J. Math. Sci. Univ. Tokyo
2 (1995), 563–588.

A duality for finite t-modules

By Yuichiro Taguchi

Abstract. An $\mathbb{F}_q[t]$-analogue of the Cartier duality is established. Applications to π-divisible groups are given. Dual Drinfeld modules are made explicit.

Introduction

In this paper, we establish a duality for finite t-modules and study its basic properties. Our duality is the $\mathbb{F}_q[t]$-analogue of the Cartier duality, where the multiplicative group $\mathbb{G}_m$ is replaced by the Carlitz module C. Finite t-modules are, roughly speaking, finite locally free group schemes which are $\mathbb{F}_q[t]$-submodules of abelian t-modules ([1]) with scalar t-action on their tangent spaces. See (2.1) for the precise definition. In fact, it is only for *finite v-modules* (Definition (3.1)) that we can define the duality (Definition (4.1)), in a way with Dieudonné theoretic flavor. See Remarks (4.4), (4.5), and Example (4.6) for accounts of the necessity of a v-module structure.

A typical case of our duality is supplied by division points of Drinfeld modules and *dual Drinfeld modules* , and is studied in some detail in Section 5. In Section 6, some results on the duality of π-divisible groups are given.

One may hope to have such a duality for a wider class of t-modules, namely, torsion points of abelian t-modules which do not have scalar t-action on the tangent spaces, such as higher Carlitz modules $C^{\otimes n}$ ([2]). But this would be possible only if the target C of the pairing was replaced by a tensor power $C^{\otimes n}$ with sufficiently large n.

Throughout the article, $\mathcal{O}_S$ denotes the structure sheaf of a scheme S. In general, we will use the following unusual

1991 *Mathematics Subject Classification.* Primary 11G09; Secondary 14L05, 14L15.

Notation. A morphism of schemes is denoted by a capital letter, and the corresponding morphism of the structure sheaves is denoted by the corresponding small letter.

1. Finite φ-modules

For the moment, let A be any commutative ring, and recall the definition of an A-module scheme. For an A-scheme S, we denote by $\alpha : A \to \Gamma(S, \mathcal{O}_S)$ the structure morphism.

If G is a commutative group scheme over a scheme S, we denote by $\mathrm{Lie}^*(G/S)$ the co-Lie module of G/S (i.e. the $\mathcal{O}_S$-module of invariant differentials of G/S); thus one has $\mathcal{O}_G \otimes_{\mathcal{O}_S} \mathrm{Lie}^*(G/S) \simeq \Omega^1_{G/S}$.

DEFINITION (1.1). An *A-module scheme* over an A-scheme S is a pair (G, Ψ) consisting of a commutative group scheme G over S and a ring homomorphism $\Psi : A \to \mathrm{End}(G/S)$; $a \mapsto \Psi_a$ such that, for each $a \in A$, Ψ_a induces multiplication by $\alpha(a)$ on the $\mathcal{O}_S$-module $\mathrm{Lie}^*(G/S)$.

A *morphism* $M : (G, \Psi) \to (G', \Psi')$ of A-module schemes is a morphism $M : G \to G'$ of group schemes such that $M \circ \Psi_a = \Psi'_a \circ M$ for all $a \in A$.

Example (1.2). A vector bundle G on S can be naturally regarded as a $\Gamma(S, \mathcal{O}_S)$-module scheme. We shall mean by a *vector group scheme* such a $\Gamma(S, \mathcal{O}_S)$-module scheme.

We will often write simply G for an A-module scheme in place of (G, Ψ).

Hereafter in this section, we consider the case where the ring A is the finite field $\mathbb{F}_q$ of q elements and S an $\mathbb{F}_q$-scheme.

For an $\mathbb{F}_q$-module scheme (G, Ψ) over S, set $\mathcal{E}_G := \underline{\mathrm{Hom}}_{\mathbb{F}_q, S}(G, \mathbb{G}_a)$. ($\underline{\mathrm{Hom}}_{\mathbb{F}_q, S}$ denotes the Zariski sheaf on S of $\mathbb{F}_q$-linear homomorphisms.) If G/S is affine (as is always the case in the following), we may confuse $\mathcal{O}_G$ and $\pi_*\mathcal{O}_G$ (where π is the structure morphism of G/S) and may think of $\mathcal{O}_G$ as an $\mathcal{O}_S$-algebra. Then $\mathcal{E}_G$ is the $\mathcal{O}_S$-submodule of the augmentation ideal $\mathcal{I}_G$ of $\mathcal{O}_G$ consisting of the local sections X which satisfy

$$\begin{cases} \delta(X) = X \otimes 1 + 1 \otimes X, & \text{and} \\ \psi_a(X) = \alpha(a)X & \text{for all } a \in \mathbb{F}_q. \end{cases}$$

Here $\delta : \mathcal{O}_G \to \mathcal{O}_G \otimes_{\mathcal{O}_S} \mathcal{O}_G$ is the coproduct of $\mathcal{O}_G$ and $\psi_a : \mathcal{O}_G \to \mathcal{O}_G$ is the $\mathcal{O}_S$-algebra homomorphism corresponding to $\Psi_a : G \to G$.

Note the correspondence $G \mapsto \mathcal{E}_G$ is similar to the "t-motive" construction ([1], §1). See also Remark (3.7) below.

Definition (1.3). An $\mathbb{F}_q$-module scheme (G, Ψ) over S is called a *finite φ-module* if $\mathcal{O}_G$ and $\mathcal{E}_G$ are locally free of finite rank over $\mathcal{O}_S$ (in particular, G/S is affine) with $\operatorname{rank}(\mathcal{O}_G) = q^{\operatorname{rank}(\mathcal{E}_S)}$, and $\mathcal{E}_G$ generates the $\mathcal{O}_S$-algebra $\mathcal{O}_G$.

A *morphism* of finite φ-modules is by definition a morphism of $\mathbb{F}_q$-module schemes.

Remarks (1.4). (i) A finite φ-module G over S can be embedded canonically into the vector group scheme $E_G := \mathbb{V}(\mathcal{E}_G) = \underline{\operatorname{Spec}}(\operatorname{Sym}^{\cdot}_{\mathcal{O}_S}\mathcal{E}_G)$ as an $\mathbb{F}_q$-submodule scheme, because $\mathcal{E}_G$ generates $\mathcal{O}_G$. Let us agree to call E_G/S the *ambient space* of G/S. It is clear that a morphism $M : G \to G'$ of finite φ-modules extends uniquely to a morphism $E_M : E_G \to E_{G'}$ of $\mathbb{F}_q$-module schemes.

(ii) The group scheme μ_p of p-th roots of unity over an $\mathbb{F}_p$-scheme is *not* a finite φ-module because $\mathcal{E}_{\mu_p} = \underline{\operatorname{Hom}}_{\mathbb{F}_q,S}(\mu_p, \mathbb{G}_a) = 0$.

Note that, if $M : G \to G'$ is a morphism of $\mathbb{F}_q$-module schemes, then the corresponding morphism $m : \mathcal{O}_{G'} \to \mathcal{O}_G$ restricts to an $\mathcal{O}_S$-module homomorphism $m : \mathcal{E}_{G'} \to \mathcal{E}_G$. Since $\mathcal{E}_{G'}$ generates $\mathcal{O}_{G'}$ if G' is a finite φ-module, we have

Lemma (1.5). *Let G and G' be finite φ-modules. Then the natural homomorphism* $\operatorname{Hom}_{\varphi,S}(G, G') \to \operatorname{Hom}_{\mathcal{O}_S\text{-mod}}(\mathcal{E}_{G'}, \mathcal{E}_G)$ *is injective.*

In the following, for an $\mathcal{O}_S$-module $\mathcal{E}$ (resp. an $\mathcal{O}_S$-module homomorphism m), $\mathcal{E}^{(q)}$ (resp. $m^{(q)}$) denotes the base extension $\mathcal{E} \otimes_{\mathcal{O}_S} \mathcal{O}_S$ (resp. $m \otimes 1$) by the q-th power map $\mathcal{O}_S \to \mathcal{O}_S$. For example, if G is a group scheme over S, then $\mathcal{O}_G^{(q)}$ is the structure sheaf of the *Frobenius group scheme* $G^{(q)}$ of G. Also, we denote by $F_G : G \to G^{(q)}$ (resp. $f_G : \mathcal{O}_G^{(q)} \to \mathcal{O}_G$) the *Frobenius morphism* . If G is an $\mathbb{F}_q$-module scheme, then so is $G^{(q)}$ and F_G is a morphism of $\mathbb{F}_q$-module schemes.

To understand the role of $\mathcal{E}_G$, recall

Definition (1.6). (Drinfeld [3], §2) A *φ-sheaf* is a pair $(\mathcal{E}, \varphi)$ consisting of a locally free $\mathcal{O}_S$-module $\mathcal{E}$ on S of finite rank and an $\mathcal{O}_S$-module homomorphism $\varphi : \mathcal{E}^{(q)} \to \mathcal{E}$.

A *morphism* $m : (\mathcal{E}, \varphi) \to (\mathcal{E}', \varphi')$ of φ-sheaves is an $\mathcal{O}_S$-module homomorphism $m : \mathcal{E} \to \mathcal{E}'$ which makes the diagram

$$\begin{array}{ccc} \mathcal{E}^{(q)} & \xrightarrow{m^{(q)}} & \mathcal{E}'^{(q)} \\ {\scriptstyle\varphi}\downarrow & & \downarrow{\scriptstyle\varphi'} \\ \mathcal{E} & \xrightarrow[m]{} & \mathcal{E}' \end{array}$$

commutative.

Let $(\mathcal{E}, \varphi)$ be a φ-sheaf and $E = \mathbb{V}(\mathcal{E})$ the vector bundle corresponding to $\mathcal{E}$. $\varphi : \mathcal{E}^{(q)} \to \mathcal{E}$ induces a morphism $\Phi : E \to E^{(q)}$ of $\mathbb{F}_q$-module schemes. Drinfeld defines then

$$\begin{aligned} \mathrm{Gr}(\mathcal{E}, \varphi) &:= \mathrm{Ker}(\Phi - F_E : E \to E^{(q)}) \\ &= \underline{\mathrm{Spec}}\left(\mathcal{S}/[(\varphi - f_{\mathcal{S}})(\mathcal{E}^{(q)})]\right), \end{aligned}$$

where $\mathcal{S} = \mathcal{O}_E$ is the symmetric algebra $\mathrm{Sym}^{\cdot}_{\mathcal{O}_S}\mathcal{E}$, $f_{\mathcal{S}} = f_E$ is the Frobenius morphism $\mathcal{S}^{(q)} \to \mathcal{S}$, and the bracket $[\cdots]$ denotes the ideal generated by its contents. This is a finite φ-module of rank $q^{\mathrm{rank}(\mathcal{E}_G)}$, with $\mathbb{F}_q$-action induced by the natural $\mathbb{F}_q$-module structure on $\mathcal{E}$. Note $\mathcal{E}_{\mathrm{Gr}(\mathcal{E},\varphi)} = \mathcal{E}$.

Conversely, if G is a finite φ-module over S, the Frobenius morphism $f_G : \mathcal{O}_G^{(q)} \to \mathcal{O}_G$ induces an $\mathcal{O}_S$-module homomorphism $\varphi_G : \mathcal{E}_G^{(q)} \to \mathcal{E}_G$. Then $(\mathcal{E}_G, \varphi_G)$ is a φ-sheaf. The natural $\mathcal{O}_S$-algerbra homomorphism $\mathrm{Sym}^{\cdot}_{\mathcal{O}_S}\mathcal{E}_G \to \mathcal{O}_G$ is surjective, and its kernel contains $(\varphi_G - f_{E_G})(\mathcal{E}_G^{(q)})$. Hence we have a surjection $\mathcal{O}_{\mathrm{Gr}(\mathcal{E}_G,\varphi_G)} \to \mathcal{O}_G$ of locally free $\mathcal{O}_S$-algebras. The equality $\mathrm{rank}(\mathcal{O}_G) = q^{\mathrm{rank}(\mathcal{E}_G)}$ implies that $\mathrm{Gr}(\mathcal{E}_G, \varphi_G) \simeq G$.

The commutativity of m and φ in the definition of a morphism $m : (\mathcal{E}, \varphi) \to (\mathcal{E}', \varphi')$ of φ-sheaves means that $m : \mathcal{E} \to \mathcal{E}'$ extends to an $\mathcal{O}_S$-Hopf algebra homomorphism

$$m : \mathcal{S}/[(\varphi - f_{\mathcal{S}})(\mathcal{E}^{(q)})] \longrightarrow \mathcal{S}'/[(\varphi' - f_{\mathcal{S}'})(\mathcal{E}'^{(q)})].$$

($\mathcal{S}'$ is the symmetric algebra made of $\mathcal{E}'$.) This is clearly compatible with the natural $\mathbb{F}_q$-actions. Noticing Lemma (1.5), we have thus

Proposition (1.7). *The category of finite φ-modules over S is anti-equivalent to the category of φ-sheaves on S.*

The set of valued points of $\mathrm{Gr}(\mathcal{E}, \varphi)$ is described as follows:

Proposition (1.8). *Let $(\mathcal{E}, \varphi)$ be a φ-sheaf on S, and let T be an S-scheme. Then the set of T-valued points of $\mathrm{Gr}(\mathcal{E}, \varphi)$ is*

$$\mathrm{Gr}(\mathcal{E}, \varphi)(T) \ = \ \mathrm{Hom}_{\varphi, \mathcal{O}_S}(\mathcal{E}, \mathcal{O}_T),$$

the set of $\mathcal{O}_S$-linear homomorphisms $f : \mathcal{E} \to \mathcal{O}_T$ such that $f(\varphi(x)) = f(x)^q$ for any local section x of $\mathcal{E}$.

Proof. This is clear from the definition of $\mathrm{Gr}(\mathcal{E}, \varphi)$. □

2. Finite t-modules

In the rest of the paper, A is the polynomial ring $\mathbb{F}_q[t]$ in one variable t over $\mathbb{F}_q$. We work over a fixed A-scheme S, and denote by θ the image of t by the structure morphism $\alpha : A \to \Gamma(S, \mathcal{O}_S)$.

Definition (2.1). A *finite t-module* (G, Ψ) over S is an A-module scheme over S such that
(1) G is killed by some $a \in A - \{0\}$; and
(2) $(G, \Psi \mid_{\mathbb{F}_q})$ is a finite φ-module over S.

A *morphism* of finite t-modules is by definition a morphism of A- module schemes.

A typical example of a finite t-module is a finite $\mathbb{F}_q[t]$-submodule of an abelian t-module ([1]) with scalar t-action on its tangent space. As is well-known, we have

Lemma (2.2). *A finite t-module G/S which is killed by $a \in A - 0$ is étale over S if a is invertible on S.*

Proof. It is enough to see $\Omega^1_{G/S} = 0$, but $a \cdot \Omega^1_{G/S} = 0$ and a is invertible. □

REMARK (2.3). If (G, Ψ) is a finite t-module, Ψ induces an action of A on the ambient space E_G (Remark (1.4), (i)). But E_G with this action is *not* in general an A-module scheme in the sense of Definition (1.1).

DEFINITION (2.4). A *t-sheaf* $(\mathcal{E}, \varphi, \psi_t)$ (or simply, $(\mathcal{E}, \varphi, \psi)$) on S is a pair consisting of a φ-sheaf $(\mathcal{E}, \varphi)$ and an endomorphism ψ_t of $(\mathcal{E}, \varphi)$ such that
(1) there exists a polynomial $a(X) \in \mathbb{F}_q[X] - \{0\}$ such that $a(\psi_t) = 0$ on $\mathcal{E}$; and
(2) ψ_t induces multiplication by θ on $\mathrm{Coker}(\varphi)$. (Recall that $\mathrm{Coker}(\varphi)$ is canonically isomorphic to $\mathrm{Lie}^*\mathrm{Gr}(\mathcal{E}, \varphi)$ ([3], Proposition 2.1, 2)).)

Equivalently, we may think that ψ is a ring homomorphism $A \to \mathrm{End}_\varphi(\mathcal{E}, \varphi)$; $a \mapsto \psi_a$ such that $\psi_a = 0$ for some $a \in A - \{0\}$ and, for each $a \in A$, ψ_a induces multiplication by $\alpha(a)$ on $\mathrm{Coker}(\varphi)$.

A *morphism* $m : (\mathcal{E}, \varphi, \psi_t) \to (\mathcal{E}', \varphi', \psi'_t)$ of t-sheaves is a morphism of φ-sheaves such that $m \circ \psi_t = \psi'_t \circ m$.

The following proposition, extending (1.7), is obvious.

PROPOSITION (2.5). *The category of finite t-modules over S is anti-equivalent to the category of t-sheaves on S.*

We write $\mathrm{Gr}(\mathcal{E}, \varphi, \psi)$ for the finite t-module corresponding to a t-sheaf $(\mathcal{E}, \varphi, \psi)$.

Example (2.6). Let (E, Ψ) be a Drinfeld A-module of rank r over S. Assume for simplicity that $S = \mathrm{Spec}\ R$ with R an A-algebra, and that the action of t is given by

$$\psi_t(X) = \theta X + a_1 X^q + \cdots + a_r X^{q^r}, \quad a_i \in R,\ a_r \in R^\times,$$

with respect to a trivialization $E \simeq \mathbb{G}_a = \mathrm{Spec}\ R[X]$. Then for $a \in A - \{0\}$, $G := \mathrm{Ker}(\Psi_a)$ is a finite t-module over R. $\mathcal{E}_G$ is a free R-module of rank $r \cdot \deg(a)$ with a basis $(X^{q^j};\ 0 \le j \le r \cdot \deg(a) - 1)$, and $\varphi : \mathcal{E}_G^{(q)} \to \mathcal{E}_G$ is given by

$$\varphi(X^{q^j} \otimes 1) = X^{q^{j+1}}.$$

Here $X^{q^{j+1}}$ for $j + 1 \ge r \cdot \deg(a)$ should be rewritten in terms of $(X^{q^j};\ 0 \le j \le r \cdot \deg(a) - 1)$ according to the relation $\psi_a(X) = 0$.

In the simple case where $a = t^k$, we can take the basis $(\psi_{t^i}(X)^{q^j};\ 0 \le i \le k-1, 0 \le j \le r-1)$ of $\mathcal{E}_G$, with respect to which ψ_t is represented by the matrix whose (i,j)-component is 1 if $i = j + r$ and 0 otherwise.

3. Finite v-modules

To establish a nice duality, we need one more structure.

Recall that a finite φ-module G is embedded canonically into its ambient space E_G (Remark (1.4), (i)), which is a vector group scheme.

DEFINITION (3.1). A *finite v-module* (G, Ψ, V) over S is a finite t-module (G, Ψ) over S together with a morphism $V : E_G^{(q)} \to E_G$ of $\mathbb{F}_q$-module schemes such that $\Psi_t = (\theta + V \circ F_{E_G})\mid_G$. (Here θ means multiplication by $\theta = \alpha(t) \in \Gamma(S, \mathcal{O}_S)$ on E_G, and F_{E_G} is the Frobenius morphism of E_G.)

A *morphism* $M : (G, \Psi, V) \to (G', \Psi', V')$ of finite v-modules is a morphism of finite φ-modules which renders the diagram

$$\begin{array}{ccc} E_G & \xrightarrow{E_M} & E_{G'} \\ {\scriptstyle V}\uparrow & & \uparrow{\scriptstyle V'} \\ E_G^{(q)} & \xrightarrow[E_M^{(q)}]{} & E_{G'}^{(q)} \end{array}$$

commutative.

DEFINITION (3.2). A *v-sheaf* $(\mathcal{E}, \varphi, v)$ on S is a pair consisting of a φ-sheaf on S and an $\mathcal{O}_S$-module homomorphism $v : \mathcal{E} \to \mathcal{E}^{(q)}$ such that $(\mathcal{E}, \varphi, \psi_t)$ with $\psi_t := \theta + \varphi \circ v$ is a t-sheaf on S. (Here θ means multiplication by θ on $\mathcal{E}$.)

A *morphism* $m : (\mathcal{E}, \varphi, v) \to (\mathcal{E}', \varphi', v')$ of v-sheaves is a morphism of φ-sheaves which renders the diagram

$$\begin{array}{ccc} \mathcal{E} & \xrightarrow{m} & \mathcal{E}' \\ {\scriptstyle v}\downarrow & & \downarrow{\scriptstyle v'} \\ \mathcal{E}^{(q)} & \xrightarrow[m^{(q)}]{} & \mathcal{E}'^{(q)} \end{array}$$

commutative.

These definitions are made so that Proposition (2.5) extends to

PROPOSITION (3.3). *The category of finite v-modules over S is anti-equivalent to the category of v-sheaves on S.*

We write $\mathrm{Gr}(\mathcal{E},\varphi,v)$ for the finite v-module corresponding to a v-sheaf $(\mathcal{E},\varphi,v)$.

Example (3.4). Let (E,Ψ) and $G=\mathrm{Ker}(\Psi_a)$ be as in Example (2.6). Then the finite t-module G is furnished with a standard v-module structure by

$$\begin{aligned} &v:\mathcal{E}_G\to\mathcal{E}_G^{(q)},\\ &\quad X^{q^i}\mapsto X^{q^{i-1}}\otimes(\theta^{q^i}-\theta)+X^{q^i}\otimes a_1^{q^i}+\cdots+X^{q^{r+i-1}}\otimes a_r^{q^i}. \end{aligned}$$

(Here $X^{q^{i-1}}\otimes(\theta^{q^i}-\theta):=0$ if $i=0$.) If $G=\mathrm{Ker}(\Psi_t)$ for example and if we regard $\mathcal{E}_G$ and $\mathcal{E}_G^{(q)}$ as the column vectors of rank r by fixing the R-basis $(X^{q^j})_{0\le j\le r-1}$ and $(X^{q^j}\otimes 1)_{0\le j\le r-1}$ respectively, then v is represented by the matrix

$$\begin{pmatrix} a_1 & -\theta & & \\ \vdots & & \ddots & \\ \vdots & & & -\theta \\ a_r & & & \end{pmatrix}.$$

(The vacant components are 0.) Note that $\psi_t=0$ on $\mathcal{E}_G$ in this case, and still v has enough information to recover the dual of G. But this v-module structure is *not* unique unless $\mathrm{Ker}(\varphi_G:\mathcal{E}_G^{(q)}\to\mathcal{E}_G)=0$.

In fact, finite v-modules over "mixed characteristic" bases are not so far from finite t-modules, since we have:

PROPOSITION (3.5). *Assume that the base scheme S is reduced. Let (G,Ψ) be a finite t-module which is étale over the generic points of S. Then (G,Ψ) has a unique v-module structure V_G extending the given t-module structure; $\Psi_t=(\theta+V_G\circ F_{E_G})\ |_G$. If G and G' are two such finite t-modules, then a morphism $G\to G'$ of finite t-modules preserves* *this v-module structure. In particular, if $\alpha : A \to \mathcal{O}_S$ is injective (cf. Lemma (2.2)), the two concepts, a finite t-module and a finite v-module, are equivalent.*

The same is valid for a t-sheaf $(\mathcal{E}, \varphi, \psi_t)$ such that $\varphi : \mathcal{E}^{(q)} \to \mathcal{E}$ is injective over the generic points.

Proof. We prove this for t-sheaves. By (2) of Definition (2.4), we have

$$\mathrm{Im}(\psi_t - \theta) \subset \mathrm{Im}(\varphi).$$

Hence $v := \varphi^{-1} \circ (\psi_t - \theta) : \mathcal{E} \to \mathcal{E}^{(q)}$ is well-defined (note that φ is in fact injective all over S by the assumption of reducedness), and gives a unique v-sheaf structure on $(\mathcal{E}, \varphi)$ extending the t-sheaf structure ψ_t.

Let $m : (\mathcal{E}, \varphi, \psi_t) \to (\mathcal{E}', \varphi', \psi'_t)$ be a morphism of t-sheaves. If φ and φ' are generically injective, we have the diagram

$$\begin{array}{ccc} \mathcal{E} & \xrightarrow{m} & \mathcal{E}' \\ {\scriptstyle v}\downarrow & & \downarrow{\scriptstyle v'} \\ \mathcal{E}^{(q)} & \xrightarrow{m^{(q)}} & \mathcal{E}'^{(q)} \\ {\scriptstyle \varphi}\downarrow & & \downarrow{\scriptstyle \varphi'} \\ \mathcal{E} & \xrightarrow[m]{} & \mathcal{E}' \end{array}$$

in which v and v' are defined as above and in which the outer and the lower squares are commutative. Since φ' is injective, the upper square is also commutative, i.e., m is a morphism of v-sheaves. □

Example (3.6). Let C be the Carlitz module over Spec A, i.e., the rank one Drinfeld A-module with underlying group scheme $\mathbb{G}_a = \mathrm{Spec}\ A[Z]$ and with t-action given by $\gamma_t : Z \mapsto \theta Z + Z^q$. (Here, one may choose another t-action $Z \mapsto \theta Z + aZ^q$ for any $a \in \mathbb{F}_q^\times$, but then $a^{-1}t$ acts by $Z \mapsto \alpha(a^{-1}t)Z + Z^q$. So in the following, we fix $t \in A$ and its action on C as above.) Let G be a finite A-submodule of C. Then over A, G has a unique v-module structure

$$\begin{aligned} v_C : \mathcal{E}_G &\to \mathcal{E}_G^{(q)}, \\ Z^{q^i} &\mapsto Z^{q^{i-1}} \otimes (\theta^{q^i} - \theta) + Z^{q^i} \otimes 1. \end{aligned}$$

In §4, we shall think of $G \times_{\mathrm{Spec}\, A} S$, over any base scheme S, as a finite v-module with v-structure induced by this canonical one. Also, it would be convenient in what follows to think of C itself as a "v-module" with $v_C : \mathcal{E}_C \to \mathcal{E}_C^{(q)}$ defined as above, though we deal in fact with its finite subgroups.

The following Remark is not used in this paper, but provides us with some feeling on $\mathcal{E}_G$.

REMARK (3.7). Let G be a finite v-module over S. Then the $\mathcal{O}_S$-module $\mathcal{E}_G$ would deserve the name the "Dieudonné module" of G, because we have $\mathcal{E}_G = \underline{\mathrm{Hom}}_{v,S}(G, CW)$. Here CW is the v-module of "Witt covectors", defined as follows (we disregard the topology): CW is, as a group scheme, the infinte direct product of $\mathbb{G}_a$'s with affine algebra $\mathcal{O}_{CW} = \mathcal{O}_S[\cdots, X_{-n}, \cdots, X_{-1}, X_0]$, and the t-module and v-module structures are defined by

$$\begin{aligned} t : X_{-n} &\mapsto \theta X_{-n} + X_{-n-1}^q, \\ v : X_{-n} &\mapsto X_{-n-1} \otimes 1 \end{aligned}$$

for all $n \geq 0$.

4. The duality

For an $\mathcal{O}_S$-module $\mathcal{E}$, put $\mathcal{E}^* := \underline{\mathrm{Hom}}_{\mathcal{O}_S}(\mathcal{E}, \mathcal{O}_S)$. If $(\mathcal{E}, \varphi, v)$ is a v-sheaf on S, then φ and v induce respectively the $\mathcal{O}_S$-module homomorphisms

$$\varphi^* : \mathcal{E}^* \to \mathcal{E}^{*(q)} \quad \text{and} \quad v^* : \mathcal{E}^{*(q)} \to \mathcal{E}^*.$$

It is easy to check that $(\mathcal{E}^*, v^*, \varphi^*)$ is a v-sheaf on S.

DEFINITION (4.1). We define the *dual* $(\mathcal{E}, \varphi, v)^*$ of a v-sheaf $(\mathcal{E}, \varphi, v)$ to be the v-sheaf $(\mathcal{E}^*, v^*, \varphi^*)$. For a finite v-module $G = \mathrm{Gr}(\mathcal{E}, \varphi, v)$, define its *dual* G^* to be $\mathrm{Gr}(\mathcal{E}^*, v^*, \varphi^*)$.

Note that if, as in Proposition (3.5), the base scheme S is reduced and (G, Ψ) is a finite t-module which is étale over the generic points (resp.

$(\mathcal{E}, \varphi, \psi_t)$ is a t-sheaf such that φ is injective over the generic points), then we can define its dual.

We have clearly the following

PROPOSITION (4.2). *Let G be a finite v-module.*
(i) G^* has the same rank as G.
(ii) The correspondence $G \mapsto G^*$ is functorial. This functor is exact.
(iii) G^{**} is canonically isomorphic to G.
(iv) $(G \times_S T)^* \simeq G^* \times_S T$ for any S-scheme T.
The same is true for the duality of v-sheaves.

THEOREM (4.3). *Let C be the Carlitz module over Spec A (cf. Example (3.6)), and let G be a finite v-module over S.*
(i) The functor

$$\underline{\mathrm{Hom}}_{v,S}(G, C) : (S\text{-}schemes) \to (A\text{-}modules)$$
$$T \mapsto \mathrm{Hom}_{v,T}(G \times_S T,\ C \times_{\mathrm{Spec}\ A} T)$$

is represented by (the underlying finite t-module of) G^*.
(ii) There exists a canonical A-bilinear pairing of A-module schemes:

$$\Pi_G : G \times_S G^* \to C$$

such that:

(ii-1) If G' is a finite t-module over S sitting in an A-bilinear pairing $\Pi' : G \times_S G' \to C$, then there exists a unique morphism $M : G' \to G^*$ of finite t-modules which makes the diagram

$$\begin{array}{ccc} G \times_S G' & \xrightarrow{\Pi'} & C \\ {\scriptstyle 1\times M}\downarrow & & \| \\ G \times_S G^* & \xrightarrow[\Pi_G]{} & C \end{array}$$

commute.

(ii-2) If $M : G \to H$ is a morphism of finite v-modules and $M^* : H^* \to G^*$ is its dual morphism induced by functoriality, then we have

$$\Pi_H \circ (M \times 1) \ = \ \Pi_G \circ (1 \times M^*) \qquad \text{on } \ G \times H^*.$$

Conversely, M^* is the unique morphism which has this property.

(ii-3) If $\alpha : A \to \mathcal{O}_S$ is injective and S is integral with function field K, then Π_G induces a non-degenerate Galois equivariant A-bilinear pairing between the A-modules of geometric points:

$$G(K^{\mathrm{sep}}) \times G^*(K^{\mathrm{sep}}) \to C(K^{\mathrm{sep}}).$$

PROOF. Recall that $\mathcal{O}_C$ is the polynomial ring $A[Z]$ with t-action $\gamma_t : Z \mapsto \theta Z + Z^q$ and v-module structure $v_C : \mathcal{E}_C \to \mathcal{E}_C^{(q)}$; $Z \mapsto Z \otimes 1$. Let $G = \mathrm{Gr}(\mathcal{E}_G, \varphi_G, v_G)$. An $\mathcal{O}_T$-algebra homomorphism $m : \mathcal{O}_C \otimes_A \mathcal{O}_T \to \mathcal{O}_G \otimes_{\mathcal{O}_S} \mathcal{O}_T$ corresponds to a morphism of v-modules $G \times_S T \to C \times_{\mathrm{Spec}\, A} T$ if and only if

$$m(Z) \in \Gamma(T, \mathcal{E}_G \otimes_{\mathcal{O}_S} \mathcal{O}_T), \qquad \text{and} \tag{4.3.1}$$

$$m^{(q)} \circ v_C(Z) = (v_G \otimes 1) \circ m(Z). \tag{4.3.2}$$

Let $\mathcal{S}^*$ be the symmetric $\mathcal{O}_S$-algebra $\mathrm{Sym}^{\cdot}_{\mathcal{O}_S} \mathcal{E}_G^*$, and Z_0 a global section of $\mathcal{E}_G \otimes_{\mathcal{O}_S} \mathcal{E}_G^*$ which gives a basis of the rank one $\mathcal{O}_S$-submodule of $\mathcal{E}_G \otimes_{\mathcal{O}_S} \mathcal{E}_G^*$ on which one has $m \otimes 1 = 1 \otimes m^*$ for all $m \in \mathrm{End}_{\mathcal{O}_S}(\mathcal{E}_G)$. A *canonical* choice for Z_0 is $\sum_i X_i \otimes X_i^*$, where $(X_i)_i$ is a local basis of $\mathcal{E}_G$ and $(X_i^*)_i$ is its dual basis. Let

$$\begin{aligned} \iota : \mathcal{E}_G \otimes_{\mathcal{O}_S} \mathcal{E}_G^* &\to \mathcal{E}_G^{(q)} \otimes_{\mathcal{O}_S} \mathcal{E}_G^{*(q)} \\ X \otimes Y &\mapsto (X \otimes 1) \otimes (Y \otimes 1) \end{aligned}$$

be the natural map. Then we have $(v \otimes 1)(Z_0) = (1 \otimes v^*) \circ \iota(Z_0)$ for all $v \in \mathrm{Hom}_{\mathcal{O}_S}(\mathcal{E}_G, \mathcal{E}_G^{(q)})$. If we take $\mathcal{O}_T = \mathcal{S}^*$ and $Z \mapsto Z_0$, then (4.3.2) reads

$$(1 \otimes f_{\mathcal{S}^*}) \circ \iota(Z_0) = (1 \otimes v_G^*) \circ \iota(Z_0).$$

($f_{\mathcal{S}^*}$ is the $\mathcal{O}_S$-linear Frobenius morphism $\mathcal{S}^{*(q)} \to \mathcal{S}^*$.) Let $\mathcal{J}^*$ be the smallest ideal of $\mathcal{S}^*$ such that

$$(1 \otimes (v_G^* - f_{\mathcal{S}^*})) \circ \iota(Z_0) \in \mathcal{E}_G^{(q)} \otimes_{\mathcal{O}_S} \mathcal{J}^*.$$

Then it follows from what we observed at the beginning of the proof that the functor $\underline{\mathrm{Hom}}_{v,S}(G,C)$ is represented by $\underline{\mathrm{Spec}}(\mathcal{S}^*/\mathcal{J}^*) = \mathrm{Gr}(\mathcal{E}_G^*, v_G^*)$, with t-action induced by ψ_t^* on $\mathcal{E}_G^*$.

REMARK (4.4). To represent the functor $\underline{\mathrm{Hom}}_{v,S}(G,C)$, the v-module structure of G^* is not needed (and in fact a v-module structure on $\mathrm{Gr}(\mathcal{E}_G^*, v_G^*)$ may not be unique (cf. Example (3.4)), but for G to represent $\underline{\mathrm{Hom}}_{v,S}(G^*,C)$, G^* must have the v-module structure φ_G^*.

PROOF CONTINUED. The pairing $G\times_S G^* \to C$ is given by

$$\begin{aligned} \pi : \mathcal{O}_C &\to \mathcal{O}_G \otimes_{\mathcal{O}_S} \mathcal{O}_{G^*}, \\ Z &\mapsto Z_1, \end{aligned}$$

where Z_1 is the image of Z_0 in $\mathcal{O}_G \otimes_{\mathcal{O}_S} (\mathcal{S}^*/\mathcal{J}^*)$. The universality of G^* (ii-1) is clear from the above discussion.

The non-degeneracy of (ii-2) is a consequence of a basic fact in linear algebra; let (X_i) and (Y_j) be $\mathcal{O}_S$-bases of $\mathcal{E}_G$ and $\mathcal{E}_H$ respectively, (X_i^*) and (Y_j^*) the dual bases, $m : \mathcal{E}_H \to \mathcal{E}_G$ an $\mathcal{O}_S$-linear map, and $m^* : \mathcal{E}_G^* \to \mathcal{E}_H^*$ its dual map. Then we have $\sum_i X_i \otimes m^*(X_i^*) = \sum_j m(Y_j)\otimes Y_j^*$ in $\mathcal{E}_G \otimes_{\mathcal{O}_S} \mathcal{E}_H^*$. Conversely, m^* is the unique $\mathcal{O}_S$-linear map with this property.

Since G is étale over K if α is injective (Lemma (2.2)), (ii-3) follows from the well-known equivalence between the category of finite étale K^{sep}-schemes and the category of finite sets. □

REMARK (4.5). If we consider only the t-module structure, we will have the following:
(i) The functor

$$\begin{aligned} \underline{\mathrm{Hom}}_{t,S}(G,C) : (S\text{-schemes}) &\to (A\text{-modules}) \\ T &\mapsto \mathrm{Hom}_{t,T}(G\times_S T,\ C\times_{\mathrm{Spec} A} T) \end{aligned}$$

is represented by an A-module scheme $\widetilde{G}^*$ over S.
(ii) Assume S is reduced. If G is étale over the generic points of S, then $\widetilde{G}^*$ is of the form $G^* \cup \widetilde{G}_0^*$, where G^* is (the underlying finite t-module of) the dual finite v-module of G, G being considered to be a finite v-module with the unique v-module structure (Proposition (3.5)), and where $\widetilde{G}_0^*$ is

supported on the locus in S over which G is not étale. In general, $\widetilde{G}_0^*$ has a positive dimension. For example:

Example (4.6). Let R be an (A/tA)-algebra (i.e., we are in the "characteristic" (t) situation in the sense that the kernel of the structure map $\alpha : A \to R$ is (t)), and let $G = \text{Spec } R[X_1, X_2]/(X_1^q, X_2^q)$ be a finite t-module with t acting by $X_i \mapsto 0$ for $i = 1, 2$. If we think of G as the t-division points of the abelian t-module $(E, \Psi) = C^{\oplus 2}$;

$$E = \text{Spec } R[X_1, X_2], \quad \psi_t \begin{pmatrix} X_1 \\ X_2 \end{pmatrix} = t \begin{pmatrix} X_1 \\ X_2 \end{pmatrix} + \begin{pmatrix} X_1^q \\ X_2^q \end{pmatrix},$$

then it is natural to make G into a finite v-module by $v : X_i \mapsto X_i \otimes 1$ for $i = 1, 2$. On the other hand, G can be regarded as the t-division points of another abelian t-module(E', Ψ') with

$$E' = \text{Spec } R[X_1, X_2], \quad \psi'_t \begin{pmatrix} X_1 \\ X_2 \end{pmatrix} = t \begin{pmatrix} X_1 \\ X_2 \end{pmatrix} + \begin{pmatrix} X_2^q \\ X_1^q \end{pmatrix}.$$

Now it is natural to make G into a finite v-module by $v : X_i \mapsto X_{3-i} \otimes 1$ for $i = 1, 2$. In the former case, we have $G^* = \text{Spec } R[Y_1, Y_2]/(Y_1 - Y_1^q, Y_2 - Y_2^q)$ (the constant group scheme $\mathbb{F}_q \oplus \mathbb{F}_q$), whereas in the latter case, we have $G^* = \text{Spec } R[Y]/(Y - Y^{q^2})$ (the étale group scheme $\mathbb{F}_{q^2}$). Of course, we could choose any v-module structure $v : X_i \mapsto X_1 \otimes a_{1i} + X_2 \otimes a_{2i}$ for $i = 1, 2$ with $a_{ji} \in R$. Without v-module structures, we will have $\widetilde{G}^* \simeq \mathbb{A}_R^2$ in this case.

Finally in this section, we describe a relation between the Frobenius and the Verschiebung over a "finite characteristic" base.

PROPOSITION (4.7). *Let (G, Ψ, V) be a finite v-module over S.*
(i) Let d be a positive integer, and $F_G^d : G \to G^{(q^d)}$ the q^d-th power Frobenius morphism. Then $G^{(q^d)}$ (resp. F_G^d) is a finite v-module (resp. a morphism of finite v-modules) if $\text{Im}(\alpha) \subset \mathbb{F}_{q^d}$. *If this is the case and $M : G \to G'$ is a morphism of finite v-modules, then we have $M^{(q^d)} \circ F_G^d = F_{G'}^d \circ M$.*
(ii) Assume $\text{Ker}(\alpha : A \to \mathcal{O}_S) = (\mathfrak{p})$ *with $\mathfrak{p} \in A$ being a monic prime element of degree d. Let $V_{G,\mathfrak{p}} : G^{(q^d)} \to G$ be the dual morphism of $F_{G^*,\mathfrak{p}} := F_{G^*}^d : G^* \to G^{*(q^d)}$. Then we have*

$$\Psi_{\mathfrak{p}} = V_{G,\mathfrak{p}} \circ F_{G,\mathfrak{p}} \quad \textit{and} \quad \Psi_{\mathfrak{p}}^{(q^d)} = F_{G,\mathfrak{p}} \circ V_{G,\mathfrak{p}}.$$

In particular, we have an exact sequence of finite t-modules

$$0 \to \mathrm{Ker}(F_{G,\mathfrak{p}}) \to \mathrm{Ker}(\Psi_{\mathfrak{p}}) \to \mathrm{Ker}(V_{G,\mathfrak{p}}) \to 0.$$

PROOF. (i) The only point we must care about is the action of $a \in A$ on $\mathrm{Lie}^*(G^{(q^d)})$, which is multiplication by $\alpha(a)^{(q^d)}$. This should be equal to $\alpha(a)$, which is the case if $\mathrm{Im}(\alpha) \subset \mathbb{F}_{q^d}$. The compatibility conditions for v-module structures and morphisms are then automatically satisfied.

(ii) Let $Z \in \mathcal{O}_C$ and $Z_1 \in \mathcal{O}_G \otimes_{\mathcal{O}_S} \mathcal{O}_{G^*}$ have the same meaning as in the proof of Theorem (4.3). Let

$$\begin{aligned} \pi : \mathcal{O}_C &\to \mathcal{O}_G \otimes_{\mathcal{O}_S} \mathcal{O}_{G^*} \\ Z &\mapsto Z_1 \end{aligned}$$

be the $\mathcal{O}_S$-algebra homomorphism corresponding to the pairing $\Pi_G : G \times_S G^* \to C$. Then the A-linearity of the pairing is written as

$$(\psi_{\mathfrak{p}} \otimes 1)(Z_1) = \pi(\gamma_{\mathfrak{p}}(Z)) = (1 \otimes \psi^*_{\mathfrak{p}})(Z_1).$$

Here $\gamma : A \to \mathrm{End}_{\mathcal{O}_S}(\mathcal{O}_C)$; $a \mapsto \gamma_a$ is the map describing the A-action on C. Since $\gamma_{\mathfrak{p}}(Z) \equiv Z^{q^d} (\mathrm{mod}.\mathfrak{p})$ (e.g. [5], Proposition 2.4), we have

$$\begin{aligned} (\psi_{\mathfrak{p}} \otimes 1)(Z_1) = \pi(Z^{q^d}) &= (f_{G,\mathfrak{p}} \otimes f_{G^*,\mathfrak{p}}) \circ \iota(Z_1) \\ &= (f_{G,\mathfrak{p}} \circ v_{G,\mathfrak{p}} \otimes 1)(Z_1). \end{aligned}$$

Hence $\psi_{\mathfrak{p}} = f_{G,\mathfrak{p}} \circ v_{G,\mathfrak{p}}$, and $\Psi_{\mathfrak{p}} = V_{G,\mathfrak{p}} \circ F_{G,\mathfrak{p}}$.

By (i), we have also the commutative diagram

$$\begin{array}{ccc} G^{(q^d)} & \xrightarrow{V_{G,\mathfrak{p}}} & G \\ {\scriptstyle F_{G^{(q^d)},\mathfrak{p}}}\downarrow & & \downarrow{\scriptstyle F_{G,\mathfrak{p}}} \\ G^{(q^{2d})} & \xrightarrow[V_{G^{(q^d)},\mathfrak{p}}]{} & G^{(q^d)}, \end{array}$$

from which follows the equality

$$\Psi_{\mathfrak{p}}^{(q^d)} = V_{G^{(q^d)},\mathfrak{p}} \circ F_{G^{(q^d)},\mathfrak{p}} = F_{G,\mathfrak{p}} \circ V_{G,\mathfrak{p}}. \ \square$$

5. Duality for Drinfeld modules

In this section, we construct explicitly the *dual* $\check{E}$ of a Drinfeld $\mathbb{F}_q[t]$-module E, and prove the compatibility of this construction and the duality of §4 for the torsion points of E and $\check{E}$. $\check{E}$ is an $(r-1)$-dimensional abelian t-module ([1]) if E is of rank $r \geq 2$.

Let $A = \mathbb{F}_q[t]$ and R an A-algebra. The image of $t \in A$ in R will be denoted by θ. (Though all constructions below work over any A-scheme S, we work over an affine $S = \mathrm{Spec}\ R$ for simplicity.)

Let (E, Ψ) be a Drinfeld module over R of rank $r \geq 2$. Suppose the action of $t \in A$ is given by

$$\psi_t(X) = \theta X + a_1 X + \cdots + a_r X^{q^r}, \quad a_i \in R,\ a_r \in R^\times$$

with respect to a coordinate X of E. (As before, we use a small letter ψ to denote a map of affine rings.) On $\check{E} := \mathbb{G}_{a\ /R}^{\oplus(r-1)}$, define an A-module scheme structure $\check{\Psi} : A \longrightarrow \mathrm{End}_R(\mathbb{G}_a^{\oplus(r-1)})$, in terms of the coordinates $\mathbb{Y} = {}^t(Y_1, \cdots, Y_{r-1})$ of $\mathbb{G}_a^{\oplus(r-1)} = \mathrm{Spec} R[Y_1, \cdots, Y_{r-1}]$, by

$$\check{\psi}_t(\mathbb{Y}) = \theta \mathbb{Y} + \mathbb{B}_1 \mathbb{Y}^{(q)} + \mathbb{B}_2^{(q)} \mathbb{Y}^{(q^2)},$$

with

$$\mathbb{B}_1 := \begin{pmatrix} & & & -a_r^{-1}a_1 \\ 1 & & & \vdots \\ & \ddots & & \vdots \\ & & 1 & -a_r^{-1}a_{r-1} \end{pmatrix}, \quad \mathbb{B}_2 := \begin{pmatrix} & & a_r^{-1} \\ & & \\ & & \end{pmatrix}.$$

Here and elsewhere, for a matrix $\mathbb{B}$, $\mathbb{B}^{(q^j)}$ denotes the matrix $\mathbb{B}$ but with entries raised to the q^j-th power. We will call this type of A-module schemes $(\check{E}, \check{\Psi})$ *dual Drinfeld modules.* Note that one can recover the Drinfeld module E starting with a dual Drinfeld module $\check{E}$, so that we may think $\check{\check{E}} = E$.

Let C be the Carlitz module on which t acts by $\gamma_t : Z \mapsto \theta Z + Z^q$ with respect to a coordinate Z of C.

THEOREM (5.1). *(i) If R is a perfect field, $\check{E}$ is an abelian t-module of t-rank $r(\check{E}) = r$, τ-rank $\rho(\check{E}) = r-1$, and weight $w(\check{E}) = (r-1)/r$ in the sense of [1].*

(ii) For a non-zero $a \in A$, the kernel ${}_a\check{E}$ of the action of a on $\check{E}$ is a finite t-module over R of rank $q^{r\cdot\deg(a)}$.
(iii) For a non-zero $a \in A$, there exists an A-bilinear pairing defined over R:

$$ {}_a\Pi_E \ : \ {}_aE \ \times_R \ {}_a\check{E} \ \longrightarrow \ {}_aC. $$

(iv) If we furnish ${}_aE$ with the standard v-module structure as in (3.4), then we have ${}_a\check{E} \simeq \ {}_aE^*$, and the pairing ${}_a\Pi_E$ of (iii) coincides with the pairing $\Pi_{{}_aE}$ of Theorem (4.3).

Remarks (5.2). (i) Anderson takes $A = \mathbb{F}_p[t]$ with a *prime* p in [1]. So we should either assume in (5.1),(i) that $q = p$, or define the t-motive $\mathrm{M}(\check{E}) = \mathrm{Hom}_R(\mathbb{G}_a^{\oplus(r-1)}, \mathbb{G}_a)$ to be the $\mathbb{F}_q$-linear homomorphisms. Here we will take the latter, and denote it, as before, by $\mathcal{E}_{\check{E}}$.
(ii) The statements of the Theorem are valid also for any d-dimensional abelian t-module (E, Ψ) if $\Psi : A \longrightarrow \mathrm{End}_R(\mathbb{G}_a^{\oplus d})$ is defined by an equation of the form

$$ \psi_t(X) \ = \ \theta X \ + \ a_1 X^{(q)} \ + \cdots + \ a_r X^{(q^r)}, \quad X \ = \ {}^t(X_1, \cdots, X_d), $$

with $a_i \in \mathrm{M}_d(R)$ and $a_r \in \mathrm{GL}_d(R)$.
(iii) For a Drinfeld module E of rank 1, there exists an ind-finite étale A-module scheme $\check{E}$ (a twist of the constant A-module scheme $\mathbb{F}_q(t)/\mathbb{F}_q[t]$), together with a pairing as in (iii) of the Theorem.
(iv) Even if E does not have good reduction over R, we can define an A-bilinear pairing between the division points of E and $\check{E}'$, a twist of $\check{E}$, with target C', a twist of C. Especially, we can take $\check{E}'$ to be the $(r-1)$-st exterior product $\wedge^{r-1}E$ of E ([1]) defined by

$$ \check{\psi}'_t(\mathbb{Y}) \ = \ \theta\mathbb{Y} \ + \ \mathbb{B}_1'^{(q)}\mathbb{Y}^{(q)} \ + \ \mathbb{B}_2'^{(q^2)}\mathbb{Y}^{(q^2)} $$

with

$$ \mathbb{B}_1' := (-1)^r \begin{pmatrix} & & & (-1)^r a_1 \\ a_r & & & \vdots \\ & \ddots & & (-1)^{r+1-i} a_i \\ & & \ddots & \vdots \\ & & a_r & a_{r-1} \end{pmatrix}, $$

$$\mathbb{B}_2' := (-1)^r \begin{pmatrix} & & a_r \\ & & \\ & & \\ & & \end{pmatrix},$$

and C' to be the r-th exterior product $\wedge^r E$ of E ([1], [4]) defined by

$$\gamma_t'(Z) \;=\; \theta Z - (-1)^r a_r X^q.$$

$\check{E}'$ and C' may have non-stable reduction. It would be interesting to seek a good model of $\check{E}'$.

PROOF OF THE THEOREM. (i) This is clear; an $R[\check{\psi}_t]$-base of $\mathcal{E}_{\check{E}} = \mathrm{Hom}_{\mathbb{F}_q,R}(\mathbb{G}_a^{\oplus(r-1)}, \mathbb{G}_a)$ is $(a_r^{-1}Y_{r-1}^q, Y_1, \cdots, Y_{r-1})$, which implies $r(\check{E}) = r$. The other assertions are obvious.

(ii) Put $G = {}_a\check{E}$. The affine ring $\mathcal{O}_G$ of G can be identified with the quotient $R[Y_1, \cdots, Y_{r-1}]/\check{\psi}_a(\mathbb{Y})$. It is enough to show that $\mathcal{O}_G$ is free over R of rank $q^{r\cdot\deg(a)}$, and that $\mathcal{E}_G$ is free over R of rank $r \cdot \deg(a)$.

We may assume $a \in A = \mathbb{F}_q[t]$ is monic of degree $k \geq 1$, and write $a = t^k + g(t)$, $g(t) = \sum_{i=0}^{k-1} g_i t^i$, $g_i \in \mathbb{F}_q$. Define elements $Y_{ij} \in \mathcal{O}_G$ for $0 \leq i \leq k-1$ and $1 \leq j \leq r-1$ by

$$Y_{k-1,j} \;=\; Y_j \qquad (1 \leq j \leq r-1),$$

and

$$\mathbb{Y}_{i-1} \;=\; \check{\psi}_t(\mathbb{Y}_i) \;+\; g_i \mathbb{Y}_{k-1} \qquad (1 \leq i \leq k-1), \tag{5.1.1}$$

where $\mathbb{Y}_i := {}^t(Y_{i1}, \cdots, Y_{i,r-1})$. Applying (5.1.1) repeatedly, we find

$$\begin{aligned} \mathbb{Y}_i = \;& \check{\psi}_{t^{k-1-i}}(\mathbb{Y}_{k-1}) \;+ g_{k-1}\check{\psi}_{t^{k-2-i}}(\mathbb{Y}_{k-1}) \;+ \cdots + g_{i+1}\mathbb{Y}_{k-1} \\ =\;& \check{\psi}_{(t^{k-1-i}+g_{k-1}t^{k-2-i}+\cdots+g_{i+1})}(\mathbb{Y}_{k-1}), \end{aligned} \tag{5.1.1a}$$

and especially,

$$\mathbb{Y}_0 \;=\; \check{\psi}_{(t^{k-1}+g_{k-1}t^{k-2}+\cdots+g_1)}(\mathbb{Y}_{k-1}),$$

whence

$$\check{\psi}_t(\mathbb{Y}_0) \ = \ \check{\psi}_a(\mathbb{Y}_{k-1}) \ - \ g_0\mathbb{Y}_{k-1}.$$

This shows that the equality $\check{\psi}_a(\mathbb{Y}_{k-1}) = 0$ (which means $G = {}_a\check{E}$) is equivalent to

$$\check{\psi}_t(\mathbb{Y}_0) \ = \ -g_0\mathbb{Y}_{k-1}. \tag{5.1.2}$$

We can thus regard $\mathcal{O}_G$ as the quotient of $R[Y_{01}, \cdots, Y_{k-1,r-1}]$ by the relations (5.1.1) and (5.1.2).

By setting $(Y'_{ij})^q := Y_{ij}$ if $j < r-1$ and $Y'_{i,r-1} := Y_{i,r-1}$, we embed $\mathcal{O}_G$ into the quotient $\mathcal{O}'$ of $R[Y'_{01}, \cdots, Y'_{k-1,r-1}]$ by the same relations (5.1.1) and (5.1.2). Then (5.1.1) and (5.1.2) read:

$$(\text{unit})(Y'_{ij})^{q^2} \ + \ (\text{lower terms}) \ = \ 0, \quad 0 \le i \le k-1, \ 1 \le j \le r-1.$$

By Lemma 1.9.1 of [2], $\mathcal{O}'$ is free of rank $q^{2k(r-1)}$ over R, with a base

$$(\prod_{i,j}(Y'_{ij})^{l_{ij}} \ ; \ 0 \le l_{ij} \le q^2 - 1).$$

Since $\mathcal{O}_G$ is the R-submodule of $\mathcal{O}'$ generated by

$$(\prod_{i,j}(Y'_{ij})^{l_{ij}} \ ; \ q|l_{ij} \text{ if } 1 \le j \le r-2),$$

it is also free, and of rank $q^{k(r-2)} \cdot q^{2k} = q^{kr}$. $\mathcal{E}_G$ is also free on the R-base $(Y_{ij};\ 0 \le i \le k-1,\ 0 \le j \le r-1)$, so we have $\operatorname{rank}(\mathcal{O}_G) = q^{\operatorname{rank}(\mathcal{E}_G)}$.

(iii) Passing to the language of affine rings, we shall give an R-algebra homomorphism

$$\pi \ : \ \mathcal{O}_{{}_aC} \ \longrightarrow \ \mathcal{O}_{{}_aE} \ \otimes_R \ \mathcal{O}_{{}_a\check{E}},$$

or more explicitly,

$$\pi \ : \ R[Z]/\gamma_a(Z) \ \longrightarrow \ R[X]/\psi_a(X) \ \otimes_R \ R[Y_1, \cdots, Y_{r-1}]/\check{\psi}_a(\mathbb{Y})$$

which is compatible with the comultiplications ($Z \mapsto Z\otimes 1 + 1 \otimes Z$, etc.) and the A-actions. Write $a = t^k + g(t)$, $g(t) = \sum_{i=0}^{k-1} g_i t^i$ and define $Y_{ij} \in \mathcal{O}_{{}_a\check{E}}$ as in the proof of (ii). Set further

$$Y_{i0} \ := \ a_r^{-1} Y_{i,k-1}^q \qquad (0 \le i \le k-1). \tag{5.1.3}$$

Simplifying the notation, we also set $X_{ij} := \psi_{t^i}(X)^{q^j}$ for $i, j \geq 0$. Then we have

$$X_{i+1,0} \;=\; \psi_t(X_{i0}) \;=\; \theta X_{i0} \;+\; \sum_{j=1}^{r} a_j X_{ij} \tag{5.1.4}$$

and

$$0 \;=\; \psi_a(X) \;=\; \psi_{t^k}(X) + \psi_{g(t)}(X) \;=\; X_{k0} + \sum_{i=0}^{k-1} g_i X_{i0} \tag{5.1.5}$$

$$= \theta X_{k-1,0} \;+\; \sum_{j=1}^{r} a_j X_{k-1,j} \;+\; \sum_{i=0}^{k-1} g_i X_{i0}. \tag{5.1.5a}$$

Now define the map π by

$$\pi \;:\; Z \;\mapsto\; \sum_{i=0}^{k-1}\sum_{j=0}^{r-1} X_{ij} \;\otimes\; Y_{ij}.$$

This is obviously compatible with the comultiplications and the actions of $\mathbb{F}_q$ $(\subset A)$; it only remains to check the commutativity of the following diagram:

$$\begin{array}{ccc} \mathcal{O}_{aC} & \xrightarrow{\;\pi\;} & \mathcal{O}_{aE} \;\otimes_R\; \mathcal{O}_{a\check{E}} \\ {\scriptstyle \gamma_t}\big\downarrow & & \big\downarrow{\scriptstyle \psi_t\otimes 1,\ 1\otimes\check{\psi}_t} \\ \mathcal{O}_{aC} & \xrightarrow[\;\pi\;]{} & \mathcal{O}_{aE} \;\otimes_R\; O_{a\check{E}}\ . \end{array}$$

The three composite maps in the diagram are calculated as follows:

$$(\psi_t \otimes 1) \circ \pi(Z) \;=\; \sum_{i=0}^{k-1}\sum_{j=0}^{r-1} X_{i+1,j} \otimes Y_{ij}$$

$$= \sum_{i=1}^{k-1}\sum_{j=0}^{r-1} X_{ij} \otimes Y_{i-1,j} \;-\; \sum_{i=0}^{k-1}\sum_{j=0}^{r-1} g_i X_{ij} \otimes Y_{k-1,j} \qquad (\text{by } (5.1.5)^{q^j})$$

$$(5.1.6) \qquad = -\sum_{j=0}^{r-1} X_{0j} \otimes g_0 Y_{k-1,j} + \sum_{i=1}^{k-1}\sum_{j=0}^{r-1} X_{ij} \otimes (Y_{i-1,j} - g_i Y_{k-1,j}) .$$

In view of (5.1.1) and (5.1.2), we find this equal to

$$(1 \otimes \check{\psi}_t) \circ \pi(Z) = \sum_{i=0}^{k-1}\sum_{j=0}^{r-1} X_{ij} \otimes \check{\psi}_t(Y_{ij}) .$$

Finally,

$$\pi \circ \gamma_t(Z) = \theta(\sum_{i=0}^{k-1}\sum_{j=0}^{r-1} X_{ij} \otimes Y_{ij}) + (\sum_{i=0}^{k-1}\sum_{j=0}^{r-1} X_{ij} \otimes Y_{ij})^q$$

$$(5.1.7) \quad = \sum_{i=0}^{k-1}\sum_{j=0}^{r-1} X_{ij} \otimes \theta Y_{ij} + \sum_{i=0}^{k-1}\sum_{j=1}^{r-1} X_{ij} \otimes Y^q_{i,j-1} + \sum_{i=0}^{k-1} X_{ir} \otimes Y^q_{i,r-1} .$$

If $0 \leq i \leq k-2$, we see from (5.1.4)

$$X_{ir} = -a_r^{-1}(\theta X_{i0} + \sum_{j=1}^{r-1} a_j X_{ij} - X_{i+1,0}).$$

For $i = k-1$, we see from (5.1.5a)

$$X_{k-1,r} = -a_r^{-1}(\theta X_{k-1,0} + \sum_{j=1}^{r-1} a_j X_{k-1,j} + \sum_{i=0}^{k-1} g_i X_{i0}).$$

Hence the companion with which X_{ij} is tensored in the above expression (5.1.7) of $\pi \circ \gamma_t(Z)$ is, if $i = j = 0$,

$$\theta Y_{00} - a_r^{-1}\theta Y^q_{0,r-1} - a_r^{-1} g_0 Y^q_{k-1,r-1} = -g_0 Y_{k-1,0} \quad \text{(by (5.1.3))};$$

if $i = 0$ and $1 \leq j \leq r-1$,

$$\theta Y_{0j} + Y^q_{0,j-1} - a_r^{-1} a_j Y^q_{0,r-1} = -g_0 Y_{k-1,j} \quad \text{(by (5.1.2))};$$

if $1 \leq i \leq k-1$ and $j=0$,

$$\theta Y_{i0} \;-\; a_r^{-1}\theta Y_{i,r-1}^q \;+\; a_r^{-1}Y_{i-1,r-1}^q \;-\; a_r^{-1}g_iY_{k-1,r-1}^q$$

$$= \; Y_{i-1,0} \;-\; g_iY_{k-1,0} \quad \text{(by (5.1.3))};$$

if $1 \leq i \leq k-1$ and $1 \leq j \leq r-1$,

$$\theta Y_{ij} \;+\; Y_{i,j-1}^q \;-\; a_r^{-1}a_jY_{i,r-1}^q \;=\; Y_{i-1,j} \;-\; g_iY_{k-1,j} \quad \text{(by (5.1.1))}.$$

Putting all these together, we find $\pi \circ \gamma_t(Z)$ is also equal to (5.1.6).

(iv) We may regard $\mathcal{E}_{aE}$ and $\mathcal{E}_{a\check{E}}$ dual to each other by making (X_{ij}) and (Y_{ij}) the dual bases. Then our construction of the pairing here coincides with the construction in §4, and we have ${}_a\check{E} \simeq {}_aE^*$. □

REMARK (5.3). In what follows, we regard ${}_aE^* = {}_a\check{E}$ by this concrete construction (iv).

PROPOSITION (5.4). *Let $M : E \to F$ be an isogeny of Drinfeld modules (resp. dual Drinfeld modules) over R of rank $r \geq 2$.*
(i) There exists a unique isogeny $\check{M} : \check{F} \to \check{E}$ of dual Drinfeld modules (resp. Drinfeld modules) such that, for all non-zero $a \in A$,

$$\text{(5.4.1)} \qquad {}_a\Pi_E \circ (1 \times \check{M}) \;=\; {}_a\Pi_F \circ (M \times 1) \qquad \textit{on } \; {}_aE \times {}_a\check{F},$$

where ${}_a\Pi_E : {}_aE \times {}_a\check{E} \to {}_aC$ and ${}_a\Pi_F : {}_aF \times {}_a\check{F} \to {}_aC$ are the duality pairings (5.1), (ii) on the a-division points.
(ii) Let $M^* : {}_aF^* \to {}_aE^*$ be the morphism of finite t-modules which $M : {}_aE \to {}_aF$ induces by functoriality of *. Then we have $M^* = \check{M}$ on ${}_aF^* = {}_a\check{F}$ (cf. (5.3)).
(iii) We have canonically $\mathrm{Ker}(\check{M}) = \mathrm{Ker}(M)^*$.

PROOF. We assume E and F are Drinfeld modules; the dual case is proved similarly.

(i) Let $\mathcal{E}_E := \mathrm{Hom}_{\mathbb{F}_q,R}(E,\mathbb{G}_a)$, the $\mathbb{F}_q$-linear homomorphisms defined over R. The rings R and A acts naturally on $\mathcal{E}_E$. It is easy to see, by the explicit form of the defining equation of E, that $\mathcal{E}_E$ is a free $R[t]$-module of rank r. For E and F (resp. $\check{E}$ and $\check{F}$), we use the common symbol

$(X_0, X_1, \cdots, X_{r-1}) = (X, X^q, \cdots, X^{q^{r-1}})$ (resp. $(Y_0, Y_1, \cdots, Y_{r-1})$) for the $R[t]$-basis of $\mathcal{E}_E$ and $\mathcal{E}_F$ (resp. $\mathcal{E}_{\check{E}}$ and $\mathcal{E}_{\check{F}}$), and regard (X_i) and (Y_i) as the dual bases each other (cf. Proof of (5.1)).

An isogeny $M : E \to F$ induces an $R[t]$-module homomorphism $m : \mathcal{E}_F \to \mathcal{E}_E$. Let $\check{m}$ be its transpose; $\check{m}$ is the unique $R[t]$-module homomorphism $\mathcal{E}_{\check{E}} \to \mathcal{E}_{\check{F}}$ such that $\sum_{i=0}^{r-1} m(X_i) \otimes Y_i = \sum_{i=0}^{r-1} X_i \otimes \check{m}(Y_i)$ in $\mathcal{E}_E \otimes_{R[t]} \mathcal{E}_{\check{F}}$. If $m(X_j) = \sum_{h=0}^{r-1} m_{hj} X_h$, $m_{hj} \in R[t]$, then $\check{m}(Y_h) = \sum_{j=0}^{r-1} m_{hj} Y_j$. Clearly $\check{m}$ defines an isogeny $\check{M} : \check{F} \to \check{E}$. We will show $\check{M}$ has the required property.

Fix a non-zero $a \in A$, and let $Z_a = \sum X_{ij} \otimes Y_{ij}$ be the element of $\mathcal{E}_{{}_aE} \otimes_R \mathcal{E}_{{}_a\check{E}}$ and $\mathcal{E}_{{}_aF} \otimes_R \mathcal{E}_{{}_a\check{F}}$ as in the proof of (5.1), (iii) (we use again the symbol Z_a in common for E and F). Then the equality (5.4.1) is equivalent to the equality

$$(5.4.2) \qquad (1 \otimes \check{m})(Z_a) = (m \otimes 1)(Z_a) \qquad \text{in } \mathcal{E}_{{}_aE} \otimes_R \mathcal{E}_{{}_a\check{F}}.$$

The uniqueness of $\check{M}$ follows from this equality, because it determines $\check{m}(Y_i) (\mathrm{mod}\ a\mathcal{E}_{\check{F}})$ for all non-zero $a \in A$.

Let us prove the equality (5.4.2). Recall that $X_{ij} = t^i X_j$ (= abbreviation of $\psi_{t^i}(X_j)$) and $Y_{ij} = b_i Y_j$ (= abbreviation of $\check{\psi}_{b_i}(Y_j)$). If $a = t^k + \sum_{l=0}^{k-1} g_l t^l$ with $g_l \in \mathbb{F}_q$, then by (5.1.1a), we see that $b_i = t^{k-1-i} + g_{k-1} t^{k-2-i} + \cdots + g_{i+1}$. Since m commutes with elements of A, we have

$$\begin{aligned}(m \otimes 1)(Z_a) &= (m \otimes 1) \sum_{i,j} (t^i \otimes b_i)(X_j \otimes Y_j) \\ &= \sum_{i,j} (t^i \otimes b_i)(\sum_{h=0}^{r-1} m_{hj} X_h) \otimes Y_j \\ &= \sum_{h,j} (m_{hj} \otimes 1)(\sum_{i=0}^{k-1} t^i \otimes b_i)(X_h \otimes Y_j).\end{aligned}$$

Similarly,

$$(1 \otimes \check{m})(Z_a) = \sum_{h,j} (1 \otimes m_{hj})(\sum_{i=0}^{k-1} t^i \otimes b_i)(X_h \otimes Y_j).$$

So the coincidence of these two elements is implied by the annihilation of $X_h \otimes Y_j$ by

$$(5.4.3) \qquad (m_{hj} \otimes 1 - 1 \otimes m_{hj}) \sum_{i=0}^{k-1} (t^i \otimes b_i)$$

Since the $\otimes$ is over R and $m_{hj} \in R[t]$, it suffices to prove this for $m_{hj} = t^n$ for all $n \geq 1$. But $t^n \otimes 1 - 1 \otimes t^n$ has the factor $t \otimes 1 - 1 \otimes t$, so we may assume $m_{hj} = t$. In that case, a simple calculation shows that (5.4.3) equals $a \otimes 1 - 1 \otimes a$. This kills $X_h \otimes Y_j$ because we are now working on a-division points.

(ii) is clear from the uniqueness of M^* as shown in (ii-2) of (4.3).

(iii) Take any non-zero $a \in A$ such that $\mathrm{Ker}(M) \subset {}_aE$. Then there exists an isogeny $N : F \to E$ such that $N \circ M = a$ on E and $M \circ N = a$ on F. Restricting the dual maps to a-division points, we have $\mathrm{Ker}(\check{M}) = \mathrm{Im}(\check{N})$ and $\mathrm{Ker}(\check{N}) = \mathrm{Im}(\check{M})$. Applying the exact functor * to the exact sequence

$$0 \longrightarrow \mathrm{Ker}(M) \longrightarrow {}_aE \xrightarrow{M} {}_aF ,$$

we find the sequence

$$0 \longleftarrow \mathrm{Ker}(M)^* \longleftarrow {}_aE^* \xleftarrow{M^*} {}_aF^*$$

exact. Using (ii), we conclude

$$\begin{aligned} \mathrm{Ker}(M)^* &\simeq {}_aE^*/\mathrm{Im}(M^*) = {}_a\check{E}/\mathrm{Im}(\check{M}) \\ &= {}_a\check{E}/\mathrm{Ker}(\check{N}) \simeq \mathrm{Im}(\check{N}) = \mathrm{Ker}(\check{M}). \square \end{aligned}$$

6. Duality for π-divisible groups

Let π be a monic prime element of $A = \mathbb{F}_q[t]$, and let G be a π-divisible group over an A-scheme S of hight h. Thus G is an inductive system $(G_n, i_n)_{n \geq 0}$ of finite v-modules G_n over S with transition maps $i_n : G_n \to G_{n+1}$ such that, for all $n \geq 0$,

(1) G_n is killed by π^n, and of rank $|\pi|^{nh} = q^{nh \cdot \deg(\pi)}$; and

(2) the sequence

$$0 \longrightarrow G_n \xrightarrow{i_n} G_{n+1} \xrightarrow{\pi^n} G_{n+1}$$

is exact.

An anti-equivalent definition can be stated in terms of v-sheaves; we call a projective system $\mathcal{E} = (\mathcal{E}_n, p_n)_{n\geq 0}$ of v-sheaves a *π-adic v-sheaf* on S of hight h if, for all $n \geq 0$,

(1) $\mathcal{E}_n$ is killed by π^n, and of rank $nh \cdot \deg(\pi)$; and

(2) the sequence

$$\mathcal{E}_{n+1} \xrightarrow{\pi^n} \mathcal{E}_{n+1} \xrightarrow{p_n} \mathcal{E}_n \longrightarrow 0$$

is exact.

It is clear that the category of π-divisible groups over S is anti-equivalent to the category of π-adic v-sheaves on S (cf. Proposition (3.3)).

The dual $G^* = (G_n^*, i_n^*)_{n\geq 0}$ of G is defined as follows: G_n^* is the dual of G_n in the sense of §4, and the transition map $i_n^* : G_n^* \to G_{n+1}^*$ is the dual morphism of the surjective morphism $\pi : G_{n+1} \to G_n$. It is clear that G^* is a π-divisible group and has the same hight as G.

Assume now that S is integral and, for all $n \geq 0$, G_n is étale over the generic point of S. Let K^{sep} be a separable closure of the function field K of S. Define two Galois modules $\Phi_\pi(G)$ and $T_\pi(G)$ as usual:

$$\Phi_\pi(G) := \varinjlim_n G_n(K^{\mathrm{sep}}),$$
$$T_\pi(G) := \varprojlim_n G_n(K^{\mathrm{sep}}),$$

where the transition maps are those induced by i_n and π respectively. If A_π denotes the π-adic completion of A, and F_π denotes the fraction field of A_π, then $\Phi_\pi(G)$ is a divisible A_π-module, and $T_\pi(G) = \mathrm{Hom}_{A_\pi}(F_\pi/A_\pi, \Phi_\pi(G))$ is a free A_π-module of rank h. Write C_n for the kernel of π^n on the Carlitz module C. Noticing the compatibility ((4.3), (ii-2)), and passing to the limit as $n \to \infty$ of the pairing ((4.3), (ii-3)): $G_n(K^{\mathrm{sep}}) \times G_n^*(K^{\mathrm{sep}}) \to C_n(K^{\mathrm{sep}})$, inductively with G_n and C_n and projectively with G_n^*, we obtain:

Proposition (6.1). *There exist canonical isomorphisms of Galois modules:*

$$\begin{aligned} T_\pi(G^*) &\simeq \mathrm{Hom}_{A_\pi}(\Phi_\pi(G), \Phi_\pi(C)) \\ &\simeq \mathrm{Hom}_{A_\pi}(T_\pi(G), T_\pi(C)). \end{aligned}$$

Assume now that $S = \mathrm{Spec}\ R$, where R is a complete noetherian local A-algebra such that the structure morphism $\alpha : A \to R$ is injective and $\alpha(\pi)$ is in the maximal ideal of R. As was shown in (1.4) of [6], the category of connected π-divisible groups over R is equivalent to the category of divisible formal A_π-modules over R. The *dimension* of a π-divisible group G over R is defined to be the dimension of the formal A_π-module corresponding to the maximal connected sub-π-divisible group G^0 of G. The following proposition is proved in the same way as Proposition 3 of [7], using Proposition (4.7).

PROPOSITION (6.2). *Let d and d^* be the dimensions of G and its dual G^* respectively. Then we have $d + d^* = h$, the hight of G and G^*.*

References

[1] Anderson, G. W., t-motives, Duke math. J. **53** (1986), 457–502.
[2] Anderson, G. W. and D. S. Thakur, Tensor powers of the Carlitz module and zeta values, Ann. of Math. **132** (1990), 159–191.
[3] Drinfeld, V. G., Moduli variety of F-sheaves, Funktsional'nyi Analiz i Ego Prilozheniya **21** (1987), 23–41.
[4] Hamahata, Y., Tensor products of Drinfeld modules and v-adic representations, Manuscripta Math. **79** (1993), 307–327.
[5] Hayes, D., Explicit class field theory for rational function fields, Trans. A.M.S. **189** (1974), 77–91.
[6] Taguchi, Y., Semi-simplicity of the Galois representations attached to Drinfeld modules over fields of "infinite characteristics", J. of Number Theory **44** (1993), 292–314.
[7] Tate, J., p-divisible groups, in: Proceedings of a conference on local fields, Driebergen, 1966, Springer-Verlag, Berlin-Heidelberg-New York, 1967, 158–183.

(Received November 24, 1992)
(Revised August 4, 1995)

Tokyo Metropolitan University
Hachioji, Tokyo, 192–03
JAPAN
taguchi@math.metro-u.ac.jp