

教授 (Professor)

小林 俊行 (KOBAYASHI Toshiyuki)

A. 研究概要

2023 年度の専攻長・数学科長と 2024 年度の副研究科長の業務で研究活動は限定されたものになったが、2020–2024 の 5 年間に於いては、主に以下の 3 テーマで約 1,000 ページの論文を著した。

1. 緩増加等質空間

リー群 G が作用する空間 X に対して、正則表現 $L^2(X)$ がいつ緩増加になるかという問題を提起した (Y. Benoist と共同) [J. Euro. Math. 2015]. まず、力学系の手法を用いて、簡約型等質空間 X $L^2(X)$ の行列要素の L^p 評価を与え、次に、一般の X に対して、群作用をもつ測度空間に新しい半順序を導入し、ユニタリ表現論の手法を援用して緩増加性の幾何的判定法を確立した (第 2 論文 [Chicago Univ. Press, 2022]). 第 3 論文 [J. Lie Theory, 2022] で非緩増加な実簡約型等質空間の完全な分類を与え、第 4 論文 [J. inst. Jussieu, 2023] では、緩増加性という解析的な性質、リー代数の極限に関する位相的性質、余随伴軌道の幾何的性質、凸多面体の組合せ論的性質の間の同値性を発見し、証明した。さらにテンソル積表現の応用を [J. Alg., 2023] で著した。

2. 対称性破れ作用素の構成と分類問題

簡約リー群の無限次元表現の「分岐則」に関して、定性的理論から定量的理論に深化させるプログラムを提起した ([8]).

2.A. (定性的理論) 離散的分岐則の理論の要となる K' -admissibility の十分条件 ([Ann. Math., 1998] の主定理の 1 つ) が、実は必要十分条件であることをシンプレクティック幾何の手法を用いて証明を与えた [Kostant 追悼論文, 2021].

2.B. (定性的理論 2) 分岐則の重複度が有限・一様有界になるための判定条件を無限次元表現の“サイズ”の言葉を用いて与え、特に、 H -distinguished な G の表現が部分群 G' に関して有界重複度をもつ 3 つ組 (H, G, G') を可視的作用の理論を援用して分類した ([JLT, 2023] 他).

2.C. (定性的理論 3) 有界重複度をもつ群の組に対し **fence** という概念を導入し、分岐則の重複度が **fence** を越えない場合に一定であるという定理を証明し、いくつかの応用を見出し始めた [5, 7].

2.D. (定量的理論 1—対称性破れ作用素) 無限次

元表現に対する対称性破れ作用素を、非局所作用素まで含めて構成するプログラムを提唱し、その手法を開発し、最初の重要な例として、総計 650 頁の長編論文で共形平坦な部分多様体の微分形式に関する対称性破れ作用素の分類を完成させた

2.E. (定量的理論 2—ホログラフィック変換) 対称性破れ作用素の族の双対として“ホログラフィック変換”の概念を導入し [Ann. Inst. Fourier, 2020], 擬リーマン空間形の離散系列表現の分岐則における離散スペクトラムをホログラフィック変換で構成し、決定した [Adv. Math., 2021].

2.F. (微分対称性破れ作用素の母関数) 標題の新しい概念の研究を開始した (文献 [1, 2, 3, 6]).

3. 不連続群

筆者の長年のモチーフである「リーマン幾何学の枠組を越えた不連続群」において、スペクトル理論の構築に踏み込んだ。幾何学的な準備として、離散群の作用の不連続性を量的に評価する sharpness という概念を導入し、高次元タイヒミュラー空間上で安定な離散スペクトラムを構成した [Adv. Math.]. さらに、第 2 論文 [JLT2019] および [Progr. Math. 2017] で隠れた対称性を表す微分作用素環の構造定理を与え、それを土台として無限次元表現の分岐則理論を援用し、標準的な擬リーマン局所対称空間に対するスペクトル分解定理の証明を完成させた [書籍 1, 2025].

For the last five years (2020-2024), I have been working on the following research topics.

1. Tempered homogeneous spaces

As a challenge to the global analysis on homogeneous spaces beyond symmetric spaces, I collaborated with Y. Benoist in 4 papers [J. Euro. Math. '15]-[J. Inst. Jussieu, 2023], and proved a geometric criterion for temperedness of the regular representation on G/H . A complete description of nontempered reductive homogeneous spaces G/H has been accomplished, and further connections with other disciplines of mathematics has been explored.

2. Restriction of representations: symmetry breaking operators

Branching problems study the restriction of irreducible representations to subgroups. I proposed a program in [Progr. Math., 2015] to ad-

vance branching problems for reductive groups, see [8] for further perspectives.

2.A Concerning the discreteness of the restriction of representations, I proved in [Kostant Memorial, 2021] using symplectic geometry, the converse of one of the main theorems in my earlier paper [Ann. Math., 1998] based on microlocal analysis.

2.B I formulated and proved a criterion for finite multiplicity/bounded multiplicity of the restriction of ‘small’ infinite-dimensional representations to reductive subgroups in a sequence of papers, including [JLT2023]. In particular, I established a classification of the triples (G, H, G') such that (G, H) is a symmetric pair and that any irreducible H -distinguished representations have bounded multiplicity when restricted to another symmetric pair (G, G') .

2.C With B. Speh, I classified *symmetry breaking operators* (SBOs) of principal series for a pair of Lorentz groups (Memoirs of AMS 2015 and [Lect. Notes Math.(2018)]), as the first complete classification of SBOs. Further, we introduced a concept **fence** to analyze the behavior of multiplicity under translations [5, 7].

2.D-F As an “inversion” of symmetry breaking, I introduced the concept of **holographic transform** in [Adv. Math. 2021] and in a joint paper with Pevzner ([Ann. Inst. Fourier 2020]). We also developed the concept of **generating operators for SBOs** in [1, 2, 3, 6].

3. Analysis on locally symmetric spaces—beyond the Riemannian case

Developing my long-standing interest in discontinuous groups beyond the Riemannian case, I initiated the study of global analysis on locally non-Riemannian symmetric spaces with F. Kassel in [Adv. Math] and proved the existence of “stable spectrum” under small deformation of discontinuous groups. Building on the structure theorem of three algebras of invariant differential operators [Progr. Math. ’17], [JLT2019], established a spectral theory on standard quotients $\Gamma \backslash G/H$ in [Book 2025].

B. 発表論文

(論文は 2024 年以降のものを記載する。2023 年以前の論文は、過去の Annual Report の各年度に記載。)

1. T. Kobayashi and M. Pevzner, A generating operator for Rankin–Cohen brackets, *Journal of Functional Analysis*, 24 pages. published on line, 18 March, 2025. DOI: 10.1016/j.jfa.2025.110944.
2. T. Kobayashi, Generating operators of symmetry breaking-from discrete to continuous, *Indagationes Mathematicae*, **36**, (2025), pp. 631-643.
3. T. Kobayashi and M. Pevzner, A short proof for Rankin-Cohen brackets and generating operators, “Lie Theory and its Applications”, Springer Proc. Math. Stat., vol 473. pages 3-15, Springer. 2025.
4. T. Kannaka and T. Kobayashi, Zariskidense deformations of standard discontinuous groups for pseudo-Riemannian homogeneous spaces, 94 pages. to appear.
5. T. Kobayashi and B. Speh, How does the restriction of representations change under translations? – A story for the general linear groups and the unitary groups, 39 pages. arxiv:2502.08479.
6. T. Kobayashi and M. Pevzner, Generating operators and normal derivatives. In Y. Tanaka, editor, 表現論と調和解析のひろがり (Expansion in Representation Theory and Harmonic Analysis), **2297** in RIMS Kôkyûroku, pages 1–15, 2024.
7. M. Harris, T. Kobayashi, and B. Speh, Translation functors and restriction of coherent cohomology of Shimura varieties. In preparation.
8. T. Kobayashi, Recent advances in branching problems of representations. *Sugaku Expositions*, **37**:129–177, 2024. DOI: 10.1090/suga/485. Published electronically: October 23, 2024. Amer. Math. Soc.; a translation by Toshihisa Kubo of

- the Japanese original article.
9. T. Kobayashi, Lie groups and Lie algebras: (3) Lie theory. Sugaku Seminar, **752**(6):52-57, 2024.
 10. T. Kobayashi, Lie groups and Lie algebras: (2) Lie algebras. Sugaku Seminar, **751**(5):49-55, 2024.
 11. T. Kobayashi, Lie groups and Lie algebras: (1) Lie groups. Sugaku Seminar, **750**(4):52-57, 2024.

著書:

1. F. Kassel and T. Kobayashi. Spectral Analysis on Standard Locally Homogeneous Spaces, Lecture Notes in Mathematics, **2367**, xi+116 pages. 2025. Springer. <https://doi.org/10.1007/978-981-96-1957-3>
2. 小林俊行, 地力をつける 微分と積分, 岩波書店, 2024 年. 278 pages. ISBN 9784000058896

C. 口頭発表

1. (分岐則における translation と fence の概念の導入) (**1.A** から **1.C** は講演タイトル, 内容は個々に異なるが, 大きなテーマとしては繋がっているので 1 つにまとめる.) **1.A.** Restricting representations of real reductive groups to reductive subgroups, Séminaire Groupes Réductifs et Formes Automorphes, France, 17 March 2025. **1.B.** Symmetry Breaking under Translations. AIM-IHP seminar. Institut Henri Poincaré, France, 11 February 2025. **1.C.** Symmetry Breaking and Translation Functor. NCTS Workshop on Representation Theory and Lie Groups. Taiwan, 11-14 December 2024.
2. (不連続群のスペクトル解析と無限次元表現論をつなげる概説講演) (**2.A** と **2.B** は講演タイトルは異なるが, 大きなテーマとしては繋がっているので 1 つにまとめる.) **2.A.** Non-commutative Harmonic Analysis, Branching Problems, and Dis-

continuous Groups. The 7th Tunisian-Japanese Conference: Geometric and Harmonic Analysis on Homogeneous Spaces and Applications in Honor of Professor Toshiyuki Kobayashi. Monastir, Tunisia, 31 October-4 November 2023. **2.B.** On the Crossroads of Global Analysis and Representation Theory. Geometry, Analysis, and Representation Theory of Lie Groups. In Honour of Professor Toshiyuki Kobayashi (organized by Y. Oshima, H. Sekiguchi, T. Kubo, T. Okuda, Y. Tanaka, and M. Kitagawa). The University of Tokyo, 5-9 September 2022.

3. (対称性破れ作用素の母作用素の理論) (**3.A.**-**3.C.** は内容は異なるが, 大きなテーマとしては繋がっているので 1 つにまとめる.) **3.A.** 29th Nordic Congress of Mathematicians with EMS. Aalborg, Denmark, 3-7 July 2023. **3.B.** (Plenary Lecture) International Workshop Lie Theory and Its Applications in Physics (LT-15). Varna, Bulgaria, 19-25 June 2023. **3.C.** Holographic Transform, 20 August, 2021, Workshop on "Actions of Reductive Groups and Global Analysis (Online Tambara), August 17-21, 2021.
4. (可視的作用と無重複性理論) "Visible actions" and "only one" — Geometric structure that produces multiplicity-free representations. 東京大学大学院数理科学研究科設立 30 周年記念講演 (The 30th Anniversary Ceremony of the Foundation of the Graduate School of Mathematical Sciences). The University of Tokyo, 15 October 2022.
5. (分岐則の重複度理論と離散的分解: Multiplicities and discrete decomposability for the restriction.) (**5.A.**-**5.E.** では講演タイトル, 内容は異なるが, 大きなテーマとしては繋がっているので 1 つにまとめる.) **5.A.** Branching in Representation Theory. Minicourses: branching problems and symmetry-breaking. Insti-

- tut Henri Poincaré, France, 13–17 January 2025. (three lectures). **5.B.** Harish-Chandra’s admissibility theorem and beyond. 18th Discussion Meeting in Harmonic Analysis (In honour of centenary year of Harish Chandra). IIT Guwahati, India, 18–21 December 2023. **5.C.** Harish-Chandra’s admissibility theorem and beyond. Harish-Chandra Centenary Celebrations 2023: Conference on Harish-Chandra. Harish-Chandra Research Institute (HRI) in Allahabad, India, 9–14 October 2023. **5.D.** Representations and Characters: Revisiting the Works of Harish-Chandra and André Weil — A satellite conference of the virtual ICM 2022 (organized by Hung Yean Loke, Tomasz Przebinda, Angela Pasquale, and Binyong Sun). the Institute for Mathematical Sciences, National University of Singapore, Singapore, 9 July 2022. **5.E.** Bounded multiplicity in the branching problems of “small” infinite-dimensional representations, 5 October 2021. リー群論・表現論セミナー (オンライン), 東京大学.
6. (固有な作用・不連続群と表現論: Proper Actions and Representation Theory.) (**6.A.**–**6.I.** では講演タイトル, 内容は個々に異なるが, 大きなテーマとしては繋がっているので1つにまとめる.) **6.A.** Local to Global in Non-Riemannian Geometry, **6.B.** Properness Criterion and its Quantification, **6.C.** Global Analysis on Locally Symmetric Spaces Beyond the Riemannian Case (**6.A.**, **6.B.**, **6.C.** は 2024 年 1 月 1 日 ~3 日に行った Global Analysis of Locally Symmetric Spaces with Indefinite-metric というテーマの 3 回の連続講演. Zariski Dense Subgroups, Number Theory and Geometric Applications. ICTS, Bangalore, India, 1–12 January 2024.) **6.D.** Properness criterion. **6.E.** Discontinuous group, Weil’s local rigidity, and deformation. **6.F.** Tempered Subgroups and tempered homogeneous spaces. (**6.D.**, **6.E.**, **6.F.** は Representations and Characters: Revisiting the Works of Harish-Chandra and André Weil — A satellite conference of the virtual ICM 2022 の 4 連続講演のうちの 3 つ. The Institute for Mathematical Sciences, National University of Singapore, Singapore, 1–15 July 2022. organized by Hung Yean Loke, Tomasz Przebinda, Angela Pasquale, and Binyong Sun). **6.G.** Discontinuous dual and properness criterion (25 April, 2022) **6.H.** The Mackey analogy and proper actions (2 May, 2022) (**6.G.**, **6.H.** は Proper Actions and Representation Theory. Mini-courses of Mini-lectures (Organizers: Pierre Clare, Nigel Higson and Birgit Speh) における 4 つのテーマの講演の 2 つ, AIM Research Community: Representation Theory & Noncommutative Geometry, online), 25 April–16 May 2022. **6.I.** A Program for Branching Problems in the Representation Theory of Real Reductive Groups: Classification Problem of Symmetry Breaking Operators. Representation Theory inspired by the Langlands Conjectures, in connection with the AMS-AWM Noether lecture by Birgit Speh. Denver, USA, 17 January 2020.
7. (緩増加空間の理論: Tempered Homogeneous Spaces.) (**7.A.**–**7.N.** では講演タイトル, 内容は個々に異なるが, 大きなテーマとしては繋がっている所以で1つにまとめる.) **7.A.** Structure of Tempered Homogeneous Spaces. III. Limit Algebras. Minicourses. Institut Henri Poincaré, France, 21 February 2025. (Day 3 of the three lectures). **7.B.** Structure of Tempered Homogeneous Spaces. II. Combinatorics Approach. Minicourses. Institut Henri Poincaré, France, 19 February 2025. (Day 2 of the three lectures). **7.C.** Structure of Tempered Homogeneous Spaces. I. Dynamical Approach. Mini-

courses. Institut Henri Poincaré, France, 17 February 2025. (Day 1 of the three lectures). **7.D.** Tempered homogeneous spaces and tempered subgroups — Dynamical approach **7.E.** Classification theory of non-tempered G/H — Combinatorics of convec polyhedra **7.F.** Tempered homogeneous spaces — Interaction with topology and geometry (**7.D.**, **7.E.**, **7.F.** は Harish-Chandra's Tempered Representations and Geometry. 18th Discussion Meeting in Harmonic Analysis (In honour of centenary year of Harish-Chandra): Workshop. IIT Guwahati, India, 12-16 December 2023) における 4 連続講演のうちの 3 つ) **7.G.** Tempered subgroups à la Margulis (9 May, 2022) **7.H.** Tempered homogeneous spaces (16 May 2022) (**7.G.**, **7.H.** は Proper Actions and Representation Theory. Mini-courses of Mini-lectures (Organizers: Pierre Clare, Nigel Higson and Birgit Speh) における 4 連続講演のうち 2 つ, AIM Research Community: Representation Theory & Noncommutative Geometry, (online), 25 April-16 May 2022. **7.I.** Symmetry in Geometry and Analysis, In honour of Professor Toshiyuki Kobayashi (organized by M. Pevzner and H. Sekiguchi). Reims University, France, 6–10 June 2022. **7.J.** 緩増加な等質空間 (Tempered Homogeneous Spaces). 日本数学会年会函数解析学分科会特別講演 (慶応大学, オンライン), 16 March 2021. **7.K.** Limit Algebras and Tempered Representations. (opening lecture). RIMS Workshop: Lie Theory, Representation Theory and Related Areas. (online), 10 August 2021. **7.L.** Limit Algebras and Tempered Representations. (plenary opening lecture). XIV. International Workshop: Lie Theory and Its Applications in Physics. Bulgaria (online), 20–26 June 2021. **7.M.** Limit algebras and tempered representation. Lie Groups and Representation Theory Seminar. The Univer-

sity of Tokyo, 15 June 2021. **7.N.** This is What I do: Limit algebras and tempered representations. Representation Theory & Noncommutative Geometry. AIM Research Community (online), 8 April 2021.

8. 極小表現をモチーフとする大域解析 Schrödinger model of minimal representations and branching problems. Minimal Representations and Theta Correspondence: (Gordan Savin 教授還暦記念研究集会). (online), The Erwin Schrödinger International Institute for Mathematics and Physics (ESI), 11–15 April 2022.
9. (等質空間上の大域解析・実球等質空間の一般理論: Regular Representations on Homogeneous Spaces.) (**9.A.**–**9.K.** では講演タイトル, 内容は個々に異なるが, 大きなテーマとしては繋がっているので 1 つにまとめる.) **9.A.** Basic Questions in Group-Theoretic Analysis on Manifolds. Introductory school at CIRM (Marseille): Methods in representation theory and operator algebras. Marseille, France, 6–10 January 2025. **9.B.** Basic Questions in Group-Theoretic Analysis on Manifolds. Colloquium at Hiroshima University. Hiroshima, Japan, 22 October 2024. **9.C.** Overview of Branching Problems in the Real Setting. Branching Problems for Representations of Real, p -Adic and Adelic Groups. the University of British Columbia - Okanagan, Kelowna, Canada. BIRS, 7–12 July 2024. **9.D.** Is representation theory useful for global analysis on a manifold? — Multiplicity: Approach from PDEs, Harish-Chandra's Tempered Representations and Geometry IIT Guwahati, India, 12-16 December 2023. (Harish-Chandra 生誕 100 年を記念した 4 回の連続講義の第 1 回目) **9.E.** Is representation theory useful for global analysis on a manifold? — Multiplicity: Approach from PDEs, **9.F.** Tempered homogeneous spaces and tempered subgroups — Dynam-

- ical approach, **9.G.** Classification theory of non-tempered G/H — Combinatorics of convex polyhedra, **9.H.** Tempered homogeneous spaces — Interaction with topology and geometry, (**9.E.**, **9.F.**, **9.G.**, **9.H.** は Analysis on Homogeneous Spaces における 4 回連続講演, Noncommutative Geometry and Analysis on Homogeneous Spaces. Williamsburg, USA, 16–20 January 2023.) **9.I.** Basic Questions in Group-Theoretic Analysis on Manifolds. MATH-IMS Joint Pure Mathematics Colloquium Series. The Chinese University of Hong Kong, 25 November 2022. **9.J.** A Foundation of Group-theoretic Analysis on Manifolds. Colloquium di dipartimento. Dipartimento di Matematica, Università di Roma “Tor Vergata” (online), 18 February 2021. **9.K.** Representation Theory of Reductive Groups from Geometric and Analytic Methods (in honour of Simon Gindikin). Kavli IPMU, Japan, 27–28 January 2020;
10. (擬リーマン対称空間におけるスペクトル解析: Global Geometry and Analysis on Locally Symmetric Spaces—Beyond the Riemannian Case.) (**10.A.**–**10.E.** では講演タイトル, 内容は個々に異なるが, 大きなテーマとしては繋がっているので 1 つにまとめる.) **10.A.** Hidden Symmetry and Spectral Analysis on Locally Pseudo-Riemannian Symmetry Spaces. Intertwining Operators and Geometry during the thematic trimester Representation Theory and Noncommutative Geometry. Institut Henri Poincaré, France, 20–24 January 2025. **10.B.** Branching Problems and Global Analysis of Locally Symmetric Spaces with Indefinite-Metric. Seminar at University of Trento. Trento, Italy, 19 September 2024. **10.C.** Branching Problems and Global Analysis of Locally Symmetric Spaces with Indefinite-Metric. International Congress of Basic Science. Beijing, China, 25 July 2024. **10.D.** Global Analysis of Locally Symmetric Spaces with Indefinite-metric. Colloquium, National University of Singapore. (online), 13 August 2021. **10.E.** Sound of an anti-de Sitter manifold. (opening lecture). Inaugural Day of the French–Kazakhstan school of Mathematics. (online), 25 June 2021.
- D. 講義
1. 数理学概論 I (文科生) (教養学部文科 1,2 年生, 2024 年度 A セメスター, 対面): フェルミ推定, 微積分, Taylor 展開, 偏微分, Lagrange の未定乗数法, 近似と概算, 微分方程式の初歩, 多変数関数の積分を講義した.
 2. 群構造論/幾何学 XF (数理大学院・4 年生共通講義, 2024 年度 S セメスター, 対面): リーマン多様体と擬リーマン多様体における等長変換群からなる離散群の作用に関して、不連続性の差異の説明を多くの例を交えておこなった。さらに、リー群の構造論 (簡約群、半単純リー群、Cartan 分解、リーマン対称空間、放物型部分群) を説明した後、等質空間への部分群の作用が固有であるかどうか (特に、作用が固有不連続であるかどうか) についての判定条件 (Properness Criterion) を原論文 (T. Kobayashi, Math. Ann. 1989 および J. Lie Theory 1996) の鍵となるアイデアを説明した。さらに、非リーマン等質空間への不連続群の作用が余コンパクトになるための判定条件を群のコホモロジーを用いて説明した。また、非リーマン等質空間における不連続群の剛性と変形理論 (高次元タイヒミュラー空間) に関する話題を紹介した。
- 参考文献
- 小林俊行「非リーマン等質空間の不連続群論」『数学の最先端 21 世紀への挑戦 I』, 丸善出版, 2002, pages 18-73 (邦訳増補版).
- T. Kobayashi, Conjectures on reductive homogeneous spaces, Mathematics Going Forward: Lecture Notes in Mathematics **2313**, pages 217–231. Springer, 2023.
3. 広島大学集中講義, 等質空間上の幾何と解析, 2024 年 10 月 21 日–25 日.

4. (フランスでの連続講義 1 : 対称性の破れ作用素の理論) Branching in Representation Theory. Minicourses: branching problems and symmetry-breaking. Institut Henri Poincaré, France, January 2025.

The concept of symmetries naturally arises in various areas of mathematics and science, including geometry, number theory, differential equations, and quantum mechanics. The more symmetries an object possesses, the better we can understand it through group-theoretic approaches.

Branching problems investigate how large symmetries break down into smaller ones, such as fusion rules, using mathematical formulations based on the language of representations and their restrictions. These problems have been studied for over 80 years. In recent years, there has been a surge of research focused on the restriction of continuous symmetries in infinite-dimensional cases, leading to the development of new geometric and analytic methods.

I provided an introduction to the branching problems of infinite-dimensional representations of real reductive groups, such as $GL(n, \mathbb{R})$, using plenty of elementary examples to make the basic concepts and key ideas more accessible.

5. (フランスでの連続講義 2 : 緩増加空間の理論) Structure of Tempered Homogeneous Spaces Institut Henri Poincaré, France, February 2025.

The lectures introduce recent theories of tempered spaces, and I plan to provide an overview of these topics, using plenty of elementary examples to make the basic concepts and key ideas more accessible.

1. I will review basic concepts such as tempered unitary representations of real reductive groups, like $GL(n, \mathbb{R})$, as well as “tempered” spaces and “tempered subgroups”. I will begin with some geomet-

ric observations of group actions, including the properness criterion for reductive homogeneous spaces. Subsequently, I will introduce a “quantification” of proper actions and incorporate a dynamical approach into analytic representation theory, including the temperedness criterion for homogeneous spaces, which was developed recently by Y. Benoist and the speaker, drawing on the Cowling-Haagerup-Howe theory and other related ideas.

2. The criterion for tempered spaces, explained in the first lecture, is computable. In this lecture, I will explain how this criterion leads to the classification theory of non-tempered reductive homogeneous spaces by breaking it down into several steps. The technical methods used in the second lecture differ from the dynamical approach presented in the first lecture. Our approach relies on elementary results from finite-dimensional representations and some combinatorics of convex polyhedral cones.

3. Recently, surprising and intriguing connections have been observed between the concept of “tempered spaces for unitary representations and various other areas of mathematics. In this lecture, we will explore different aspects of tempered spaces from the perspectives of topology and geometry, including limit algebras (collapsing Lie algebras) and geometric quantization.

6. 数学講究 XB (数理科学概論)「対称性の数学」, (理学部数学科 4 年生), 2024 年 5 月 14 日.
7. Striving for Universal Truth: Discovery and Creation. The 138th Public Lecture Series at the University of Tokyo (Spring 2024): Constraints and Creativity in the Arts and Sciences. Yasuda Auditorium, The University of Tokyo, 22 June 2024.
8. (インドでの講義) Global Analysis of Locally Symmetric Spaces with Indefinite-

Metric. Zariski Dense Subgroups, Number Theory and Geometric Applications. ICTS, Bangalore, India, 1–12 January 2024 (1月1日から3日にわたる3回の連続講義). Summary: The local to global study of geometries was a major trend of 20th century geometry, with remarkable developments achieved particularly in Riemannian geometry. In contrast, in areas such as pseudo-Riemannian geometry, familiar to us as the space-time of relativity theory, and more generally in pseudo-Riemannian geometry of general signature, surprising little was known about global properties of the geometry even if we impose a locally homogeneous structure. This theme has been developed rapidly in the last three decades. In the series of lectures, I discussed two topics by the general theory and some typical examples.

(a) Global geometry: Properness criterion and its quantitative estimate for the action of discrete groups of isometries on reductive homogeneous spaces, existence problem of compact manifolds modeled on homogeneous spaces, and their deformation theory.

(b) Spectral analysis: Construction of periodic L^2 -eigenfunctions for the Laplacian with indefinite signature, stability question of eigenvalues under deformation of geometric structure, and spectral decomposition on the locally homogeneous space of indefinite metric.

E. 修士・博士論文

1. (修士) 宮内俊輔 (MIYAUCHI Shunsuke): 粗幾何学における従順性及びアファイン保測変換における連結固有対の分類問題.

F. 対外研究サービス

1. 日仏数学連携拠点 FJ-LMI (French-Japanese Laboratory of Mathematics and its Interactions), 日本側代表 (co-director) (2023年9月1日–).

2. Kavli IPMU(数物宇宙連携機構), 上席科学研究員併任 (2009.8–2011.5); 主任研究員 (Principal Investigator) 併任 (2011.6–2022.3); 連携研究員 (2022.4–).

[ジャーナルのエディター]

3. Editor in Chief, Japanese Journal of Mathematics (日本数学会, Springer-Nature) (2006–)
4. Editor, International Mathematics Research Notices (Oxford 大学出版) (2002–2021)
5. Editor in Chief, Takagi Booklet, vol. 1–22 (日本数学会) (2006–)
6. Editor, Geometriae Dedicata (Springer) (2000–)
7. Editor, Advances in Pure and Applied Mathematics (de Gruyter) (2008–)
8. Editor, International Journal of Mathematics (World Scientific) (2004–)
9. Editor, Journal of Mathematical Sciences, The University of Tokyo (2007–)
10. Editor, Kyoto Journal of Mathematics (2010–)
11. Editor, Representation Theory (アメリカ数学会) (2015–2019)
12. Editor, AMS Translation Series (アメリカ数学会) (2016–)
13. Editor, Tunijian Journal of Mathematics (2017–)
14. Editor, Special Issue in commemoration of Professor Kunihiko Kodaira's centennial birthday (J. Math. Sciences, The University of Tokyo).
15. Editor, Special Issue in honor of Professor Masaki Kashiwara's 70th birthday (Publ. RIMS) 2017–2021.
16. Chief Editor, Mikio Sato's Collected Papers, (Springer-Nature).
17. 共立出版, 『共立講座 数学探検 (全18巻)』, 『共立講座 数学の魅力 (全14巻 + 別巻1)』, 『共立講座 数学の輝き (全40巻予定)』の3シリーズ編集委員
18. 編集委員, 数学の現在 i, e, π , (with 斎藤毅, 河東泰之), 東京大学出版会.

[学会・他大学の委員など]

19. ある国際賞 (数学部門) の授賞委員会: Prize Committee (mathematics), 2020–2021, 国外.
20. ある国際賞の授賞委員会: Prize Committee (mathematics), 2019 および 2020, 国外.
21. ICM2022 における招待講演者および Plenary lecturers の Selection に関する国際委員会の議長 (Chair, Lie Theory and its generalizations, ICM2022), 2019–2022.
22. 国際数学者賞の授賞委員会: Prize Committee (International Prize, 数学部門) 2018, 国外.
23. ICM2030 招致招致委員 (2025–).
24. 国内の数学の賞 (複数) の授賞委員会 (anonymous, various years).
25. 京都大学数理解析研究所運営委員 (2015–2017; 2017–2019).
26. 京都大学数理解析研究所専門委員 (2007–2009; 2009–2011; 2015–2017; 2017–2019; 2021–2023).
27. 科学研究費等の審査委員: 日本 (JSPS), 米国 (NSF-AMS), EU, ドイツ, ルクセンブルク, 中華人民共和国・香港 (various years).
28. OIST (沖縄科学技術大学大学院) の数学部門に対する国際 Advisory Board Member (2021–).

[国際研究集会のオーガナイザーなど]

29. オーガナイザー, 日仏数学連携拠点開設記念コンファランス (Opening Conference of French-Japanese Laboratory of Mathematics and its Interactions), April 4–5, 2024 (with M. Pevzner).
30. オーガナイザー, Periods and Branching Problems for Representations of Real, p -adic and Adelic Groups, BIRS Conference, Canada, July 2024 (with M. Pevzner, B. Speh).
31. オーガナイザー, Summer School on Representation Theory, リー群の群作用と大域解析に関するセミナー, 玉原国際セミナーハウス, 19–23 August 2024.
32. オーガナイザー, Summer School on Representation Theory, リー群の群作用と大域解

析に関するセミナー, (virtual 玉原国際セミナーハウス), August 19–23, 2023 (オンライン).

33. オーガナイザー, Summer School on Representation Theory, リー群の群作用と大域解析に関するセミナー, (virtual 玉原国際セミナーハウス), August 17–21, 2022 (オンライン).
34. オーガナイザー, Summer School on Representation Theory, リー群の群作用と大域解析に関するセミナー, (virtual 玉原国際セミナーハウス), August 17–21, 2021(オンライン).
35. オーガナイザー, Integral Geometry, Representation Theory and Complex Analysis, Kavli Institute for the Physics and Mathematics of the Universe, 27–28 January 2020.
36. オーガナイザー, 高木レクチャー, 創設責任者. オーガナイザー第1回-第24回 (currently with Y. Kawahigashi, T. Kumagai, H. Nakajima, K. Ono and T. Saito).
37. オーガナイザー, Summer School on Representation Theory, リー群の群作用と大域解析に関するセミナー, (virtual 玉原国際セミナーハウス), 18–22 August 2020 (オンライン).
38. オーガナイザー, リー群論・表現論セミナー (2007–present 東大; 2003–2007 RIMS; 1987–2001 東大)
39. オーガナイザー, 日仏数学連携拠点, FJ-LMI セミナー, 2023.10– (with M. Pevzner).
40. 群馬県立太田高等学校の大学訪問, 模擬講義, 2023 年 11 月 8 日.
41. Scientific Committee, “Representation Theory and Non Commutative Geometry” at IHP 2025, Paris, France.

G. 受賞他

1. W. M. Goldman et al, The Mathematical Work of Toshiyuki Kobayashi, Symmetry in Geometry and Analysis, Volume 1, Progress in Mathematics **357** (2025), Springer-Nature, pages 1–102.

2. Doctrat Honoris Causa (University of Reims), 2022, France.
3. 日本数学会出版賞 (2019) 『数学の現在 i, e, π 』東京大学出版会, (斎藤毅氏, 河東泰之氏との共同受賞).
4. アメリカ数学会フェロー (2017) 「簡約リー群の構造論と表現論に対する貢献」 (Contribution to Structure Theory and Representation Theory of Reductive Lie groups).
5. 紫綬褒章 (Medal with Purple Ribbon)(2014) 「数学研究」.
6. [学生の受賞] 東京大学学位記授与式における総代, Víctor Pérez-Valdés, (2024 年 3 月, 総代),
東京大学学位記授与式における総代, 甘中一輝 (2021 年 3 月, 総代・答辞),
田森宥好 (2020 年 3 月, 総代).

actions における招待講演 (Mathematicians and Spinors) を行う.

連携併任講座

H. 海外からのビジター

1. M. Pevzner (University of Reims, France), September 2023–, French-Japanese Laboratory of Mathematics and its Interactions, director.
2. Wentao Teng (JSPS PosDoc), October 2023–March 2025. He is working on the (k, a) -generalized Fourier transforms.
3. 郡山幸雄 (エコールポリテクニーク, France), (2024.4). Giving an invited lecture at Opening Conference of French-Japanese Laboratory of Mathematics and its Interactions における招待講演 (Deriving Egalitarian and Proportional Principles from Individual Monotonicity) を行う.
4. Étienne Ghys, (CNRS, École Normale Supérieure de Lyon, France), (2024.4). Giving an invited lecture at Opening Conference of French-Japanese Laboratory of Mathematics and its Interactions における招待講演 (Linking numbers of modular knots) を行う.
5. Jean-Pierre Bourguignon (IHES, France), (2024.4). Giving an invited lecture at Opening Conference of French-Japanese Laboratory of Mathematics and its Inter-