J. Math. Sci. Univ. Tokyo
2 (1995), 637–655.

Singularities of the Bergman kernel for a two-dimensional pseudoconvex tube domain with corners

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Dedicated to Professor Hikosaburo Komatsu on his sixtieth birthday

Abstract. We consider singularities of the Bergman kernel at corner point for a two-dimensional tube pseudoconvex domain with corners and obtain an asymptotic expansion from the microlocal point of view.

Introduction

The study on the Bergman kernel has a long history and contains enormous works. Especially, the regularity of the Bergman kernel has investigated by many people. Let $\Omega = \{z; f(z,\overline{z}) < 0\} \Subset \mathbb{C}^n$ be a strictly pseudoconvex bounded domain with $\mathcal{C}^\infty$ (resp. analytic) boundary; that is, f is a $\mathcal{C}^\infty$ (resp. analytic) function satisfying that $df \neq 0$ at $f = 0$ and that the matrix $\left(\frac{\partial^2 f}{\partial z_j \partial \overline{z}_k}\right)$ is positive definite at every point of $\partial\Omega$. We denote the Bergman kernel for Ω by $B(z, \overline{w})$. In 1974, C. Fefferman proved the following:

0.1 Theorem. ([F]) *Assume that $\Phi : \Omega \to \widetilde{\Omega}$ is a biholomorphic mapping between bounded strictly pseudoconvex domains with $\mathcal{C}^\infty$ boundary. Then, Φ can be extended smoothly up to the boundary.*

In order to prove this theorem, he obtained a new precise result on singularities of $B(z, \overline{z})$ near the boundary. In fact $B(z, \overline{z})$ has a form of

1991 *Mathematics Subject Classification.* Primary 32H10; Secondary 32A45, 32F15, 41A60.

typical asymptotic expansion appearing in the theory of pseudodifferential operators. Seeing his result, L. Boutet de Monvel and J. Sjöstrand found out the following Fourier integral representation of the Bergman kernel in 1976:

0.2 THEOREM. ([B-Sj]) *Assume that $f(z,\overline{z})$ is $\mathcal{C}^\infty$, then $B(z,\overline{w})$ has the following asymptotic expansion:*

$$B(z,\overline{w}) \equiv \int_0^\infty e^{\sqrt{-1}tg(z,\overline{w})} b(z,\overline{w},t)\,dt \qquad \text{mod. } \mathcal{C}^\infty\text{kernels.} \tag{0.1}$$

Here $g(z,\overline{w})$ is an almost holomorphic extension of the function $g(z,\overline{z}) = -\sqrt{-1}f(z,\overline{z})$, and the amplitude $b(z,\overline{w},t)$ is an element of $S^n(\overline{\Omega}\times\overline{\Omega^a}\times\mathbb{R}_+)$ and allows an asymptotic expansion at $t = \infty$ of the form: $\sum_{k=0}^\infty t^{n-k} b_k(z,\overline{w})$ where Ω^a denotes the complex conjugate of Ω.

Inspired by their result, M. Kashiwara obtained a holonomic system satisfied by the Bergman kernel when Ω has analytic boundary:

0.3 THEOREM. ([Kash]) *Assume that $f(z,\overline{z})$ is analytic. Then The Bergman kernel $B(z,\overline{w})$ satisfies the following microdifferential equations near the hypersurface $\{f(z,\overline{w}) = 0\}$ which is the complexification of the boundary $\partial\Omega$: For any microdifferential operators P, Q satisfying*

$$P(z,\partial_z)Y(-f(z,\overline{w})) = Q(w,\partial_{\overline{w}})Y(-f(z,\overline{w})), \tag{0.2}$$

it follows that

$$^tP(z,\partial_z)B(z,\overline{w}) = {}^tQ(w,\partial_{\overline{w}})B(z,\overline{w}). \tag{0.3}$$

Here ^{t} denotes the formal adjoint of operators, Y denotes the Heaviside function, and the equalities (0.2) *and* (0.3) *hold as holomorphic microfunctions.*

He also showed that the Bergman kernel can be determined locally modulo functions holomorphic at the boundary. Precisely he obtained the following theorem by using the microlocal Bergman kernel (that is, a microlocalization of the Bergman kernel):

0.4 Theorem. (see also [Kash]) *Under the same condition and notation, for any $z_0 \in \partial\Omega$ there exist some neighborhood $U \ni z_0$ and $a(z,\overline{w})$, $b(z,\overline{w}) \in \mathcal{O}(U \times U^a)$ such that the Bergman kernel has the following form in $U \times U^a$:*

$$B(z,\overline{w}) = \frac{a(z,\overline{w})}{f(z,\overline{w})^{n+1}} + b(z,\overline{w})\log(-f(z,\overline{w})). \tag{0.4}$$

Here $\mathcal{O}(U \times U^a)$ denotes the set of holomorphic functions on $U \times U^a$.

(See [Kan2] for proofs of theorems (0.3) (0.4) and further study, and also note that if we replace the Bergman kernel and the Heaviside function with the Szegö kernel and Dirac's δ-function respectively, similar results hold). Hence, if $\partial\Omega$ is analytic and strictly pseudoconvex on some neighborhood of $y^1 \in \partial\Omega$, we see that the Bergman kernel has the form above at y^1. However, when a domain has non-smooth boundary, it seems that the study of singularities of the Bergman kernel is not so satisfactory. Therefore in this paper, as a simple example in non-smooth boundary cases we will consider singularities of the Bergman kernel for a two-dimensional pseudoconvex tube domain $\Omega = \mathbb{R}^2 + \sqrt{-1}W$, with $W = W_1 \cap W_2$, where each W_j is strictly convex domain as follows:

$$\begin{cases} W_j = \{y \in \mathbb{R}^2; \varphi_j(y) < 0\} \text{ with an analytic function} \\ \qquad \varphi_j \ (j = 1,2) \text{ such that} \\ (1)\partial W_1 \text{ and } \partial W_2 \text{ intersect transversally,} \\ (2) \text{ if } \varphi_j(y_0) = 0, \text{ then } d\varphi_j(y_0) \neq 0 \text{ and the Hessian matrix} \\ \qquad \left(\frac{\partial^2\varphi_j}{\partial y_k \partial y_\ell}(y_0)\right)_{1\le k,\ell\le 2} \text{ is positive definite for } j = 1,2. \end{cases} \tag{0.5}$$

In this paper, we interpret the Bergman kernel as a microfunction on some conormal bundle. We will denote the Bergman kernel for Ω by $B(z,\overline{w})$. In Section 1, we first recall the integral representation of the Bergman kernel for the pseudoconvex tube domain. The Bergman kernel is holomorphic except for the diagonal points $\{z = w\}$ at the boundary. (Note that this fact was already known before Fefferman's work). Hence, setting $z := x + \sqrt{-1}y$, $w := u + \sqrt{-1}v$ we study the singularity of $B(z,\overline{w})$ at $\{x = u,\ y + v = 0,\ y = y^1\}$ with $y^1 \in \partial W_1 \cap \partial W_2$ as a hyperfunction. Precisely, we set a holomorphic function $f(z,\overline{w}) := B(z + \sqrt{-1}y^1, \overline{w} - \sqrt{-1}y^1)$ and investigate singularity of $f(z,\overline{w})$ at $\{x=u=0, y=v=0\}$. We can see that hyperfunction $f((x,u)+\sqrt{-1}(\Gamma\times\Gamma^a)0)$ is well-defined, with $\Gamma := \{y; y+y^1\in W\}$, $\Gamma^a := -\Gamma$. We define θ_j^1 $(j=1,2)$ by the relation

$$\frac{-d\varphi_j(y^1)}{|d\varphi_j(y^1)|} = \omega(\theta_j^1) \quad (j=1,2). \tag{0.6}$$

where and hereafter $\omega(\theta)$ denotes $(\cos\theta, \sin\theta)$. Without loss of generality, we may assume that

$$0<\theta_1^1<\theta_2^1<2\pi, \text{ and } 0<\theta_2^1-\theta_1^1<\pi. \tag{0.7}$$

Thus we have the following:

$$\begin{aligned}(0.8)\quad &\mathrm{SS}(f((x,u)+\sqrt{-1}(\Gamma\times\Gamma^a)0))\cap\{x=u\}\\ &\subset\{(x,u\,;\sqrt{-1}(\omega(\theta),-\omega(\theta)))\in\sqrt{-1}T^*\mathbb{R}^4; x=u,\ \theta_1^1\le\theta\ \le\theta_2^1\}\\ &=\{(x,u\,;\sqrt{-1}\sum_{j=1}^{2}(t\frac{\partial\varphi_1}{\partial y_j}(y^1)+(1-t)\frac{\partial\varphi_2}{\partial y_j}(y^1))(du_j-dx_j))\in\sqrt{-1}T^*\mathbb{R}^4;\\ &\qquad x=u,\ 0\le t\le 1\}.\end{aligned}$$

where $\mathrm{SS}(\cdot)$ denotes the singularity spectrum of a hyperfunction. Moreover we can define a function $g(z,\overline{w},\theta)$ and also see that

$$\begin{aligned}(0.9)\quad &\mathrm{sp}(f((x,u)+\sqrt{-1}(\Gamma\times\Gamma^a)0))\\ &=\mathrm{sp}\left(\int_{\theta_1^1}^{\theta_2^1} g((x,u)+\sqrt{-1}(\Gamma\times\Gamma^a)0,\theta)\,d\theta\right).\end{aligned}$$

Here sp: $\mathcal{B}_{\mathbb{R}^4}\xrightarrow{\sim}\pi_*\mathcal{C}_{\mathbb{R}^4}$ denotes the spectral isomorphism from the sheaf of hyperfunctions to that of microfunctions. In Section 2 we obtain an asymptotic expansion of $f(z,\overline{w})$ above from the microlocal point of view using the result of Section 1. Under the notations above, our main theorem is the following:

MAIN THEOREM. *There exists a sequence $\{R_j(\theta)\}_{j=0}^{\infty}$ such that for any $\varepsilon>0$ the boundary value of*

$$f(z,\overline{w})-\frac{1}{(2\pi)^2}\int_{\theta_1^1+\varepsilon}^{\theta_2^1-\varepsilon}d\theta\int_{A_j}^{\infty}\sum_{j=0}^{\infty}R_j(\theta)\,r^{3-j}e^{\sqrt{-1}\langle z-\overline{w},\omega(\theta)\rangle\,r}dr \tag{0.10}$$

is microanalytic at

$$\{(x, u\,; \sqrt{-1}(\omega(\theta), -\omega(\theta))) \in \sqrt{-1}T^*\mathbb{R}^4; x = u, \theta_1^1{+}\varepsilon < \theta < \theta_2^1{-}\varepsilon\}. \tag{0.11}$$

Here $\{R(\theta)\}_{j=0}^{\infty}$ *satisfies following conditions:*
(1) *there exists a complex neighborhood* U *of* $]\theta_1^1, \theta_2^1[$ *such that each* $R_j(\theta)$ *is a holomorphic function of* U *with*

$$R_0(\theta) = \frac{\tan(\theta_2 - \theta)\tan(\theta - \theta_1)}{\tan(\theta_2 - \theta) + \tan(\theta - \theta_1)}, \tag{0.12}$$

(2) *for any* $V \Subset U$ *there exist constants* $\widetilde{C}$, $\widetilde{M}$ *such that*

$$\sup_{\theta \in V} |R_j(\theta)| \le j!\widetilde{C}\widetilde{M}^j \qquad (\forall j \ge 0). \tag{0.13}$$

R Where in (0.10) $A_j := \max\{0,\, (j-3)A\}$ *with* A *is some positive constant depending on* $\widetilde{M}$*. In other words, on the set* (0.11) *the following equality holds as a microfunction:*

$$f((x, u) + \sqrt{-1}(\Gamma \times \Gamma^a)0) = R(D_x)\delta(x - u). \tag{0.14}$$

Here $R(D_x)$ *denotes a microdifferential operator defined by the symbol* $\sum_{j=0}^{\infty} R_j(\theta) r^{2-j}$ *where* $r\omega(\theta)$ *denotes the symbol of* $-\sqrt{-1}(\partial/\partial x_1, \partial/\partial x_2)$ *by the polar coordinates. Moreover the second term of* (0.10) *is calculated as follows:*

$$\begin{aligned} &\frac{3}{2\pi^2}\int_{\theta_1^1+\varepsilon}^{\theta_2^1-\varepsilon} \frac{R_0(\theta)}{\langle z - \overline{w}, \omega(\theta)\rangle^4} d\theta \\ &+ \frac{1}{(2\pi)^2}\sum_{j=1}^{3}\int_{\theta_1^1+\varepsilon}^{\theta_2^1-\varepsilon} d\theta \int_0^{\infty} R_j(\theta)\, r^{3-j} e^{\sqrt{-1}\langle z - \overline{w}, \omega(\theta)\rangle r} dr \\ &+ \frac{1}{(2\pi)^2}\sum_{j=4}^{\infty}\int_{\theta_1^1+\varepsilon}^{\theta_2^1-\varepsilon} d\theta \int_{(j-3)A}^{\infty} R_j(\theta)\, r^{3-j} e^{\sqrt{-1}\langle z - \overline{w}, \omega(\theta)\rangle r} dr. \end{aligned} \tag{0.15}$$

Acknowledgements. I would like to express my sincere gratitude to Professor Kiyoomi Kataoka for he Professor Shinya Moritoh for fruitful discussions. Last but not least, it is a pleasure to dedicate this paper to Professor Hikosaburo Komatsu as a token of thanks for encouragement.

§1. Preliminary

In this paper we will use the following notation: for $z = (z_1, \dots, z_n) = x + \sqrt{-1}y$, $w = (w_1, \dots, w_n) = u + \sqrt{-1}v \in \mathbb{C}^n$, $\langle z, w\rangle$ denotes $\sum_{j=1}^{n} z_j w_j$. $\omega(\theta)$ denotes $(\cos\theta, \sin\theta)$. If U is an open subset of $\mathbb{C}^n$, then we denote the Hilbert space of square integrable and holomorphic functions on U by $\mathcal{O}_{\mathcal{L}_2}(U)$.
Let $W \subset \mathbb{R}^n$ be a bounded domain, and

$$\Omega := \mathbb{R}^n + \sqrt{-1}W = \{z \in \mathbb{C}^n \,;\, \operatorname{Im} z \in W\} \tag{1.1}$$

be a pseudoconvex tube domain. It is well-known that this condition is equivalent to the convexity of W. We denote the Bergman kernel for Ω (that is, the reproducing kernel function for $\mathcal{O}_{\mathcal{L}_2}(\Omega)$) by $B(z, \overline{w})$.

1.1 PROPOSITION. (cf. [Kor]) *The Bergman kernel $B(z, \overline{w})$ for Ω has the following form:*

$$B(z, \overline{w}) = \frac{1}{(2\pi)^n} \int_{\mathbb{R}^n} \frac{e^{\sqrt{-1}\langle z-\overline{w}, \xi\rangle}}{\int_W e^{-2\langle v, \xi\rangle} dv} d\xi. \tag{1.2}$$

Though it is a well-known fact, we will show this formula for the convenience of the reader.

1.2 LEMMA. *For any $f(w) \in \mathcal{O}_{\mathcal{L}_2}(\Omega)$ and $(\xi, v) \in \mathbb{R}^n \times W$, the integral*

$$\begin{aligned} e^{\langle v, \xi\rangle} &\int_{\mathbb{R}^n} f(u + \sqrt{-1}v)\, e^{-\sqrt{-1}\langle u, \xi\rangle} du \\ &= \int_{\mathbb{R}^n} f(u + \sqrt{-1}v)\, e^{-\sqrt{-1}\langle u+\sqrt{-1}v, \xi\rangle} du \end{aligned} \tag{1.3}$$

does not depend on v.

Note that the integral (1.3) is well-defined in $\mathcal{L}_2$ sense, and that $\mathcal{L}_2$ convergence implies locally uniform convergence on $\mathcal{O}_{\mathcal{L}_2}$. Hence (1.3) is a holomorphic function of (ξ, v) variables.

Proof of Lemma. For any $v_0, v_1 \in W$, we set

$$v_t := tv_1 + (1-t)v_0.$$

Since W is convex, v_t is an element of W for any $t \in [0,1]$. For any positive number ℓ, we define (n+1) chain γ_ℓ by

$$\gamma_\ell := \{u + \sqrt{-1}v_t\,; t \in [0,1], |u| \le l\}.$$

Hence, by the Cauchy-Poincaré theorem we have

$$\int_{\partial\gamma_\ell} f(w)\, e^{-\sqrt{-1}\langle w,\xi\rangle} dw = 0.$$

Here we classify the parts of $\partial\gamma_\ell$ as follows:

$$\partial\gamma_\ell = \underset{\{t=0\}}{\alpha_0} - \underset{\{t=1\}}{\alpha_1} + \underset{\{|u|=l\}}{\beta_\ell}.$$

For any $g(\xi) \in \mathcal{C}_0^\infty(\mathbb{R}^n)$, we have

$$\int_{\mathbb{R}^n} g(\xi)\, d\xi \int_{\beta_\ell} f(w)\, e^{-\sqrt{-1}\langle w,\xi\rangle} dw = \int_{\beta_\ell} f(w)\, \widehat{g}(w)\, dw$$

by the Fubini theorem, where

$$\widehat{g}(w) := \int_{\mathbb{R}^n} g(\xi)\, e^{-\sqrt{-1}\langle w,\xi\rangle} d\xi,$$

that is, the Fourier-Laplace transform of $g(\xi)$.
Set

$$L := \text{convex hull of } \operatorname{supp} g \Subset \mathbb{R}^n,$$

$$H_L(w) := \sup_{x\in L} \operatorname{Im}\langle x, w\rangle = \sup_{x\in L}\langle x, \operatorname{Im} w\rangle.$$

Therefore, by the Paley-Wiener theorem, for any positive integer N there exists a positive constant C_N such that

$$|\widehat{g}(w)| \le \frac{C_N}{(1+|w|)^N}\, e^{H_L(w)}.$$

Because $\operatorname{Im} w = v \in W$ on β_ℓ, there exists a positive constant C_1 such that

$$e^{H_L(w)} \le C_1 \qquad \text{for any } w \in \beta_\ell.$$

On the other hand, there exists positive number r such that

$$\bigcup\{\varDelta_r(w); w = u + \sqrt{-1}v_t, u \in \mathbb{R}^n, t \in [0,1]\} \subsetneq \Omega$$

where $\varDelta_r(w)$ denotes the polydisc of radii $(r, \dots, r)$ with center w. Hence if we apply Cauchy's inequality to $f(w)^2$, we have

$$|f(u + \sqrt{-1}v_t)| \le \frac{1}{(r\sqrt{\pi})^n}\|f\|_{\mathcal{L}_2} \qquad t \in [0,1],\ u \in \mathbb{R}^n.$$

Thus we have

$$\left|\int_{\beta_\ell} f(w)\,\widehat{g}(w)\,dw\right| \le \frac{C_1 C_N}{(r\sqrt{\pi})^n}\|f\|_{\mathcal{L}_2}\int_{\beta_\ell}\frac{1}{(1+|w|)^N}dw, \tag{1.4}$$

and if we choose N so that $N \ge n+1$, we can easily see that the right-hand-side of (1.4) converges to zero as ℓ tends to infinity. Thus, as a distribution, we have

$$\lim_{\ell\to\infty}\int_{\beta_\ell} f(w)\,e^{-\sqrt{-1}\langle w,\xi\rangle}dw = 0. \tag{1.5}$$

Because the equality (1.5) holds in $\mathcal{L}_2$ sense, we conclude the following equality:

$$\begin{aligned}&\int_{\mathbb{R}^n} f(u+\sqrt{-1}v_0)\,e^{-\sqrt{-1}\langle u+\sqrt{-1}v_0,\xi\rangle}du\\ &= \int_{\mathbb{R}^n} f(u+\sqrt{-1}v_1)\,e^{-\sqrt{-1}\langle u+\sqrt{-1}v_1,\xi\rangle}du. \qquad \square\end{aligned}$$

PROOF OF PROPOSITION. By virtue of the preceding Lemma, we can write (1.3) as $I(f;\xi) = e^{\langle v,\xi\rangle}\widehat{f}(\xi;v)$, where $\widehat{f}(\xi;v)$ denotes the partial Fourier transform with respect to u variables. Thus we have

$$e^{-\langle y,\xi\rangle}I(f;\xi) = \widehat{f}(\xi;y) \qquad \text{for any } y \in W. \tag{1.6}$$

Hence, by Fourier's inversion formula, the Fubini theorem and (1.6) we obtain the following equality in $\mathcal{L}_2$ sense for any $f(z) \in \mathcal{O}_{\mathcal{L}_2}(\Omega)$:

$$
\begin{aligned}
f(z) =& \frac{1}{(2\pi)^n} \int_{\mathbb{R}^n} e^{\sqrt{-1}\langle x,\xi\rangle} \widehat{f}(\xi; y)\, d\xi \\
=& \frac{1}{(2\pi)^n} \int_{\mathbb{R}^n} e^{\sqrt{-1}\langle x,\xi\rangle} e^{-\langle y,\xi\rangle} I(f;\xi)\, d\xi \\
=& \frac{1}{(2\pi)^n} \int_{\mathbb{R}^n} e^{\sqrt{-1}\langle z,\xi\rangle} d\xi \int_W \frac{e^{-\langle 2v,\xi\rangle}}{\int_W e^{-2\langle v,\xi\rangle} dv} dv \\
& \cdot \int_{\mathbb{R}^n} f(u+\sqrt{-1}v)\, e^{-\sqrt{-1}\langle u,\xi\rangle}\, e^{\langle v,\xi\rangle} du \\
=& \frac{1}{(2\pi)^n} \int_{\mathbb{R}^n \times W} \frac{e^{\sqrt{-1}\langle z,\xi\rangle}}{\int_W e^{-2\langle v,\xi\rangle} dv} d\xi dv \\
& \cdot \int_{\mathbb{R}^n} f(u+\sqrt{-1}v)\, e^{-\sqrt{-1}\langle u,\xi\rangle}\, e^{-\langle v,\xi\rangle} du \\
=& \int_{\mathbb{R}^n+\sqrt{-1}W} du dv\, f(w)\, \frac{1}{(2\pi)^n} \int_{\mathbb{R}^n} \frac{e^{\sqrt{-1}\langle z-\overline{w},\xi\rangle}}{\int_W e^{-2\langle v,\xi\rangle} dv} d\xi .
\end{aligned} \tag{1.7}
$$

We easily see that both sides of (1.7) are holomorphic. Hence this equality holds as holomorphic functions. Thus we prove the Proposition by the fact that the Bergman kernel is uniquely determined by its reproducing property on the space $\mathcal{O}_{\mathcal{L}_2}(\Omega)$. □

Hereafter we assume that the dimension $n = 2$, $W = W_1 \cap W_2$, where each W_j is strictly convex domain as follows:

$$
\left\{
\begin{array}{l}
W_j = \{y \in \mathbb{R}^2; \varphi_j(y) < 0\} \text{ with an analytic function} \\
\qquad \varphi_j\ (j = 1,2) \text{ such that} \\
(1) \partial W_1 \text{ and } \partial W_2 \text{ intersect transversally,} \\
(2) \text{ if } \varphi_j(y_0) = 0, \text{ then } d\varphi_j(y_0) \neq 0 \text{ and the Hessian matrix} \\
\qquad \left(\frac{\partial^2 \varphi_j}{\partial y_k \partial y_\ell}(y_0) \right)_{1 \le k,\ell \le 2} \text{ is positive definite for } j = 1,2.
\end{array}
\right. \tag{1.8}
$$

As mentioned in Introduction the Bergman kernel $B(z, \overline{w})$ is holomorphic except for the diagonal points $\{z = w\}$ at the boundary and our main concern is singularities at corner points. Thus, we investigate singularities on the set

$$\{(x+\sqrt{-1}y, u+\sqrt{-1}v);\, x=u,\, y+v=0,\, y=y_0\}. \tag{1.9}$$

Here $y_0 \in \partial W_1 \cap \partial W_2$. Set $\partial W_1 \cap \partial W_2 = \{y^1, y^2, \dots, y^{2N}\}$ by the assumption on W, and define θ_j^ℓ by the following relation:

$$\frac{-d\varphi_j(y^\ell)}{|d\varphi_j(y^\ell)|} = \omega(\theta_j^\ell) = (\cos\theta_j^\ell, \sin\theta_j^\ell) \tag{1.10}$$

(recall that $\omega(\theta) = (\cos\theta, \sin\theta)$). Without loss of generality, we may assume

$$\begin{cases} 0 < \theta_1^1 < \theta_2^1 < \theta_2^2 < \theta_1^2 < \cdots < \theta_2^{2N} < \theta_1^{2N} < 2\pi, \\ 0 < \theta_2^{2\ell-1} - \theta_1^{2\ell-1},\ \ \theta_1^{2\ell} - \theta_2^{2\ell} < \pi \quad (\ell = 1, 2, \dots, N). \end{cases} \tag{1.11}$$

Thus, we can define the continuous surjection $y(*) : [0, 2\pi] \ni \theta \mapsto y(\theta) \in \partial W$ by

$$y(\theta) := \begin{cases} y^{2\ell-1} & (\theta_1^{2\ell-1} \le \theta \le \theta_2^{2\ell-1}), \\ y^{2\ell} & (\theta_2^{2\ell} \le \theta \le \theta_1^{2\ell}), \\ y \ \text{ satisfying } \frac{-d\varphi_2(y)}{|d\varphi_2(y)|} = \omega(\theta) & (\theta_2^{2\ell-1} < \theta < \theta_2^{2\ell}), \\ y \ \text{ satisfying } \frac{-d\varphi_1(y)}{|d\varphi_1(y)|} = \omega(\theta) & (\theta_1^{2\ell} < \theta < \theta_1^{2\ell+1}), \end{cases} \tag{1.12}$$

where $\ell = 1, 2, \dots, N$ and we set $\theta_1^{2N+1} := \theta_1^1$. Hence we have

$$\begin{aligned} B(z, \overline{w}) &= \frac{1}{(2\pi)^2} \int_{\mathbb{R}^2} \frac{e^{\sqrt{-1}\langle z-\overline{w}, \omega(\theta)\rangle r}}{\int_W e^{-2\langle v, \omega(\theta)\rangle r} dv} r dr d\theta \\ &= \frac{1}{(2\pi)^2} \int_{\mathbb{R}^2} \frac{e^{\sqrt{-1}\langle z-\overline{w}-2\sqrt{-1}y(\theta), \omega(\theta)\rangle r}}{\int_W e^{-2\langle v-y(\theta), \omega(\theta)\rangle r} dv} r dr d\theta. \end{aligned} \tag{1.13}$$

From now on we consider the singularity of $B(z, \overline{w})$ at $\{x = u,\, y+v = 0,\, y = y^1\}$ since singularities at $\{y^2, \dots, y^N\}$ are similar. For any $\theta \in [0, 2\pi]$, $s \ge 0$, define

$$H(s, \theta) := \{v \in W; \langle v - s\omega(\theta) - y(\theta), s\omega(\theta)\rangle = 0\} \tag{1.14}$$

and let $dv(s,\theta)$ denote the volume element of the line $\{v \in W; \langle v - s\omega(\theta) - y(\theta), s\omega(\theta)\rangle = 0\}$. Thus by setting

$$\rho(s,\theta) := \int_{H(s,\theta)} dv(s,\theta) \tag{1.15}$$

we have

$$\begin{aligned}\int_W e^{-2\langle v-y(\theta),\omega(\theta)\rangle r} dv &= \int_0^{c_\theta} ds \int_{H(s,\theta)} e^{-2\langle v-y(\theta),\omega(\theta)\rangle r} dv(s,\theta) \\ &= \int_0^{c_\theta} e^{-2sr}\rho(s,\theta)\,ds,\end{aligned} \tag{1.16}$$

where c_θ is a constant depending on θ such that

$$\int_0^{c_\theta} ds \int_{H(s,\theta)} dv(s,\theta) = \mu(W), \tag{1.17}$$

and $\mu(W)$ denotes the Lebesgue measure of W. Note that by the assumption on W we easily see

$$\begin{cases} 0 < \inf\limits_\theta c_\theta < \sup\limits_\theta c_\theta < \infty, \\ 0 < \mu(W) < \infty. \end{cases} \tag{1.18}$$

Set

$$K(r,\theta) := \int_0^{c_\theta} e^{-2sr}\rho(s,\theta)\,ds. \tag{1.19}$$

Define a holomorphic function $f(z,\overline{w})$ by

$$\begin{aligned} f(z,\overline{w}) :=& B(z+\sqrt{-1}y^1, \overline{w}-\sqrt{-1}y^1) \\ =& \frac{1}{(2\pi)^2}\int_{\mathbb{R}^2} \frac{e^{\sqrt{-1}\langle z-\overline{w}+2\sqrt{-1}(y^1-y(\theta)),\omega(\theta)\rangle r}}{K(r,\theta)} r\,dr\,d\theta. \end{aligned} \tag{1.20}$$

Then, $f(z,\overline{w})$ is holomorphic when $\mathrm{Im}\ z$, $\mathrm{Im}\ w \in \Gamma := \{y \in \mathbb{R}^2;\ y + y^1 \in W\}$, and the singularity of $B(z,\overline{w})$ at $\{x = u,\ y + v = 0,\ y = y^1\}$ is equivalent to that of $f(z,\overline{w})$ at $\{x = u,\ y = v = 0\}$. Hence, a hyperfunction

$f((x,u)+\sqrt{-1}(\Gamma\times\Gamma^a)0)$ is well-defined, where $\Gamma^a:=-\Gamma$ (see [Kan1], [K-K-K], and [S-K-K] for hyperfunctions and microfunctions theory). Define a continuous function $g(z,\overline{w},\theta)$ by

$$g(z,\overline{w},\theta):=\frac{1}{(2\pi)^2}\int_0^\infty \frac{e^{\sqrt{-1}\langle z-\overline{w}+2\sqrt{-1}(y^1-y(\theta)),\omega(\theta)\rangle r}}{K(r,\theta)}r\,dr. \tag{1.21}$$

Thus we can define a hyperfunction $g((x,u)+\sqrt{-1}(\Gamma\times\Gamma^a)0,\theta)$ similarly to
$f((x,u)+\sqrt{-1}(\Gamma\times\Gamma^a)0)$. Clearly we have

$$f(z,\overline{w})=\int_0^{2\pi} g(z,\overline{w},\theta)\,d\theta, \tag{1.22}$$

or equivalently,

$$f((x,u)+\sqrt{-1}(\Gamma\times\Gamma^a)0)=\int_0^{2\pi} g((x,u)+\sqrt{-1}(\Gamma\times\Gamma^a)0,\theta)\,d\theta. \tag{1.22'}$$

By using the estimate of the singularity spectrum (= the analytic wave front set) for a hyperfunction we have the following:

$$\begin{aligned}
&\mathrm{SS}(f((x,u)+\sqrt{-1}(\Gamma\times\Gamma^a)0))\cap\{x=u\} \\
&\quad\subset\{(x,u\,;\sqrt{-1}(\omega(\theta),-\omega(\theta)))\in\sqrt{-1}T^*\mathbb{R}^4; x=u,\ \theta_1^1\le\theta\ \le\theta_2^1\} \\
&=\{(x,u\,;\sqrt{-1}\sum_{j=1}^{2}(t\frac{\partial\varphi_1}{\partial y_j}(y^1)+(1-t)\frac{\partial\varphi_2}{\partial y_j}(y^1))(du_j-dx_j))\in\sqrt{-1}T^*\mathbb{R}^4; \\
&\qquad\qquad x=u,\ 0\le t\le 1\}.
\end{aligned} \tag{1.23}$$

where $\mathrm{SS}(\cdot)$ denotes the singularity spectrum of a hyperfunction. On the other hand, by the definition of $g(z,\overline{w},\theta)$ we can also see that

$$\begin{aligned}
&\mathrm{sp}(f((x,u)+\sqrt{-1}(\Gamma\times\Gamma^a)0)) \\
&\qquad=\mathrm{sp}\left(\int_{\theta_1^1}^{\theta_2^1} g((x,u)+\sqrt{-1}(\Gamma\times\Gamma^a)0,\theta)\,d\theta\right).
\end{aligned} \tag{1.24}$$

Here sp: $\mathcal{B}_{\mathbb{R}^4}\xrightarrow{\sim}\pi_*\mathcal{C}_{\mathbb{R}^4}$ denotes the spectral isomorphism from the sheaf of hyperfunctions to that of microfunctions.

§2. Asymptotic expansion

In this section, we will obtain an asymptotic expansion of $g(z,\overline{w},\theta)$ from the microlocal point of view. For $\omega(\theta)=(\cos\theta,\sin\theta)$, we set ${}^t\omega(\theta):=(-\sin\theta,\cos\theta)$. Hence we have $\langle\omega(\theta),{}^t\omega(\theta)\rangle=0$. For any $v\in H(s,\theta)$, there exists a unique real number t such that

$$v=y(\theta)+s\cdot\omega(\theta)+t\cdot{}^t\omega(\theta), \tag{2.1}$$

that is, $H(s,\theta)$ is parametrized by t. For $s\geq 0$, we have $H(s,\theta)\cap\partial W=\{w_1^s,w_2^s\}$. By the assumption on W and definition of $y(\theta)$, we can find the function $w_j^s=w_j(s,\theta)$ and we have

$$\rho(s,\theta)=|w_1(s,\theta)-w_2(s,\theta)|. \tag{2.2}$$

On the other hand, we will consider equations

$$\varphi_j(y(\theta)+s\cdot\omega(\theta)+t\cdot{}^t\omega(\theta))=0 \qquad \theta\in]\theta 1^1,\theta_2^1[,\ s\geq 0,\ (j=1,2), \tag{2.3}$$

Thus, we can apply analytic version of the implicit function theorem by the assumption that $0<\theta_1^2-\theta_1^1<\pi$; that is, we can find a strictly positive constant δ and analytic functions $t_j=t_j(s,\theta)$ ($j=1,2$, $\theta\in]\theta 1^1,\theta_2^1[$, $0\leq s\leq\delta$) such that

$$\varphi_j(y^1+s\cdot\omega(\theta)+t_j(s,\theta)\cdot{}^t\omega(\theta))=0, \tag{2.4}$$

$$t_j(0,\theta)=0. \tag{2.5}$$

We note that there exist complex neighborhoods L and U of $\{s;\ 0\leq s\leq\delta\}$ and $]\theta 1^1,\theta_2^1[$ respectively such that t_j's are holomorphic on $L\times U$ ($j=1,2$). The lemma below is proved by direct calculation so we omit the proof:

2.1 Lemma. *If $0\leq s\leq\delta$ and $\theta\in]\theta 1^1,\theta_2^1[$, then*

$$\rho(s,\theta)=t_1(s,\theta)-t_2(s,\theta), \tag{2.6}$$

$$w_j(s,\theta)=y^1+s\cdot\omega(\theta)+t_j(s,\theta)\cdot{}^t\omega(\theta). \tag{2.7}$$

By replacing δ small enough, we can obtain for $|s| \leq \delta$, $\theta \in U$ the Taylor expansion

$$(t_1 - t_2)(s,\theta) = \sum_{j=1}^{\infty} \frac{a_j(\theta)}{j!} s^j. \tag{2.8}$$

We can see by the implicit function theorem that

$$a_1(\theta) = \frac{\partial(t_1 - t_2)}{\partial s}(0,\theta) = \frac{1}{\tan(\theta - \theta_1)} + \frac{1}{\tan(\theta_2 - \theta)}. \tag{2.9}$$

Hence by shrinking L and U, we may assume that

$$\begin{cases} a_1(\theta) \neq 0 & (\forall \theta \in U), \\ |t_j(s,\theta)|\text{'s are bounded on } s \in L,\ \theta \in V & (\forall V \Subset U\ (j = 1,2)). \end{cases} \tag{2.10}$$

Thus there exist constants C and M such that

$$\begin{cases} \sup_{\theta \in V} |a_j(\theta)| \leq j! C M^j \quad (\forall j \geq 1), \\ M\delta < 1. \end{cases} \tag{2.11}$$

Thus we can prove the following Lemma:

2.2 LEMMA. *$K(r,\theta)$ has the following asymptotic expansion :*

$$K(r,\theta) \sim \sum_{j=1}^{\infty} \frac{a_j(\theta)}{(2r)^{j+1}} \qquad (r \longrightarrow \infty). \tag{2.12}$$

Precisely, for any $V \Subset U$, there exist positive numbers r_0 and C_V such that for every $r \geq r_0$, $\theta \in V \cap \mathbb{R}$ and $N \geq 1$ the following inequality holds:

$$\left| K(r,\theta) - \sum_{j=1}^{N} \frac{a_j(\theta)}{(2r)^{j+1}} \right| < \frac{C_V^{N+1}(N+1)!}{r^{N+2}}. \tag{2.13}$$

PROOF. By (2.8) we can write on $V \cap \mathbb{R}$

$$
\begin{aligned}
(2.14)\quad K(r,\theta) &= \int_0^\delta \rho(s,\theta)\, e^{-2sr}\, ds + \int_\delta^{c_\theta} \rho(s,\theta)\, e^{-2sr} ds \\
&= \int_0^\delta \sum_{j=1}^{\infty} \frac{a_j(\theta)}{j!} s^j\, e^{-sr}\, ds + \int_\delta^{c_\theta} \rho(s,\theta)\, e^{-2sr} ds \\
&= \int_0^\delta \sum_{j=1}^{N} \frac{a_j(\theta)}{j!} s^j\, e^{-sr}\, ds + \int_0^\delta \sum_{j=N+1}^{\infty} \frac{a_j(\theta)}{j!} s^j\, e^{-sr}\, ds \\
&\quad + \int_\delta^{c_\theta} \rho(s,\theta)\, e^{-2sr} ds.
\end{aligned}
$$

First, by (2.11) we have for $1 \le j \le N$

$$
\begin{aligned}
(2.15)\quad & \left| \sum_{j=1}^{N} \frac{a_j(\theta)}{(2r)^{j+1}} - \int_0^\delta \sum_{j=1}^{N} \frac{a_j(\theta)}{j!} s^j\, e^{-sr}\, ds \right| \\
&= \left| \sum_{j=1}^{N} \frac{a_j(\theta)}{(2r)^{j+1}} - \int_0^{2r\delta} \sum_{j=1}^{N} \frac{a_j(\theta)}{j!} \left(\frac{\ell}{2r}\right)^j e^{-\ell}\, \frac{d\ell}{2r} \right| \\
&\le \sum_{j=1}^{N} \frac{CM^j}{(2r)^{j+1}} \left(j! - \int_0^{2r\delta} \ell^j e^{-\ell} dl \right) = \sum_{j=1}^{N} \frac{CM^j}{(2r)^{j+1}} \int_{2r\delta}^{\infty} \ell^j e^{-\ell} d\ell.
\end{aligned}
$$

For any $j \ge 1$ and $2r\delta \ge 1$ we have

$$
(2.16\text{-}j)\qquad \int_{2r\delta}^{\infty} \ell^j e^{-\ell} d\ell \le (j+1)!(2r\delta)^j e^{-2r\delta}.
$$

In fact, if $j = 1$ then

$$
(2.16\text{-}1)\qquad \int_{2r\delta}^{\infty} l e^{-\ell} d\ell = (2r\delta + 1) e^{-2r\delta} \le 2\,(2r\delta) e^{-2r\delta},
$$

and assuming (2.16-j) we get

$$
\begin{aligned}
(2.16\text{-}(j{+}1))\quad \int_{2r\delta}^{\infty} \ell^{j+1} e^{-\ell} d\ell &= (2r\delta)^{j+1} e^{-2r\delta} + (j+1) \int_{2r\delta}^{\infty} \ell^j e^{-\ell} d\ell \\
&\le (1 + (j+1)(j+1)!)(2r\delta)^{j+1} e^{-2r\delta} \\
&\le (j+2)!(2r\delta)^{j+1} e^{-2r\delta}.
\end{aligned}
$$

Hence we get (2.16-j) by induction. Thus we have

$$\left|\sum_{j=1}^{N}\frac{a_j(\theta)}{(2r)^{j+1}}-\int_0^{2r\delta}\sum_{j=1}^{N}\frac{a_j(\theta)}{j!}\left(\frac{\ell}{2r}\right)^j e^{-\ell}\,\frac{d\ell}{2r}\right| \le \sum_{j=1}^{N}C(M\delta)^j(j+1)!\frac{e^{-2r\delta}}{2r}\le CN(N+1)!\frac{e^{-2r\delta}}{2r}. \tag{2.17}$$

Next, we have by (2.11)

$$\begin{aligned}\left|\int_0^{\delta}\sum_{j=N+1}^{\infty}\frac{a_j(\theta)}{j!}s^j\,e^{-sr}\,ds\right| &\le \sum_{j=N+1}^{\infty}CM^j\int_0^{\delta}s^j\,e^{-sr}\,ds\\ &=CM^{N+1}\sum_{j=0}^{\infty}M^j\int_0^{\delta}s^{N+1+j}\,e^{-sr}\,ds\\ &\le CM^{N+1}\sum_{j=0}^{\infty}(M\delta)^j\int_0^{\delta}s^{N+1}\,e^{-sr}\,ds\\ &=CM^{N+1}\sum_{j=0}^{\infty}(M\delta)^j\int_0^{2r\delta}\left(\frac{\ell}{2r}\right)^{N+1}e^{-\ell}\,\frac{d\ell}{2r}\\ &<\frac{CM^{N+1}}{1-M\delta}\int_0^{\infty}\left(\frac{\ell}{2r}\right)^{N+1}e^{-\ell}\,\frac{d\ell}{2r}=\frac{CM^{N+1}}{1-M\delta}\frac{(N+1)!}{(2r)^{N+2}}\end{aligned} \tag{2.18}$$

Lastly, we have

$$0<\int_{\delta}^{c_\theta}\rho(s,\theta)\,e^{-2sr}\,ds\le e^{-2r\delta}\int_{\delta}^{c_\theta}\rho(s,\theta)\,ds<e^{-2r\delta}\mu(W). \tag{2.19}$$

Thus we prove the Lemma by (2.17), (2.18) and (2.19). □

On the other hand, by virtue of (2.10) we easily see that $\sum_{j=1}^{\infty}a_j(\theta)/(2r)^{j+1}$ has formal inverse $\sum_{j=0}^{\infty}R_j(\theta)r^{2-j}$: that is, as formal power series with $\mathcal{O}(U)$ coefficients the following equality holds:

$$\left(\sum_{j=1}^{\infty}\frac{a_j(\theta)}{(2r)^{j+1}}\right)^{-1}=\sum_{j=0}^{\infty}R_j(\theta)r^{2-j}. \tag{2.20}$$

Here, $R_0(\theta) := a_1(\theta)^{-1}$ and $R_j(\theta)$'s are determined inductively by (2.20). Therefore, we can see that for any $V \Subset U$ each $R_j(\theta)$ satisfies a similar inequality to (2.11) ($j = 0, 1, 2, \dots$). Thus, we obtain the following theorem (cf. [A], [B1] and [Kat] for symbolic calculus):

2.3 THEOREM. *There exists a sequence $\{R_j(\theta)\}_{j=0}^{\infty}$ such that for any $\varepsilon > 0$ the boundary value of*

$$f(z, \overline{w}) - \frac{1}{(2\pi)^2} \int_{\theta_1^1+\varepsilon}^{\theta_2^1-\varepsilon} d\theta \int_{A_j}^{\infty} \sum_{j=0}^{\infty} R_j(\theta)\, r^{3-j} e^{\sqrt{-1}\langle z-\overline{w}, \omega(\theta)\rangle\, r} dr \tag{2.21}$$

is micro-analytic at

$$\{(x, u\,; \sqrt{-1}(\omega(\theta), -\omega(\theta))) \in \sqrt{-1}T^*\mathbb{R}^4; x = u, \theta_1^1+\varepsilon < \theta < \theta_2^1-\varepsilon\,\}. \tag{2.22}$$

Here $\{R(\theta)\}_{j=0}^{\infty}$ satisfies following conditions:
(1) there exists a complex neighborhood U of $]\theta_1^1, \theta_2^1[$ such that each $R_j(\theta)$ is a holomorphic function of U with

$$R_0(\theta) = \frac{\tan(\theta_2 - \theta)\tan(\theta - \theta_1)}{\tan(\theta_2 - \theta) + \tan(\theta - \theta_1)}, \tag{2.23}$$

(2) for any $V \Subset U$ there exist constants $\widetilde{C}$, $\widetilde{M}$ such that

$$\sup_{\theta \in V} |R_j(\theta)| \le j!\widetilde{C}\widetilde{M}^j \qquad (\forall j \ge 0). \tag{2.24}$$

Where in (2.21) $A_j := \max\{0, (j-3)A\}$ with A is some positive constant depending on $\widetilde{M}$. In other words, on the set (2.22) the following equality holds as a microfunction:

$$f((x, u) + \sqrt{-1}(\Gamma \times \Gamma^a)0) = R(D_x)\delta(x - u), \tag{2.25}$$

Here $R(D_x)$ denotes a microdifferential operator defined by the symbol $\sum_{j=0}^{\infty} R_j(\theta) r^{2-j}$ where $r\omega(\theta)$ denotes the symbol of $-\sqrt{-1}(\partial/\partial x_1, \partial/\partial x_2)$ by *the polar coordinates. Moreover the second term of* (2.21) *is calculated as follows:*

$$
\begin{aligned}
(2.26)\qquad & \frac{3}{2\pi^2}\int_{\theta_1^1+\varepsilon}^{\theta_2^1-\varepsilon}\frac{R_0(\theta)}{\langle z-\overline{w},\omega(\theta)\rangle^4}d\theta \\
&+\frac{1}{(2\pi)^2}\sum_{j=1}^{3}\int_{\theta_1^1+\varepsilon}^{\theta_2^1-\varepsilon}d\theta\int_0^{\infty}R_j(\theta)\,r^{3-j}e^{\sqrt{-1}\langle z-\overline{w},\omega(\theta)\rangle r}dr \\
&+\frac{1}{(2\pi)^2}\sum_{j=4}^{\infty}\int_{\theta_1^1+\varepsilon}^{\theta_2^1-\varepsilon}d\theta\int_{(j-3)A}^{\infty}R_j(\theta)\,r^{3-j}e^{\sqrt{-1}\langle z-\overline{w},\omega(\theta)\rangle r}dr.
\end{aligned}
$$

2.3 Remark. By (2.26), we can see that

$$B(z,\overline{z})=O\left(\operatorname{dist}(y,y^1)^{-4}\right)\quad (W\ni y\longrightarrow y^1).$$

On the other hand, if $v_0\in\partial(W_1\cap W_2)\setminus(\partial W_1\cap\partial W_2)$, then we can apply the proof of Kashiwara's theorem (0.4) and obtain that

$$B(z,\overline{z})=O\left(\operatorname{dist}(y,v_0)^{-3}\right)\quad (W\ni y\longrightarrow v_0).$$

Hence we can hardly expect that similar result to theorem (0.3) holds at $(x,u\,;\sqrt{-1}\sum_{j=1}^{2}(t\frac{\partial\varphi_1}{\partial y_j}(y^1)+(1-t)\frac{\partial\varphi_2}{\partial y_j}(y^1))(du_j-dx_j))$, $(x=u,\ t=0,1)$.

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(Received October 28, 1994)
(Revised June 21, 1995)

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