Spreading and extinction in a multidimensional shifting environment

Thomas Giletti, University of Lorraine

In this talk we will consider a heterogeneous reaction-diffusion equation in a multidimensional or cylindrical domain, where the reaction term depends on a moving variable $x - ct$. More precisely, we study the large time behaviour of solutions of the following Cauchy problem

$$\begin{cases} \partial_t u = \Delta u + f(x - ct, y, u), \\ u(t = 0) = u_0, \end{cases}$$

in the spatial domain $(x, y) \in \mathbb{R} \times \omega$, where $\omega \subset \mathbb{R}^{N-1}$ and $N \geq 1$.

Such an equation arises in the modelling of the effect of climate change on species ranges in ecology. In such a context, the unknown u stands for a population density, and the parameter $c > 0$ stands for the climate velocity in the x-direction (the variable x may correspond for instance to the latitude). Our key assumptions will be that the reaction term is increasing with respect to $x-ct$, and that the limit of $f(x-ct, y, u)$ as $x-ct \to +\infty$ (respectively $x - ct \to -\infty$) is a monostable function of u (respectively a negative function of u). This means that the favourable habitat zone is receding to the right with a constant speed as time passes.

We will see that, under the joint influence of the heterogeneity and a weak Allee effect, whether the solution spreads (i.e. converges to a positive steady state which is close to 1 in the favourable zone) depends not only on the shfiting parameter c but also on the size of the initial datum u_0. This is in sharp contrast with both the KPP (no Allee effect) equation in a shifting environment [2], and with the monostable equation in a homogeneous environment [1], where a hair-trigger effect typically appears. We will further analyze the situation by showing the existence of several sharp thresholds between two reasonable outcomes, namely extinction (i.e. convergence to 0) and spreading of the solution, in terms of both the climate velocity and the initial datum. This is a joint work with Juliette Bouhours.

References

[1] D. G. Aronson, H. F. Weinberger, Multidimensional nonlinear diffusion arising in population genetics, Adv. in Math. 30 (1978), 33-76.

[2] H. Berestycki, O. Diekmann, C. J. Nagelkerke, P. A. Zegeling, Can a species keep up with a shifting climate?, Bull. Math. Biol. 71 (2009), 399-429.