Lie Groups and Representation Theory Seminar at the University of Tokyo

リー群論・表現論セミナー

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Title A positive product formula of integral kernels of k-Hankel transforms

Abstract Let R be a root system in $\mathbb{R}^N$ and G be the finite subgroup generated by the reflections associated to the root system. We establish a positive radial product formula for the integral kernels $B_{k,1}(x,y)$ of $(k,1)$-generalized Fourier transforms (or the k-Hankel transforms) $F_{k,1}$

$$B_{k,1}(x,z)j_{2\langle k\rangle+N-2}\left(2\sqrt{t\,|z|}\right)=\int_{\mathbb{R}^N}B_{k,1}(\xi,z)\,d\sigma_{x,t}^{k,1}(\xi),$$

where j_λ is the normalized Bessel function, and $\sigma_{x,t}^{k,1}(\xi)$ is the unique probability measure. Such a product formula is equivalent to the following representation of the generalized spherical mean operator $f\mapsto M_f,\ f\in C_b(\mathbb{R}^N)$ in $(k,1)$-generalized Fourier analysis

$$M_f(x,t)=\int_{\mathbb{R}^N}f\,d\sigma_{x,t}^{k,1},\ (x,t)\in\mathbb{R}^N\times\mathbb{R}_+.$$

We will then analyze the representing measure $\sigma_{x,t}^{k,1}(\xi)$ and show that the support of the measure is contained in

$$\left\{\xi\in\mathbb{R}^N:\sqrt{|\xi|}\geq|\sqrt{|x|}-\sqrt{t}|\right\}\cap\left(\bigcup_{g\in G}\{\xi\in\mathbb{R}^N:d(\xi,gx)\leq\sqrt{t}\}\right),$$

where $d\left(x,y\right)=\sqrt{|x|+|y|-\sqrt{2\left(|x|\,|y|+\langle x,y\rangle\right)}}$. Based on the support of the representing measure $\sigma_{x,t}^{k,1}$, we will get a weak Huygens's principle for the deformed wave equation in $(k,1)$-generalized Fourier analysis. Moreover, for $N\geq 2$, if we assume that $F_{k,1}\left(\mathcal{S}(\mathbb{R}^N)\right)$ consists of rapidly decreasing functions at infinity, then we get two different results on $\mathrm{supp}\sigma_{x,t}^{k,1}$, which indirectly denies such assumption.