CONTROLLABILTY OF THE HEAT EQUATION IN A STRATIFIED MEDIA : A CONSEQUENCE OF ITS SPECTRAL STRUCTURE

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Abstract. We suppose that $\Omega' \subset \mathbb{R}^{n-1}$ is a bounded regular open set and that ω is a small open set such that $\omega \subset\subset \Omega := \Omega' \times (0, H)$. We introduce the function $\Omega \ni (x', x_n) \to c(x_n) \in BV(0, H), c, c^{-1} \in L^\infty, c > 0$, and we consider the following system

$$\begin{cases} \partial_t q - \nabla.c\nabla q = 1_\omega u \text{ in } (0, T) \times \Omega, \\ q(t, x) = 0 \text{ if } t \in (0, T),\ x \in \partial\Omega, \\ q(0, x) = q_0(x) \text{ in } \Omega. \end{cases}$$

We will explain, that for arbitrary time $T > 0$ and initial condition $q_0 \in L^2(\Omega)$, there exists $u \in L^2((0, T) \times \Omega)$ such that the corresponding solution q satifies $q(T) = 0$ a.e. in Ω. In order to obtain this result we need the coefficient $c \in BV(0, H)$ is also of class C^1 in a some nonempty open subset ω_n such that there exists $\omega' \neq \emptyset, \omega' \times \omega_n \subset \omega$. The proof uses the spectral properties of the selfadjoint operator $\nabla.c\nabla$.
We will give some nonlinear extensions.

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