Lie Groups and Representation Theory Seminar at the University of Tokyo

リー群論・表現論セミナー

DATE Nov 27 (Wed), 2024, 13:30–14:30

PLACE Room 122

SPEAKER **Hidenori Fujiwara (藤原英徳)** (OCAMI/Kindai University)

TITLE Inductions and restrictions of unitary representations for exponential solvable Lie groups

ABSTRACT Let $G = \exp \mathfrak{g}$ be a connected and simply connected real nilpotent Lie group with Lie algebra $\mathfrak{g}$, $H = \exp \mathfrak{h}$ an analytic subgroup of G with Lie algebra $\mathfrak{h}$, χ a unitary character of H and $\tau = \mathrm{ind}_H^G \chi$ the monomial representation of G induced from χ. Let $D_\tau(G/H)$ be the algebra of the G-invariant differential operators on the line bundle over G/H associated to the data (H, χ). We denote by C_τ the center of $D_\tau(G/H)$. We know that χ is written as χ_f, where $\chi_f(\exp X) = e^{if(X)}$ $(X \in \mathfrak{h})$ with a certain $f \in \mathfrak{g}^*$ verifying $f([\mathfrak{h}, \mathfrak{h}]) = \{0\}$. Let $S(\mathfrak{g})$ be the symmetric algebra of $\mathfrak{g}$ and $\mathfrak{a}_\tau = \{X + \sqrt{-1}f(X); X \in \mathfrak{h}\}$. We regard $S(\mathfrak{g})$ as the algebra of polynomial functions on $\mathfrak{g}^*$ by $X(\ell) = \sqrt{-1}\ell(X)$ for $X \in \mathfrak{g}$, $\ell \in \mathfrak{g}^*$. Now, $S(\mathfrak{g})$ possesses the Poisson structure $\{,\}$ well determined by the equality $\{X, Y\} = [X, Y]$ if X,Y are in $\mathfrak{g}$. Let us consider the algebra $(S(\mathfrak{g})/S(\mathfrak{g})\overline{\mathfrak{a}_\tau})^H$ of the H-invariant polynomial functions on the affine subspace $\Gamma_\tau = \{\ell \in \mathfrak{g}^* : \ell(X) = f(X), X \in \mathfrak{h}\}$ of $\mathfrak{g}^*$. This inherits the Poisson structure from $S(\mathfrak{g})$. We denote by Z_τ its Poisson center. Michel Duflo asked: the two algebras C_τ and Z_τ, are they isomorphic? Here we provide a positive answer to this question.