# The Semi-Wave Solutions of the KPP Equations with Free Boundaries in Almost Periodic Media.

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Consider the following diffusive KPP equation with a free boundary,

$$\begin{cases} u_t = u_{xx} + u(a(x) - u), & -\infty < x < h(t) \\ u(h(t), t) = 0, h'(t) = -\mu u_x(h(t), t) \end{cases} \tag{0.1}$$

where $\mu > 0$, $a(x)$ is a positive alomost periodic function in $x \in \mathbb{R}$. Here, as in previous works, we use the concept "semi-wave" to replace "traveling wave" since the profile function of the wave is only defined on the half real line $(-\infty, 0]$.

**Definition 0.1.** *Let $(u, h) = (u(x, t), h(t))$ be one positive entire solution of (0.1). If $u$ can be written as $u(x, t) = v(x - h(t), h(t))$, where $h(\pm\infty) = \pm\infty$, $v(\xi, \tau) \in C^2((-\infty, 0] \times \mathbb{R})$ , $v(\cdot, \tau)$ is an almost periodic function in $\tau$ from $\mathbb{R}$ to $C((-\infty, 0]) \cap L^\infty((-\infty, 0))$, then $u$ is called an almost periodic traveling wave solution.*

In my work, I prove the existence and uniqueness of the semi-wave solutions.

# References

[1] H.Berestycki and G. Nadin, Spreading speeds for one-dimensional monostable reaction-diffusion equations, *J. Math. Physics*, **53**(2012),115619.

[2] Y. Du and Z. Lin, Spreading-vanishing dichotomy in the diffusive logistic model with a free boundary, *SIAM J. Math. Anal.*, **42** (2010), 377-405.

[3] Y. Du and X. Liang, Pulsating semi-waves in periodic media and spreading speed determined by a free boundary model, *Ann. I. H. Poincaré-AN.*, **32** (2015), no. 2, 279-305.

[4] H. Matano, Traveling waves in spatially inhomogeneous diffusive media : The non-periodic case, preprint.