Convergence rates for nonlinear ill-posed problems based on variational inequalities expressing source conditions

BERND HOFMANN

Chemnitz University of Technology, Department of Mathematics,

Reichenhainer Str. 39/41, D-09107 Chemnitz, Germany

Abstract: Twenty years ago ENGL, KUNISCH and NEUBAUER presented the fundamentals of a systematic theory for convergence rates in Tikhonov regularization

$$\|F(u) - v^\delta\|^2 + \alpha\|u - u^*\|^2 \to \min$$

of nonlinear ill-posed problems $F(u) = v$ with solutions $u^\dagger$ and noisy data u^δ in a Hilbert space setting. On the one hand, results are based on structural conditions concerning the nonlinearity of F, in principle Lipschitz continuity of a Fréchet derivative $F'(u)$ in a neighbourhood of $u^\dagger$. On the other hand, source conditions $u^\dagger - u^* = F'(u^\dagger)^* w$ concerning the solution smoothness with some additional smallness assumption on $\|w\|$ are required. In this talk, following the lines of [2], [4] and [5], we show that both nonlinearity and smoothness conditions can be expressed by variational inequalities in a unified manner characterizing the capability of yielding convergence rates. In this context, we also extend the ideas of approximate source conditions presented in [1] for linear ill-posed problems to the nonlinear case. For handling the different types of nonlinearity we adapt the concept of a degree of nonlinearity which was originally developed in [3]. To extend the results to a Banach space setting we use Bregman distances for measuring the regularization error.

References:

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