamma \to 1 + p\mathbf{Z}_p$ (or $1 + 4\mathbf{Z}_2$ if $p = 2$) be the p-cyclotomic character, i.e. it is the unique character satisfying $\gamma(\zeta) = \zeta^{\kappa(\gamma)}$ for every $\zeta \in W$. We define the element $\dot{T} \in \Lambda$ by

$$\dot{T} = \kappa(\gamma_0)(1+T)^{-1} - 1,$$

where γ_0 is the topological generator which is fixed in the above. For each $a \geq 0$, let $W^{(a)}$ be the subgroup of all p^a-th roots of unity in W, and we will consider

$$\varprojlim W^{(a)}.$$

This is isomorphic to $\mathbf{Z}_p$ as a $\mathbf{Z}_p$-module, and Γ acts on $\varprojlim W^{(a)}$ via the character κ. Hence we have the following Λ-isomorphism:

$$\varprojlim W^{(a)} \cong \Lambda/(1 + T - \kappa(\gamma_0)) = \Lambda/(\dot{T}).$$

Iwasawa has determined the structure of the Λ-module X as follows ([Iw2] Theorem 25):

(i) Suppose that $k_\infty = k(W)$, i.e. $W_\infty = W$. Then

$$X \cong \Lambda^d \oplus \Lambda/(\dot{T}),\ Y = \Lambda/(\dot{T}),\ X' \cong \Lambda^d$$

(ii) Suppose that $k_\infty \neq k(W)$, i.e. W_∞ is finite. Then

$$X \subseteq \Lambda^d,\ \Lambda^d/X \cong W_\infty.$$

We shall prove the following:

Proposition 1. (i) *Suppose that* $k_\infty = k(W)$. *Then we have an exact sequence*

$$0 \longrightarrow X \longrightarrow \Lambda[D] \oplus \Lambda/(\dot{T}) \longrightarrow F \longrightarrow 0$$

of $\Lambda[D]$*-modules, where* F *is a* $\Lambda[D]$*-module such that* $p^a F = 0$ *for some* $a \geq 0$.

(ii) *Suppose that* $k_\infty \neq k(W)$. *Then we have an exact sequence*

$$0 \longrightarrow X \longrightarrow \Lambda[D] \longrightarrow F \longrightarrow 0$$

of $\Lambda[D]$*-modules, where* F *is a* $\Lambda[D]$*-module such that* $p^a F = 0$ *for some* $a \geq 0$.

Proof. (i) Since $X' \cong \Lambda^d$, the exact sequence

$$0 \longrightarrow Y \longrightarrow X \longrightarrow X' \longrightarrow 0 \tag{1}$$

of $\Lambda[D]$-modules induces an exact sequence

$$0 \longrightarrow Y/TY \longrightarrow X/TX \longrightarrow X'/TX' \longrightarrow 0$$

of $\mathbf{Z}_p[D]$-modules. Furthermore we have that $X/TX = X/\omega_0 X = \mathrm{Gal}(M_0/k_\infty)$, and that $Y/TY = (\Lambda/(\dot{T}))/T(\Lambda/(\dot{T})) = \Lambda/(\dot{T}, T)$ is a finite $\mathbf{Z}_p[D]$-module. Hence we obtain a $\mathbf{Q}_p[D]$-isomorphism

$$\mathrm{Gal}(M_0/k_\infty) \otimes_{\mathbf{Z}_p} \mathbf{Q}_p \cong (X'/TX') \otimes_{\mathbf{Z}_p} \mathbf{Q}_p.$$

On the other hand, from the exact sequence

$$0 \longrightarrow \mathrm{Gal}(M_0/k_\infty) \longrightarrow \mathrm{Gal}(M_0/k) \longrightarrow \mathrm{Gal}(k_\infty/k) \longrightarrow 0,$$

we have an exact sequence

$$0 \longrightarrow (X'/TX') \otimes_{\mathbf{Z}_p} \mathbf{Q}_p \longrightarrow A_k \otimes_{\mathbf{Z}_p} \mathbf{Q}_p \longrightarrow \mathbf{Q}_p \longrightarrow 0$$

of $\mathbf{Q}_p[D]$-modules. Since $A_k = \pi^{\mathbf{Z}_p} \times U_k$, we have $A_k \otimes_{\mathbf{Z}_p} \mathbf{Q}_p \cong \mathbf{Q}_p \oplus \mathbf{Q}_p[D]$ as $\mathbf{Q}_p[D]$-modules, hence as representation spaces over $\mathbf{Q}_p$ for D. Therefore we obtain a $\mathbf{Q}_p[D]$-isomorphism

$$(X'/TX') \otimes_{\mathbf{Z}_p} \mathbf{Q}_p \cong \mathbf{Q}_p[D].$$

We need now the following lemma (Greengerg[Gr2] Lemma):

Lemma 1. *Let Δ be a finite abelian group, and let $\Im$ denote the quotient field of Λ. Let both M and M' be $\Lambda[\Delta]$-modules such that both of them are finitely generated and torsion-free as Λ-modules.*

Suppose that $M \otimes_\Lambda \Im$ and $M' \otimes_\Lambda \Im$ are isomorphic as representation spaces over $\Im$ for Δ, equivalently that $(M/TM) \otimes_{\mathbf{Z}_p} \mathbf{Q}_p$ and $(M'/TM') \otimes_{\mathbf{Z}_p} \mathbf{Q}_p$ are isomorphic as representation spaces over $\mathbf{Q}_p$ for Δ. Then there exists an injective $\Lambda[\Delta]$-homomorphism $\varphi : M \to M'$ such that $p^a M' \subseteq \varphi(M)$ for some integer $a \geq 0$.

Since $X' \cong \Lambda^d$, using Lemma 1 for the above isomorphism, we obtain there exists an injective $\Lambda[\Delta]$-homomorphism $\varphi : \Lambda[D] \to X'$ such that cokernel(φ) is annihilated by p^a. Let X^0 be the inverse image of $\Lambda[D]$ by the map $X \to X'$ at (1), and let cokernel(φ) $= F'$. Then we have a commutative diagram of $\Lambda[D]$-modules

$$\begin{array}{ccccccccc}
 & & & & 0 & & 0 & & \\
 & & & & \downarrow & & \downarrow & & \\
0 & \longrightarrow & \Lambda/(\dot{T}) & \longrightarrow & X^0 & \longrightarrow & \Lambda[D] & \longrightarrow & 0 \\
 & & \| & & \downarrow & & \downarrow & & \\
0 & \longrightarrow & \Lambda/(\dot{T}) & \longrightarrow & X & \longrightarrow & X' & \longrightarrow & 0 \\
 & & & & \downarrow & & \downarrow & & \\
 & & & & F' & \xrightarrow{\sim} & F' & & \\
 & & & & \downarrow & & \downarrow & & \\
 & & & & 0 & & 0. & &
\end{array}$$

Therefore we obtain an isomorphism : $X^0 \cong \Lambda[D] \oplus \Lambda/(\dot{T})$ of $\Lambda[D]$-modules. Thus, the cokernel of the map $X \to \Lambda[D] \oplus \Lambda/(\dot{T})$ defined by multiplication by p^a is annihilated by p^a. This completes the proof of (`i`).

(`ii`) Since $X/TX = \mathrm{Gal}(M_0/k_\infty)$, similarly as in the case (`i`), we obtain

$$(X/TX) \otimes_{\mathbf{Z}_p} \mathbf{Q}_p \cong \mathbf{Q}_p[D].$$

Since X is a torsion-free Λ-module, we can also use Lemma 1 in a similar manner as above to complete the proof of (`ii`). □

1.3. Let U_n be the principal units of k_n, and let

$$U_\infty = \varprojlim U_n,$$

where the projective limit is defined by means of the norm maps $k_m \to k_n$ for $m \geq n \geq 0$. Γ acts on U_∞ in the obvious manner; so as in §1.2, we can make U_∞ into a $\Lambda[D]$-module. We now study its structure as a $\Lambda[D]$-module in the following.

For each $n \geq 0$, let $\mathcal{O}_n$ be the ring of integers in k_n, and π_n a uniformizing parameter of k_n. Take the p-adic completion from the exact sequence

$$0 \longrightarrow (\mathcal{O}_n)^\times \longrightarrow {k_n}^\times \longrightarrow < \pi_n > \longrightarrow 0,$$

to obtain the exact sequence

$$0 \longrightarrow U_n \longrightarrow A_{k_n} \longrightarrow \mathbf{Z}_p \longrightarrow 0,$$

of $\mathbf{Z}_p[\mathrm{Gal}(k_n/\mathbf{Q}_p)]$-modules. For $m \geq n \geq 0$, we consider the maps $A_{k_m} \to A_{k_n}$ induced by the norm maps. For $m \geq n \geq n_0$, since k_m/k_n is a totally ramified extension, π_m maps to π_n by the norm map for a suitable choice of uniformizing parameters. Therefore we obtain the following commutative diagram for $m \geq n \geq n_0$:

$$\begin{array}{ccccccccc} 0 & \longrightarrow & U_m & \longrightarrow & A_{k_m} & \longrightarrow & \mathbf{Z}_p & \longrightarrow & 0 \\ & & \downarrow & & \downarrow & & \| & & \\ 0 & \longrightarrow & U_n & \longrightarrow & A_{k_n} & \longrightarrow & \mathbf{Z}_p & \longrightarrow & 0. \end{array}$$

Since $\varprojlim A_{k_n} \cong \varprojlim \mathrm{Gal}(M_n/k_n) = X$, taking the projective limit, we obtain the exact sequence

$$0 \longrightarrow U_\infty \longrightarrow X \longrightarrow \mathbf{Z}_p \longrightarrow 0 \qquad (2)$$

of $\Lambda[D]$-modules. We note that $\Gamma \times D$ act on $\mathbf{Z}_p$ trivially by definition.

New, we shall prove the following:

Proposition 2. (i) *Suppose that $k_\infty = k(W)$. Then we have an exact sequence*

$$0 \longrightarrow U_\infty \longrightarrow \Lambda[D] \oplus \Lambda/(\dot{T}) \longrightarrow F \longrightarrow 0$$

of $\Lambda[D]$*-modules, where* F *is a* $\Lambda[D]$*-module such that* $p^aF = 0$ *for some* $a \geq 0$.

(ii) *Suppose that* $k_\infty \neq k(W)$. *Then we have an exact sequence*

$$0 \longrightarrow U_\infty \longrightarrow \Lambda[D] \longrightarrow F \longrightarrow 0$$

of $\Lambda[D]$*-modules, where* F *is a* $\Lambda[D]$*-module such that* $p^aF = 0$ *for some* $a \geq 0$.

PROOF. (i) U_∞ contains $\varprojlim W^{(a)} \cong \Lambda/(\dot{T})$, and we set $U'_\infty = U_\infty/(\Lambda/(\dot{T}))$. From the sequences (1) and (2), we have a commutative diagram

$$\begin{array}{ccccccccc}
 & & & & 0 & & & & \\
 & & & & \downarrow & & & & \\
0 & \longrightarrow & \Lambda/(\dot{T}) & \longrightarrow & U_\infty & \longrightarrow & U'_\infty & \longrightarrow & 0 \\
 & & \| & & \downarrow & & \downarrow & & \\
0 & \longrightarrow & \Lambda/(\dot{T}) & \longrightarrow & X & \longrightarrow & X' & \longrightarrow & 0 \\
 & & & & \downarrow & & \downarrow & & \\
 & & & & \mathbf{Z}_p & \xrightarrow{\sim} & \mathbf{Z}_p & & \\
 & & & & \downarrow & & \downarrow & & \\
 & & & & 0 & & 0. & &
\end{array}$$

It follows that $U'_\infty \to X'$ is injective, and hence $U_\infty{}'$ is a torsion-free Λ-module, since $X' \cong \Lambda^d$. Tensoring $\Im$ over Λ for the sequence right vertical, we obtain an $\Im[D]$-isomorphism

$$U'_\infty \otimes_\Lambda \Im \cong X' \otimes_\Lambda \Im.$$

In the proof of Proposition 1 (i), we have seen that $X' \otimes_\Lambda \Im \cong \Im[D]$. Hence we have

$$U'_\infty \otimes_\Lambda \Im \cong \Im[D].$$

Using Lemma 1 to this situation, similarly as in Proposition 1 (i), one can prove (i).

(ii) U_∞ is a torsion-free Λ-module, since $U_\infty \subseteq X \subseteq \Lambda^d$. Tensoring $\Im$ over Λ for the sequence (2), we obtain an $\Im[D]$-isomorphism

$$U_\infty \otimes_\Lambda \Im \cong X \otimes_\Lambda \Im.$$

Using Lemma 1 and the isomorphism $X \otimes_\Lambda \Im \cong \Im[D]$, one can also prove (ii). This completes the proof of Proposition 2. □

REMARK 1. One can extend Propositions 1 and 2 to a more general situation as follows. Let k' be any finite algebraic extension of $\mathbf{Q}_p$ with the ring of integers $\mathcal{O}$, and let k'_∞/k' be a $\mathbf{Z}_p$-extension different from k'_{ur}/k'. Let k/k' be a finite abelian extension such that $k \cap k'_\infty = k'$, and let $k_\infty = kk'_\infty$. Let $D = \mathrm{Gal}(k/k') \cong \mathrm{Gal}(k_\infty/k'_\infty)$. Both X and U_∞, defined as above for k_∞/k are $\Lambda[D]$-modules. Then we have the same exact sequences of $\Lambda[D]$-modules as in Propositions 1 and 2, if we replace the terms $\Lambda[D] = \mathbf{Z}_p[D][[T]]$ by $\mathcal{O}[D][[T]]$.

I would like to thank Professor Masato Kurihara for supplying the following:

REMARK 2. Here we mention that there exists a $\mathbf{Z}_p$-extension k_∞/k such that U_∞ is not isomorphic to $\Lambda[D] \oplus \Lambda/(\dot{T})$ (i.e. $F \neq \{0\}$ in Proposition 2).

Let p be an odd prime. Let H be the unramified cyclic p-extension of $\mathbf{Q}_p$ with the ring of integers $\mathcal{O}_H$. Let $k = H(\zeta_p)$ and $k_\infty = H(W)$. Thus k_∞ is the cyclotomic $\mathbf{Z}_p$-extension of k. Note that, since $\mathcal{O}_H \cong \mathbf{Z}_p[\mathrm{Gal}(H/\mathbf{Q}_p)]$, we have $\mathcal{O}_H[[\mathrm{Gal}(k_\infty/H)]] \cong \mathbf{Z}_p[[\mathrm{Gal}(k_\infty/\mathbf{Q}_p)]] \cong \Lambda[D]$. By Coleman[C1], [C2] and Greither[G], there is an exact sequence of $\Lambda[D]$-modules

$$0 \longrightarrow \mathbf{Z}_p(1) \longrightarrow U_\infty \longrightarrow \mathcal{O}_H[[\mathrm{Gal}(k_\infty/H)]] \longrightarrow \mathbf{Z}_p(1) \longrightarrow 0$$

where $\mathbf{Z}_p(1) = \varprojlim W^{(a)}$. (In fact, if $H/\mathbf{Q}_p$ is any unramified extension, then such a sequence exists.) The first map is inclusion map, and $\mathbf{Z}_p(1) \cong \Lambda/(\dot{T})$. Then we will consider the kernel of the third map $\phi : \mathcal{O}_H[[\mathrm{Gal}(k_\infty/H)]] \longrightarrow \mathbf{Z}_p(1)$. The map ϕ is defined as following:

Let $\kappa : \mathrm{Gal}(k_\infty/H) \to \mathbf{Z}_p{}^\times$ be the p-cyclotomic character and fix a generator $\zeta = (\zeta_{p^a})$ of $\mathbf{Z}_p(1) = \varprojlim W^{(a)}$. Then ϕ is given by $\phi(\sigma) = \zeta^{-p\kappa(\sigma)}$ for $\sigma \in \mathrm{Gal}(k_\infty/H)$ and $\phi(v) = \zeta^{-\mathrm{Tr}(v)}$ for $v \in \mathcal{O}_H$, where Tr is the trace map from $\mathcal{O}_H$ to $\mathbf{Z}_p$.

Since $\mathcal{O}_H \cong \mathbf{Z}_p[\mathrm{Gal}(H/\mathbf{Q}_p)]$, identifying a generator τ of $\mathrm{Gal}(H/\mathbf{Q}_p)$ with the element $1+S$ in the formal power series ring $\mathbf{Z}_p[[S]]$, we obtain an isomorphism

$$\mathcal{O}_H \cong \mathbf{Z}_p[[S]]/((1+S)^p - 1) \cong \mathbf{Z}_p[S]/((1+S)^p - 1).$$

Therefore we have

$$\mathcal{O}_H[[\mathrm{Gal}(k_\infty/H)]] \cong (\mathbf{Z}_p[S]/((1+S)^p-1)\ )[\mathrm{Gal}(k/H)][[T]]\ (\ \cong \Lambda[D]\ ).$$

Then by the definition of ϕ, we have $S = \tau - 1$, $\dot{T} = \kappa(\gamma_0)\gamma_0^{-1} - 1 \in \ker(\phi)$. Hence the ideal $(S,\ \dot{T})$ which generated by S and $\dot{T}$ over $(\mathbf{Z}_p[S]/((1+S)^p - 1)\ )[\mathrm{Gal}(k/H)][[T]] \cong \Lambda[D]$ is contained in $\ker(\phi)$. Consequently $\ker(\phi)$ and hence, $U_\infty/(\Lambda/(\dot{T}))$ is not isomorphic to $\Lambda[D]$.

2. Semi-Local Units

We will denote by $\mathbf{Q}_\infty$ the cyclotomic $\mathbf{Z}_p$-extension of $\mathbf{Q}$. Let K be an abelian extension of $\mathbf{Q}$. Assume that K is of the first kind, that is, its conductor is not divisible by p^2 (or 8 if $p = 2$). Then we obviously have $K \cap \mathbf{Q}_\infty = \mathbf{Q}$. Let K_∞ be the cyclotomic $\mathbf{Z}_p$-extension over K, i.e. $K_\infty = K\mathbf{Q}_\infty$; and for each $n \geq 0$, let K_n denote the intermediate field of K_∞/K such that K_n is a cyclic extension of degree p^n over K. Then one can easily see that, under our assumption on K, every prime divisor of K, lying above p, is totally ramified in K_∞. Put $\Delta = \mathrm{Gal}(K/\mathbf{Q})$ and $\Gamma = \mathrm{Gal}(K_\infty/K)$.

Let v be a finite prime divisor of K_∞, lying above p. For each $n \geq 0$, let $K_{n,v}$ be the completion of K_n with respect to the restriction of v to K_n. Let $U_{n,v}$ denote the principal units of $K_{n,v}$, and let

$$\mathcal{U}_n = \prod_{v|p} U_{n,v},$$

where v runs over all the prime divisors of K_∞ lying above p. Let

$$\mathcal{U}_\infty = \varprojlim \mathcal{U}_n,$$

where the projective limit is defined by means of the maps $\mathcal{U}_m \to \mathcal{U}_n$ for $m \geq n \geq 0$ induced by the norm maps. Γ act on $\mathcal{U}_\infty$ in the obvious manner. Identifying Δ with $\mathrm{Gal}(K_\infty/\mathbf{Q}_\infty)$, we make $\mathcal{U}_\infty$ into a $\mathbf{Z}_p[\Delta][[T]]$-module as in §1.2. Then we will consider the structure of the $\mathbf{Z}_p[\Delta][[T]]$-module $\mathcal{U}_\infty$ in the following. Put $\Lambda[\Delta] = \mathbf{Z}_p[\Delta][[T]]$.

Let D be the decomposition group of p in $K/\mathbf{Q}$. Fixing a finite prime divisor v of K_∞, we have

$$\mathcal{U}_n = \prod_{\sigma \in D\backslash\Delta} \mathcal{U}_{n,v^\sigma} = \prod_{\sigma \in D\backslash\Delta} (\mathcal{U}_{n,v})^\sigma$$

where σ runs over a set of left representatives for the cosets of $D\backslash\Delta$. Therefore we obtain

$$\mathcal{U}_n \xrightarrow{\sim} U_{n,v} \otimes_{\mathbf{Z}_p[D]} \mathbf{Z}_p[\Delta]$$

as $\mathbf{Z}_p[\mathrm{Gal}(K_n/\mathbf{Q})]$-modules since every prime divisor of K, lying above p, is totally ramified in K_∞. Furthermore, since $\mathbf{Z}_p[\Delta]$ is a free $\mathbf{Z}_p[D]$-module, taking the projective limit, we obtain a $\Lambda[\Delta]$-isomorphism

$$\mathcal{U}_\infty \cong U_{\infty,v} \otimes_{\mathbf{Z}_p[D]} \mathbf{Z}_p[\Delta],$$

where $U_{\infty,v} = \varprojlim U_{n,v}$.

Tensoring $\mathbf{Z}_p[\Delta]$ over $\mathbf{Z}_p[D]$ on the sequences in Proposition 2, we obtain the following result:

Proposition 3. (i) *Suppose that K_v contains a primitive p-th root ζ_p of unity (or i if $p = 2$). Then we have an exact sequence*

$$0 \longrightarrow \mathcal{U}_\infty \longrightarrow \Lambda[\Delta] \oplus (\Lambda/(\dot{T}) \otimes_{\mathbf{Z}_p[D]} \mathbf{Z}_p[\Delta]) \longrightarrow F \longrightarrow 0$$

of $\Lambda[\Delta]$-modules, where F is a $\Lambda[\Delta]$-module such that $p^a F = 0$ for some $a \geq 0$.

(ii) *Suppose that K_v contains no primitive p-th root ζ_p of unity (or i if $p = 2$). Then we have an exact sequence*

$$0 \longrightarrow \mathcal{U}_\infty \longrightarrow \Lambda[\Delta] \longrightarrow F \longrightarrow 0$$

of $\Lambda[\Delta]$-modules, where F is a $\Lambda[\Delta]$-module such that $p^a F = 0$ for some $a \geq 0$.

Remark 3. Assume that $[K : \mathbf{Q}]$ is not divisible by p. Let Φ be an irreducible character of Δ over $\mathbf{Q}_p$, and e_Φ the corresponding idempotent in $\mathbf{Z}_p[\Delta]$:

$$e_\Phi = \frac{1}{[k : \mathbf{Q}]} \sum_{\delta \in \Delta} \Phi(\delta)\delta^{-1}.$$

Choose an absolutely irreducible component χ of Φ, and let $\mathcal{O}_\Phi$ denote the ring of integers in Ω_p generated by the values of χ over $\mathbf{Z}_p$. Then we obtain a $\mathbf{Z}_p[\Delta]$-isomorphism

$$e_\Phi \mathbf{Z}_p[\Delta] \xrightarrow{\sim} \mathcal{O}_\Phi.$$

Thus $e_\Phi\mathcal{U}_\infty$ is regarded as a $\mathcal{O}_\Phi$-module.

The following result had been shown by Gillard ([Gi] Proposition 1): As $\mathcal{O}_\Phi[[T]]$-modules

$$\begin{array}{lll} e_\Phi\mathcal{U}_\infty & \cong & \mathcal{O}_\Phi[[T]], \qquad\qquad \omega\chi^{-1}(p) \neq 1 \\ e_\Phi\mathcal{U}_\infty & \cong & \mathcal{O}_\Phi[[T]] \oplus \mathcal{O}_\Phi[[T]]/(\dot{T}), \qquad \omega\chi^{-1}(p) = 1, \end{array}$$

where ω is the Teichmüller character.

Hence if $[K:\mathbf{Q}]$ is not divisible by p, then we have the following:

(i) If ζ_p (or i if $p=2$) $\in K_v$, then

$$\mathcal{U}_\infty \cong \Lambda[\Delta] \oplus (\Lambda/(\dot{T}) \otimes_{\mathbf{Z}_p[D]} \mathbf{Z}_p[\Delta]).$$

(ii) If ζ_p (or i if $p=2$) $\notin K_v$, then

$$\mathcal{U}_\infty \cong \Lambda[\Delta].$$

3. Kummer Duality and Adjoint Modules

3.1. We keep the notion as in the previous section. But we assume here that p is an odd prime number, and that K contains a primitive p-th root ζ_p of unity. Let $J \in \Delta$ be the complex conjugation ($J \neq 1$).

For a $\Lambda[\Delta]$-module M, we define

$$M^+ = \{m \in M | J(m) = m\}, \quad M^- = \{m \in M | J(m) = -m\}.$$

Then $M^+ = (1+J)M$, $M^- = (1-J)M$ and

$$M = M^+ \oplus M^-$$

since $p \neq 2$. Also K_∞ contains the group W of all p-th power roots of unity, by our assumption on K.

Let M_∞ be the maximal abelian p-extension unramified outside p, and let

$$\mathcal{X} = \mathrm{Gal}(M_\infty/K_\infty).$$

It is known that there is a subgroup $\mathcal{m}$ of the discrete abelian group $K_\infty{}^\times \otimes_{\mathbf{Z}} (\mathbf{Q}_p/\mathbf{Z}_p)$ such that the usual pairing induces the Pontryagin duality:

$$<\,,\,>:\ \mathcal{X} \times \mathcal{m} \longrightarrow W.$$

It also has the property that

$$< \xi\sigma, \xi v > = \xi < \sigma, v >$$

for any $\xi \in \mathrm{Gal}(K_\infty/\mathbf{Q}) \cong \Gamma \times \Delta$ (cf.Iwasawa[Iw2] §7). Clearly the pairing $< \, , \, >$ induces a $\mathbf{Z}_p$-isomorphism

$$\mathscr{X} \ \tilde{\longrightarrow} \ \mathrm{Hom}_{\mathbf{Z}_p}(\mathscr{m}, W).$$

We define $\xi \circ \varphi$ for $\varphi \in \mathrm{Hom}_{\mathbf{Z}_p}(\mathscr{m}, W)$ by

$$(\xi \circ \varphi)(v) = \xi\varphi(\xi^{-1}v), \qquad \xi \in \Gamma \times \Delta, \ v \in \mathscr{m}.$$

Then the above $\mathbf{Z}_p$-isomorphism becomes a $\Lambda[\Delta]$-isomorphism.

Let A_n be the p-Sylow subgroup of the ideal class group of K_n, and let

$$A_\infty = \varinjlim A_n$$

where the inductive limit is defined by means of the natural maps $A_n \to A_m$ for $m \geq n \geq 0$. Clearly A_∞ is a $\Lambda[\Delta]$-module. It is known that

$$A_\infty{}^- \cong \mathscr{m}^-$$

as $\Lambda[\Delta]$-modules (cf.Iwasawa[Iw2] Lemma10). Then defining also $\xi \circ \varphi$ for $\varphi \in \mathrm{Hom}_{\mathbf{Z}_p}(A_\infty{}^-, W)$ by

$$(\xi \circ \varphi)(v) = \xi\varphi(\xi^{-1}v), \qquad \xi \in \Gamma \times \Lambda, \ v \in A_\infty{}^-,$$

we have a $\Lambda[\Delta]$-isomorphism

$$\mathscr{X}^+ \cong \mathrm{Hom}_{\mathbf{Z}_p}(A_\infty{}^-, W). \tag{3}$$

3.2. Let both M and M' be finitely generated Λ-modules. A morphism

$$f \ : \ M \ \longrightarrow \ M'$$

is call a pseudo-isomorphism if the kernel and the cokernel of f are both finite modules. When there exists such a pseudo-isomorphism, we write

$$M \sim M'.$$

If both M and M' are torsion Λ-modules, then $M \sim M'$ implies $M' \sim M$. However, this is not true in general. The structure theorem of finitely generated Λ-modules states that given such a Λ-module M, there exists a unique Λ-module of the form

$$E(M) = \Lambda^r \oplus \Big(\bigoplus_{i=1}^{t} \Lambda/(p)^{e_i}\Big) \oplus \Big(\bigoplus_{j=1}^{s} \Lambda/(f_j)^{e_j}\Big)$$

where f_j is an irreducible distinguished polynomial, $r,\ s,\ t\ \geq 0$ and $e_i,\ e_j > 0$, with $M \sim E(M)$ as Λ-modules. We call $E(M)$ the elementary Λ-module associated with M. Also Iwasawa invariants associated with M are defined by

$$\mu(M) = \sum_{i=1}^{t} e_i, \quad \lambda(M) = \sum_{j=1}^{s} e_j \deg(f_j).$$

Let $\wp$ be a prime ideal of height 1 in Λ. Then either $\wp = (p)$, the principal ideal generated by p, or there exists a unique irreducible distinguished polynomial $f(T)$ such that $\wp = (f(T))$.

For each prime ideal $\wp$ of height 1 in Λ, we will set

$$M_\wp = M \otimes_\Lambda \Lambda_\wp$$

where $\Lambda_\wp$ denotes the localization of Λ at $\wp$. Now, let Δ be a finite abelian group, and let M be a $\Lambda[\Delta]$-module which is finitely generated and torsion as a Λ-module. If

$$E(M) = \bigoplus_{i=1}^{t} \Lambda/{\wp_i}^{e_i}$$

with prime ideals $\wp_i$ of height 1 in Λ and $e_i > 0$, then $M_\wp = \{0\}$ if and only if $\wp \neq \wp_i,\ 1 \leq i \leq t$. Let X^0 and Y denote the kernel and the cokernel, respectively, of the morphism

$$M \ \longrightarrow\ \prod_\wp M_\wp$$

induced by the canonical map $M \to M_\wp$, the product being taken over all $\wp$. Then X^0 is the maximal finite Λ-submodule of M. We define

$$\alpha(M) = \mathrm{Hom}_{\mathbf{Z}_p}(Y, \mathbf{Q}_p/\mathbf{Z}_p).$$

and make $\alpha(M)$ into a $\Lambda[\Delta]$-module by defining

$$(\xi \cdot \varphi)(y) = \varphi(\xi y)$$

for $\xi \in \Delta \times \Lambda$, $\varphi \in \alpha(M)$ and $y \in Y$. We call $\alpha(M)$ the adjoint module of M.

Let both M and M' be finitely generated torsion Λ-modules. The following properties of Λ-modules are known (cf. Federer[F]):

1) $\alpha(M)$ is a finitely generated torsion Λ-module.
2) If E is an elementary torsion Λ-module then $E \cong \alpha(E)$.
3) If $M \sim M'$ then $\alpha(M) \sim \alpha(M')$.
4) $\alpha(M) \sim M$.

LEMMA 2. *Let M be a $\Lambda[\Delta]$-module which is finitely generated and torsion as a Λ-module. Suppose $\mu(M) = 0$. Then we have a pseudo-isomorphism of $\Lambda[\Delta]$-modules*

$$\alpha(M) \sim M.$$

PROOF. Let Φ be an irreducible character of Δ over $\mathbf{Q}_p$, and e_Φ the corresponding idempotent in $\mathbf{Q}_p[\Delta]$:

$$e_\Phi = \frac{1}{|\Delta|} \sum_{\delta \in \Delta} \Phi(\delta)\delta^{-1}.$$

Then we have

$$|\Delta| \sum_{\Phi} e_\Phi \mathbf{Z}_p[\Delta] \ \subseteq\ \mathbf{Z}_p[\Delta] \ \subseteq\ \sum_{\Phi} e_\Phi \mathbf{Z}_p[\Delta]$$

in $\mathbf{Q}_p[\Delta]$, where Φ runs over all distinct irreducible characters. Choose an absolutely irreducible component χ of Φ, and let $\mathcal{O}_\Phi$ denote the ring of integers in Ω_p generated by the values of χ over $\mathbf{Z}_p$. Then we obtain a $\mathbf{Z}_p[\Delta]$-isomorphism

$$e_\Phi \mathbf{Z}_p[\Delta] \xrightarrow{\sim} \mathcal{O}_\Phi.$$

Let F_1 and F_2 denote the cokernels of the inclusion map $\mathbf{Z}_p[\Delta] \to \bigoplus_{\Phi} \mathcal{O}_\Phi$ and the map $\bigoplus_{\Phi} \mathcal{O}_\Phi \to \mathbf{Z}_p[\Delta]$, defined by multiplication by p^N for large

$N \geq 0$, respectively. Then both F_1 and F_2 are finite modules, and the following diagram is commutative:

$$\begin{array}{ccccccccc} & & 0 & \longrightarrow & \mathbf{Z}_p[\Delta] & \longrightarrow & \bigoplus_{\Phi} \mathcal{O}_\Phi & \longrightarrow & F_1 & \longrightarrow & 0 \\ & & & & \downarrow p^N & & \| & & & & \\ 0 & \longleftarrow & F_2 & \longleftarrow & \mathbf{Z}_p[\Delta] & \longleftarrow & \bigoplus_{\Phi} \mathcal{O}_\Phi & \longleftarrow & 0. & & \end{array}$$

Then the above diagram induces a commutative diagram of $\Lambda[\Delta]$-module s

$$\begin{array}{ccccccccc} & & M & \longrightarrow & \bigoplus_{\Phi}(M \otimes_{\mathbf{Z}_p[\Delta]} \mathcal{O}_\Phi) & \longrightarrow & M \otimes_{\mathbf{Z}_p[\Delta]} F_1 & \longrightarrow & 0 \\ & & \downarrow p^N & & \| & & & & \\ 0 \longleftarrow M \otimes_{\mathbf{Z}_p[\Delta]} F_2 & \longleftarrow & M & \longleftarrow & \bigoplus_{\Phi}(M \otimes_{\mathbf{Z}_p[\Delta]} \mathcal{O}_\Phi), & & & & \end{array}$$

where $\delta \in \Delta$ acts on $M \otimes_{\mathbf{Z}_p[\Delta]} \mathcal{O}_\Phi$ as $\chi(\delta)$. Since $\mu(M) = 0$, both $M \otimes_{\mathbf{Z}_p[\Delta]} F_1$ and $M \otimes_{\mathbf{Z}_p[\Delta]} F_2$ are finite modules. Hence we have a pseudo-isomorphism

$$M \sim \bigoplus_{\Phi}(M \otimes_{\mathbf{Z}_p[\Delta]} \mathcal{O}_\Phi)$$

of $\Lambda[\Delta]$-modules. By the properties 3) and 4), we obtain

$$\begin{aligned} \alpha(M) &\sim \bigoplus_{\Phi} \alpha(M \otimes_{\mathbf{Z}_p[\Delta]} \mathcal{O}_\Phi) \\ &\sim \bigoplus_{\Phi}(M \otimes_{\mathbf{Z}_p[\Delta]} \mathcal{O}_\Phi) \\ &\sim M \end{aligned}$$

as $\Lambda[\Delta]$-modules. This completes the proof of Lemma 2. □

REMARK 4. If $|\Delta|$ is not divisible by p then

$$\mathbf{Z}_p[\Delta] = \sum_{\Phi} e_\Phi \mathbf{Z}_p[\Delta] \cong \bigoplus_{\Phi} \mathcal{O}_\Phi.$$

Hence we obtain

$$M \cong \bigoplus_{\Phi}(M \otimes_{\mathbf{Z}_p[\Delta]} \mathcal{O}_\Phi).$$

In this case, one can prove the above lemma without the assumption that $\mu(M)=0$.

3.3. We again assume that p is an odd prime number, and that K contains ζ_p here. Let L_n, $0 \le n \le \infty$, denote the maximal unramified abelian p-extension over K_n, and let

$$X = \mathrm{Gal}(L_\infty/K_\infty), \quad Y_0 = \mathrm{Gal}(L_\infty/K_\infty L_0).$$

We make $\mathrm{Hom}_{\mathbf{Z}_p}(A_\infty, \mathbf{Q}_p/\mathbf{Z}_p)$ into a $\Lambda[\Delta]$-module by defining

$$(\xi \cdot \varphi)(v) = \varphi(\xi v)$$

for $\xi \in \Gamma \times \Delta$, $\varphi \in \mathrm{Hom}_{\mathbf{Z}_p}(A_\infty, \mathbf{Q}_p/\mathbf{Z}_p)$ and $v \in A_\infty$. Then, it is known that there is a $\Lambda[\Delta]$-isomorphism

$$\alpha(Y_0) \;\cong\; \mathrm{Hom}_{\mathbf{Z}_p}(A_\infty, \mathbf{Q}_p/\mathbf{Z}_p)$$

(cf. Iwasawa[Iw2] Theorem 11). Now, we have $\mu(X) = 0$ when K is abelian over $\mathbf{Q}$ by Ferrero-Washington[F-W]. So, using Lemma 2 we obtain a pseudo-isomorphism

$$X^- \sim \mathrm{Hom}_{\mathbf{Z}_p}({A_\infty}^-, \mathbf{Q}_p/\mathbf{Z}_p) \tag{4}$$

of $\Lambda[\Delta]$-modules.

Fixing a $\mathbf{Z}_p$-isomorphism

$$W \xrightarrow{\sim} \mathbf{Q}_p/\mathbf{Z}_p$$

we have a $\mathbf{Z}_p$-isomorphism

$$\mathrm{Hom}_{\mathbf{Z}_p}({A_\infty}^-, W) \xrightarrow{\sim} \mathrm{Hom}_{\mathbf{Z}_p}({A_\infty}^-, \mathbf{Q}_p/\mathbf{Z}_p).$$

By (3) and (4), we obtain a pseudo-isomorphism

$$\mathscr{X}^+ \sim X^-,$$

of $\mathbf{Z}_p$-modules. We will consider the action of Γ and Δ on the above groups. Since $K_\infty \supset W$, we may consider the p-cyclotomic character $\kappa : \Gamma \to$

$1 + p\mathbf{Z}_p$. Recall $\dot{T} = \kappa(\gamma_0)(1+T)^{-1} - 1$. For $\varphi \in \mathrm{Hom}_{\mathbf{Z}_p}({A_\infty}^-, W) \cong \mathrm{Hom}_{\mathbf{Z}_p}({A_\infty}^-, \mathbf{Q}_p/\mathbf{Z}_p)$, we have

$$\begin{aligned} (\gamma_0 \circ \varphi)(v) &= \gamma_0 \varphi(\gamma_0^{-1} v) \\ &= \kappa(\gamma_0)\varphi(\gamma_0^{-1} v) \\ &= ((\kappa(\gamma_0)\gamma_0^{-1}) \cdot \varphi)(v). \end{aligned}$$

Thus the following diagram is commutative:

$$(5) \qquad \begin{array}{ccc} \mathcal{X}^+ & \longrightarrow & X^- \\ {\scriptstyle (1+T)}\big\downarrow & & \big\downarrow{\scriptstyle (1+\dot{T})} \\ \mathcal{X}^+ & \longrightarrow & X^- \end{array}$$

where the lefthand map and the righthand map are the action of $(1+T)$ and $(1+\dot{T})$, respectively.

Next we will consider the action of Δ. We note that, for $\zeta \in W$ and $\delta \in \Delta$, we have

$$\delta(\zeta) = \zeta^{\omega(\delta)}.$$

For $\varphi \in \mathrm{Hom}_{\mathbf{Z}_p}({A_\infty}^-, W) \cong \mathrm{Hom}_{\mathbf{Z}_p}({A_\infty}^-, \mathbf{Q}_p/\mathbf{Z}_p)$,

$$\begin{aligned} (\delta \circ \varphi)(v) &= \delta \varphi(\delta^{-1} v) \\ &= \omega(\delta)\varphi(\delta^{-1} v) \\ &= ((\omega(\delta)\delta^{-1}) \cdot \varphi)(v). \end{aligned}$$

Thus the following diagram is commutative, for any $\delta \in \Delta$:

$$(6) \qquad \begin{array}{ccc} \mathcal{X}^+ & \longrightarrow & X^- \\ {\scriptstyle \delta}\big\downarrow & & \big\downarrow{\scriptstyle \omega(\delta)\delta^{-1}} \\ \mathcal{X}^+ & \longrightarrow & X^- \end{array}$$

where the lefthand map and the righthand map are the action of δ and $\omega(\delta)\delta^{-1}$, respectively.

3.4. In this subsection, we let K be a totally imaginary finite abelian extension of $\mathbf{Q}$ of the first kind; but we assume that K does not contain ζ_p.

Letting $K' = K(\zeta_p)$, K' is also an abelian extension of $\mathbf{Q}$ of the first kind. Then $\mathrm{Gal}(K'/K)$ is a cyclic group of degree $d(\neq 1)$, which is a divisor of $(p-1)$. Fix a generator σ of $\mathrm{Gal}(K'/K)$. Let K'_∞ be the cyclotomic $\mathbf{Z}_p$-extension over K', i.e. $K'_\infty = K'\mathbf{Q}_\infty = K(W)$, and let $\Delta' = \mathrm{Gal}(K'/\mathbf{Q})$.

Let L'_∞ be the maximal unramified abelian p-extension over K'_∞, and M'_∞ the maximal abelian p-extension over K'_∞ unramified outside p. Let

$$X' = \mathrm{Gal}(L'_\infty/K'_\infty),\ \mathscr{X}' = \mathrm{Gal}(M'_\infty/K'_\infty).$$

Clearly both X' and $\mathscr{X}'$ are $\Lambda[\Delta']$-modules. We let

$$e_i = \frac{1}{d}\sum_{j=1}^{d} \omega^i(\sigma^j)\sigma^{-j} \in \mathbf{Z}_p[\Delta'].$$

On the other hand we have seen in §3.2 that

$$\mathscr{X}'^{+} \sim\ X'^{-}$$

as $\Lambda[\Delta']$-modules, in the sense of (5) and (6). By (6), we obtain

$$e_i\mathscr{X}'^{+} \sim\ e_{1-i}X'^{-}.$$

Let M^0 be the maximal abelian extension of K_∞ contained in M'_∞. Then one can see easily that M^0 corresponds to $(\sigma-1)\mathscr{X}'$, and $\mathscr{X} = \mathrm{Gal}(M_\infty/K_\infty)$ is the p-Sylow subgroup of $\mathrm{Gal}(M^0/K_\infty)$. Hence

$$\mathscr{X} \cong \mathscr{X}'/(\sigma-1)\mathscr{X}'$$

since the order of $\mathrm{Gal}(K'_\infty/K_\infty) \cong <\sigma>$ is prime to p. Similarly for X', we have

$$X \cong X'/(\sigma-1)X'.$$

Therefore we obtain

$$X = e_0X \cong e_0\Big(X'/(\sigma-1)X'\Big) \cong e_0X'.$$

Summarizing the above results, we see that

$$X^{-} \sim e_1\mathscr{X}'^{+}$$

as $\Lambda[\Delta]$-modules. (The action of $\Lambda[\Delta]$ is the same as above.)

4. Pseudo-Cyclic $\Lambda[\Delta]$-Modules and Cyclic $\Lambda_\wp[\Delta]$-Modules

Let M be a finitely generated torsion Λ-module. We will call M a pseudo-cyclic Λ-module if there exists a cyclic Λ-module M' with

$$M \sim M'$$

as Λ-modules. Assume that $g,\ h \in \Lambda$ are relatively prime. One can easily see that

$$\begin{aligned} \Lambda/(g\cdot h) &\sim \Lambda/(g) \oplus \Lambda/(h) \\ \Lambda/(g) \oplus \Lambda/(h) &\sim \Lambda/(g\cdot h). \end{aligned}$$

Let

$$E(M) = \bigoplus_{i=1}^{t} \Lambda/{\wp_i}^{e_i}$$

with prime ideals $\wp_i$ of height 1 in Λ and $e_i > 0$. Then M is a pseudo-cyclic Λ-module if and only if $\wp_i \neq \wp_j$ for all $i \neq j,\ 1 \leq i,\ j \leq t$. Furthermore this is equivalent to saying that $M_\wp = M \otimes_\Lambda \Lambda_\wp$ is a cyclic $\Lambda_\wp$-module for every $\wp$.

Let $\mathcal{O}$ be the ring of integers in a finite algebraic extension of $\mathbf{Q}_p$, and let

$$\Lambda_{\mathcal{O}} = \mathcal{O}[[T]], \quad M_{\mathcal{O}} = M \otimes_{\mathbf{Z}_p} \mathcal{O} = M \otimes_\Lambda \Lambda_{\mathcal{O}}.$$

Noting the remark above, M is a pseudo-cyclic Λ-module if and only if $M_{\mathcal{O}}$ is a pseudo-cyclic $\Lambda_{\mathcal{O}}$-module.

Now, let Δ be a finite abelian group, and let M be a $\Lambda[\Delta]$-module which is finitely generated and torsion as a Λ-module. We will call M a pseudo-cyclic $\Lambda[\Delta]$-module if there exists a cyclic $\Lambda[\Delta]$-module M' with

$$M \sim M'.$$

as $\Lambda[\Delta]$-modules. Since both M and M' are torsion Λ-modules, we also have $M' \sim M$. Therefore M is a pseudo-cyclic $\Lambda[\Delta]$-module if and only if M contains a cyclic $\Lambda[\Delta]$-submodule of finite index.

We will assume that $\mu(M) = 0$. We have seen, in the proof of Lemma 2, that

$$M \sim \bigoplus_{\Phi} M \otimes_{\mathbf{Z}_p[\Delta]} \mathcal{O}_\Phi$$

as $\Lambda[\Delta]$-modules, where Φ runs over all distinct irreducible characters of Δ over $\mathbf{Q}_p$. Note that $\delta \in \Delta$ acts on $M \otimes_{\mathbf{Z}_p[\Delta]} \mathcal{O}_\Phi$ as $\chi(\delta)$, where χ is an absolutely irreducible component of Φ. Then M is a pseudo-cyclic $\Lambda[\Delta]$-module if and only if $M \otimes_{\mathbf{Z}_p[\Delta]} \mathcal{O}_\Phi$ is a pseudo-cyclic $\mathcal{O}_\Phi[[T]]$-module for every irreducible character Φ of Δ over $\mathbf{Q}_p$. Furthermore, let $\mathcal{O}$ be the ring of integers in a finite algebraic extension of $\mathbf{Q}_p$, containing all the values of χ, which will be fixed throughout the following, and let

$$\begin{aligned} M_{\Phi,\mathcal{O}} &= M \otimes_{\mathbf{Z}_p[\Delta]} \mathcal{O}_\Phi \otimes_{\mathcal{O}_\Phi} \mathcal{O} \\ &= M \otimes_{\mathbf{Z}_p[\Delta]} \mathcal{O}. \end{aligned}$$

Note that $\delta \in \Delta$ acts on $M_{\Phi,\mathcal{O}}$ as $\chi(\delta)$. Summarizing the argument above, M is a pseudo-cyclic $\Lambda[\Delta]$-module if and only if $M_{\Phi,\mathcal{O}}$ is pseudo-cyclic $\Lambda_{\mathcal{O}}$-module for every irreducible character Φ of Δ over $\mathbf{Q}_p$.

We shall prove following two lemmas:

LEMMA 3. *Let M be a $\Lambda[\Delta]$-module which is finitely generated and torsion as a Λ-module. Suppose $\mu(M) = 0$. Then the following two conditions are equivalent:*

a) *M is a pseudo-cyclic $\Lambda[\Delta]$-module.*

b) *$M_\wp$ is a cyclic $\Lambda_\wp[\Delta]$-module for every prime ideal $\wp$ of height 1 in Λ.*

PROOF. a) $\Rightarrow$ b) is clear.

a) is equivalent to saying that $M_{\Phi,\mathcal{O}}$ is pseudo-cyclic $\Lambda_{\mathcal{O}}$-module for every irreducible character Φ of Δ over $\mathbf{Q}_p$. In addition, this is also equivalent to saying that $M_{\Phi,\mathcal{O}} \otimes_{\Lambda_{\mathcal{O}}} \Lambda_{\mathcal{O},\wp}$ is a cyclic $\Lambda_{\mathcal{O},\wp}$-module for every irreducible character Φ of Δ over $\mathbf{Q}_p$ and for every prime ideal $\wp$ of height 1 in $\Lambda_{\mathcal{O}}$, where $\Lambda_{\mathcal{O},\wp}$ denote the localization of $\Lambda_{\mathcal{O}}$ at $\wp$. Since

$$\begin{aligned} \bigoplus_\Phi (M_{\Phi,\mathcal{O}} \otimes_{\Lambda_{\mathcal{O}}} \Lambda_{\mathcal{O},\wp}) &= (\bigoplus_\Phi M_{\Phi,\mathcal{O}}) \otimes_{\Lambda_{\mathcal{O}}} \Lambda_{\mathcal{O},\wp} \\ &\cong (M \otimes_{\mathbf{Z}_p} \mathcal{O}) \otimes_{\Lambda_{\mathcal{O}}} \Lambda_{\mathcal{O},\wp} \\ &= M_{\mathcal{O},\wp}, \end{aligned}$$

the above statement is equivalent to saying that $M_{\mathcal{O},\wp}$ is a cyclic $\Lambda_{\mathcal{O},\wp}[\Delta]$-module for every $\wp$. Therefore a necessary and sufficient condition for a) is that $M_{\mathcal{O},\wp}$ is cyclic $\Lambda_{\mathcal{O},\wp}[\Delta]$-module for every prime ideal $\wp$ of height 1 in $\Lambda_{\mathcal{O}}$.

Now, let $\wp$ be a prime ideal of height 1 in Λ, and write $\wp = \wp_1 \cdots \wp_s$ in $\Lambda_{\mathcal{O}}$, where $\wp_i$, $1 \le i \le s$, are prime ideals of height 1 in $\Lambda_{\mathcal{O}}$. Since

$$\begin{aligned} M_{\mathcal{O},\wp_i} &= (M \otimes_{\mathbf{Z}_p} \mathcal{O}) \otimes_{\Lambda_{\mathcal{O}}} \Lambda_{\mathcal{O},\wp_i} \\ &= M_\wp \otimes_{\Lambda_\wp} \Lambda_{\mathcal{O},\wp_i}, \end{aligned}$$

if $M_\wp$ is a cyclic $\Lambda_\wp[\Delta]$-module, then $M_{\mathcal{O},\wp_i}$ is a cyclic $\Lambda_{\mathcal{O},\wp_i}[\Delta]$-module for $1 \le i \le s$.

Hence we have b) $\Rightarrow$ a), this completes the proof of Lemma 3. □

Lemma 4. *Let M be a pseudo-cyclic $\Lambda[\Delta]$-module. Suppose $\mu(M) = 0$. Then a $\Lambda[\Delta]$-submodule of M is also a pseudo-cyclic $\Lambda[\Delta]$-module.*

Proof. $M_{\mathcal{O},\Phi}$ is a pseudo-cyclic $\Lambda_{\mathcal{O}}$-module for every irreducible character Φ of Δ over $\mathbf{Q}_p$. By the structure theorem of finitely generated Λ-modules, for every prime ideal $\wp$ of height 1 in $\Lambda_{\mathcal{O}}$, there exist some $e \ge 0$ such that

$$M_{\mathcal{O},\Phi} \otimes_{\Lambda_{\mathcal{O}}} \Lambda_{\mathcal{O},\wp} \cong (\Lambda_{\mathcal{O}}/\wp^e) \otimes_{\Lambda_{\mathcal{O}}} \Lambda_{\mathcal{O},\wp}.$$

Any $\Lambda_{\mathcal{O},\wp}$-submodule of $(\Lambda_{\mathcal{O}}/\wp^e) \otimes_{\Lambda_{\mathcal{O}}} \Lambda_{\mathcal{O},\wp}$ is also a cyclic $\Lambda_{\mathcal{O},\wp}$-module. Hence a $\Lambda_{\mathcal{O}}$-submodule of $M_{\mathcal{O},\Phi}$ is also a pseudo-cyclic $\Lambda_{\mathcal{O}}$-module, which completes the proof of the lemma. □

Remark 5. If $|\Delta|$ is not divisible by p then we have seen that

$$M = \sum_{\Phi} e_\Phi M \cong \bigoplus_{\Phi} (M \otimes_{\mathbf{Z}_p[\Delta]} \mathcal{O}_\Phi)$$

in Remark 4. Therefore, in this case, one can prove the above lemmas without the assumption that $\mu(M) = 0$.

5. The Proof of the Main Theorem and Its Corollary

Let E_n be the group of all units in K_n, and identify it with the image of the embedding $K_n \hookrightarrow \prod_{v|p} K_{n,v}$. Let $\mathcal{E}_n$ be the closure of $E_n \cap \mathcal{U}_n$ in $\mathcal{U}_n$, and let

$$\mathcal{E}_\infty = \varprojlim \mathcal{E}_n,$$

where the projective limit is defined in the same manner as $\mathcal{U}_\infty$.

The main theorem of this paper is the following:

THEOREM. *Let p be an odd prime and let K be a totally imaginary abelian extension of $\mathbf{Q}$. Suppose that K is of the first kind. Then $(\mathcal{U}_\infty/\mathcal{E}_\infty)^+$ contains a cyclic $\Lambda[\Delta]$-submodule of finite index.*

We will show that $\mu(\ (\mathcal{U}_\infty/\mathcal{E}_\infty)^+) = 0$, in order to apply Lemma 3 to $(\mathcal{U}_\infty/\mathcal{E}_\infty)^+$.

By class field theory, there is the exact sequence

$$0 \longrightarrow (\mathcal{U}_\infty/\mathcal{E}_\infty)^+ \longrightarrow \mathfrak{X}^+ \longrightarrow X^+ \longrightarrow 0 \tag{7}$$

of $\Lambda[\Delta]$-modules (cf.Washington[W] Corollary 13.6).

By $\mu(X^+) = 0$, we have $\mu(\ (\mathcal{U}_\infty/\mathcal{E}_\infty)^+) = \mu(\mathfrak{X}^+)$. If $\zeta_p \in K$, then we have seen that

$$\mathfrak{X}^+ \sim X^-$$

in §3.3.Therefore we obtain $\mu(\mathfrak{X}^+) = \mu(X^-) = 0$.

If $\zeta_p \notin K$ then we have seen that

$$\mathfrak{X}^+ \cong \mathfrak{X}'^+/(\sigma - 1)\mathfrak{X}'^+$$

in §3.4. Therefore we have $\mu(\mathfrak{X}^+) \leq \mu(\mathfrak{X}'^+) = 0$.

Hence we have proved that $\mu(\ (\mathcal{U}_\infty/\mathcal{E}_\infty)^+) = 0$.

By Lemma 3, it is enough to prove the following claim, for each prime ideal $\wp$ of height 1 in Λ:

CLAIM. $((\mathcal{U}_\infty/\mathcal{E}_\infty)^+)_\wp$ is a cyclic $\Lambda_\wp[\Delta]$-module.

First we will consider the case where $\zeta_p \in K$.

- $\wp \neq (\dot{T}),\ (p)$. By Proposition 3 (i), we obtain a $\Lambda_\wp[\Delta]$-isomorphism

$$(\mathcal{U}_\infty)_\wp \cong \Lambda_\wp[\Delta].$$

The result follows immediately.

- $\wp = (p)$. Since $\mu(\ (\mathcal{U}_\infty/\mathcal{E}_\infty)^+) = 0$, we have $((\mathcal{U}_\infty/\mathcal{E}_\infty)^+)_{(p)} = \{0\}$.
- $\wp = (\dot{T})$. By (7), we have that $((\mathcal{U}_\infty/\mathcal{E}_\infty)^+)_{(\dot{T})}$ is isomorphic to a $\Lambda_{(\dot{T})}[\Delta]$-submodule of $(\mathfrak{X}^+)_{(\dot{T})}$. Thus it is enough to prove that $(\mathfrak{X}^+)_{(\dot{T})}$ is a

cyclic $\Lambda_{(\dot{T})}[\Delta]$-module by Lemma 4. Furthermore, by the diagram (5), this is equivalent to saying that $(X^-)_{(T)}$ is a cyclic $\Lambda_{(T)}[\Delta]$-module. So we will consider the cyclicity of $(X^-)_{(T)}$.

In general for any CM-field K, Iwasawa[Iw2] and Greenberg[Gr1] have shown that the characteristic polynomial $f(T)$ of X^- is divisible by $T^{r(K)}$, where $r(K)$ is the number of prime ideals dividing p in the maximal totally real subfield K^+ of K which split in K. Let $g(T) = f(T)/T^{r(K)}$ Greenberg[Gr1] has then shown that $g(0)$ is non-zero when K is an abelian field.

Let L'_∞ be the maximal subextension of K_∞ in L_∞ in which every prime divisor of K_∞, lying above p splits completely. Sinnott ([S] Proposition 6.1) has shown that

$$\mathrm{Gal}(L_\infty/L'_\infty)^- \cong (\Lambda/(T))^{r(K)}.$$

Summarizing the above results, the characteristic polynomial of $\mathrm{Gal}(L'_\infty/K_\infty)^-$ is prime to T. Then we have $\mathrm{Gal}(L'_\infty/K_\infty)^-{}_{(T)} = X^-{}_{(T)}$. In the proof of the above result, Sinnott has also shown that

$$\mathrm{Gal}(L_\infty/L'_\infty)^- \cong \Big(\bigoplus_v \mathbf{Z}_p \cdot v\Big)^- .$$

where v runs over all the primes of K_∞ lying above p, using Iwasawa[Iw2] Lemma 24. Since K is of the first kind, the direct sum is running over all the primes of K lying above p. Therefore we have

$$\mathrm{Gal}(L_\infty/L'_\infty)^- \cong \mathbf{Z}_p[D\backslash\Delta]^-,$$

as desired.

This proves the claim for all prime ideals $\wp$ of height 1 in Λ when $\zeta_p \in K$.

Next we will consider the case where $\zeta_p \notin K$.

Let $\mathcal{U}_\infty{}'$ and $\mathcal{E}_\infty{}'$ be the modules defined as before for K'_∞/K', where $K' = K(\zeta_p)$. Then $e_0(\mathcal{U}_\infty{}'/\mathcal{E}_\infty{}')^+ = (\mathcal{U}_\infty/\mathcal{E}_\infty)^+$ and we have just proved that $(\mathcal{U}_\infty{}'/\mathcal{E}_\infty{}')^+$ is a pseudo-cyclic $\Lambda[\Delta']$-module. Hence we see that $(\mathcal{U}_\infty/\mathcal{E}_\infty)^+$ is a pseudo-cyclic $\Lambda[\Delta]$-module. This completes the proof of the main Theorem.

We remark that, when $\zeta_p \notin K_v$, one can prove the claim using Proposition 3 (`ii`), in a manner similar to the case where $\zeta_p \in K$.

Let X' and $\mathscr{X}'$ denote the modules for $K' = K(\zeta_p)$ defined in §3.4, and let e_i denote the element in $\mathbf{Z}_p[\Delta']$ defined in §3.4. Under the same condition as in Theorem, we obtain the following:

COROLLARY. (i) *When* $\zeta_p \in K$, *suppose that* $\lambda(X^+) = 0$. *Then* X^- *contains a cyclic* $\Lambda[\Delta]$*-submodule of finite index.*

(ii) *When* $\zeta_p \notin K$, *suppose that* $\lambda(e_1 X'^+) = 0$. *Then* X^- *contains a cyclic* $\Lambda[\Delta]$*-submodule of finite index.*

PROOF. (i) By $\mu(X^+) = \lambda(X^+) = 0$ and (7), we have that $(\mathcal{U}_\infty/\mathcal{E}_\infty)^+ \sim \mathfrak{X}^+$. However, we have seen that $\mathfrak{X}^+ \sim X^-$ in §3.3, which proves (i).

(ii) By $\mu(X'^+) = \lambda(e_1 X'^+) = 0$ and (7), we have that $e_1(\mathcal{U}_\infty{}'/\mathcal{E}_\infty{}')^+ \sim e_1\mathfrak{X}'^+$. And, we have seen that $e_1\mathfrak{X}'^+ \sim e_0 X'^- = X^-$ in §3.4, which completes the proof of (ii). □

References

[C1] Coleman, R., Division values in local fields, Invent. Math. **53** (1979), 91–116.

[C2] Coleman, R., Local units modulo circular units, Proc. Amer. Math. Soc. **89** (1983), 1–7.

[F] Federer, L. J., Neotherian $\mathbf{Z}_p[[T]]$-modules, adjoints, and Iwasawa theory, Illinois J. Math. **30** (1986), 636–652.

[F-W] Ferrero, B. and L. C. Washington, The Iwasawa invariant μ_p vanishes for abelian number fields, Ann. of Math. **109** (1979), 377–395.

[Gi] Gillard, R., Unités cyclotomiques,unités semi-locales et $\mathbf{Z}_l$-extensions II, Ann. Inst. Fourier, Grenoble **29** (1979), fasc.4 1–15.

[Gr1] Greenberg, R., On a certain l-adic repesentation, Invent. Math. **21** (1973), 117–124.

[Gr2] Greenberg, R., On Iwasawa invariants of totally real number fields, Amer. J. Math. **98** (1976), 263–284.

[G] Greither, C, Class group of abelian fields, and the main conjecture, Ann. Inst. Fourier, Grenoble **42** (1992), fasc.3 449–499.

[Iw1] Iwasawa, K., On some modules in the theory of the cyclotomic fields, J. Math. Soc. Japan **16** (1964), 42–82.

[Iw2] Iwasawa, K., On $\mathbf{Z}_l$-extensions of algebraic number fields, Ann. of Math. **98** (1973), 246–326.

[S] Sinnott, W., Appendix to L. J. Federer and B. H. Gross: Regulaters and Iwasawa modules, Invent. Math. **62** (1981), 443–457.

[W] Washington, L. C., Introduction to Cyclotomic Fields, GTM 83, Springer-Verlag: New York, 1983.

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