Lie Groups and Representation Theory Seminar at the University of Tokyo

リー群論・表現論セミナー

DATE Oct 25 (Thu), 16:30–18:00; Oct 30 (Tue), 15:00–16:30; Nov 1 (Thu), 17:00–18:30; Nov 6 (Tue), 15:00–16:30

PLACE Room 002 (Oct 25), 126 (Oct 30 & Nov 6) or 052 (Nov 1) Graduate School of Mathematical Sciences

SPEAKER **Michaël Pevzner** (Université de Reims and University of Tokyo)

TITLE Quantization of symmetric spaces and representation

ABSTRACT The goal of this series of lectures will be to describe and compare two intimately related but nevertheless fundamentally different methods of quantization of symmetric spaces : on the one hand deformation quantization and symbolic calculus on the other hand. We shall also discuss interesting connections with the representation theory of semi-simple Lie groups.
Undergraduate students are welcome.

- General setting and historic background : mathematical formulation of the Quantization procedure : Weyl symbolic calculus and Moyal star product.
- Kontsevich' deformation quantization of a smooth Poisson mani-fold.
- Quantization of a linear Poisson structure and Duflo isomorphism.
- Non flat case, covariant symbolic calculus on co-adjoint orbits of conformal Lie algebras.
- Quantization of para-Hermitian symmetric spaces : spectral approach.
- Particular case of causal symmetric spaces of Cayley type and Rankin-Cohen brackets.

Lecture 1

The first and introductory lecture of a series of four will be devoted to the discussion of fundamental principles of the Quantum mechanics and their mathematical formulation. This part is not essential for the rest of the course but it might give a global vision of the subject to be considered.

We shall introduce the Weyl symbolic calculus, that relates classical and quantum observables, and will explain its relationship with the so-called deformation quantization of symplectic manifolds.

Afterwards, we will pay attention to a more algebraic question of formal deformation of an arbitrary smooth Poisson manifold and will define the Kontsevich star-product.

Lecture 2

Back to Mathematics. Two methods of quantization.
We'll start with a discussion on

- Weyl symbolic calculus on a symplectic vector space and its asymptotic behavior.

In the second part, as a consequence of previous considerations, we'll define the notion of deformation quantization.

Lecture 3

Kontsevich's formality theorem and applications in Representation theory.
We shall first give

- an explicit construction of an associative star-product on an arbitrary smooth finite-dimensional Poisson manifold.

As application, we'll consider in details the crucial example of the dual of a finite-dimensional Lie algebra and will sketch a generalization of the Duflo isomorphism describing the set of infinitesimal characters of irreducible unitary representations of the corresponding Lie group.

Lecture 4

The last lecture will be devoted to following subjects:

- Application of Kontsevich's star-product to the case of the dual of a Lie algebra.
- Duflo Isomorphism through deformation quantization.
- Covariant symbolic calculus on symmetric spaces.

We shall finish by discussing merits and demerits of these two approaches to the quantization problem of homogeneous spaces.

Pevzner 教授の連続講義のご案内

Michaël Pevzner 先生（フランス）が東京大学に 5 ヶ月間滞在されます。
Pevzner 先生は表現論，調和解析，変形量子化などで活躍されている方です。
4 年生，修士課程の方を対象に易しいトピックから未解決問題までをゆっくりと連続講義していただくことになりました。特に，学部生の皆さんを歓迎します。奮ってご参加ください。

題目　対称空間の量子化と表現論
(Quantization of symmetric spaces and representation)

場所　東大数理研究科棟 126 号室（火曜日），052 号室（木曜日）

日時　第 1 回：10 月 25 日（木）16:30–18:00
第 2 回：10 月 30 日（火）15:00–16:30
第 3 回：11 月 1 日（木）16:30–18:00
第 4 回：11 月 6 日（火）15:00–16:30

要旨　この連続講義の目標は対称空間の量子化の相異なる二つの手法を記述し，比較することである。一つの手法は量子変形で，もう一つは記号計算法である。これらの二つの手法は互いに深い関係をもっているが基本的には異なっている。
また，半単純リー群の表現論との関連についても議論する予定である。

- 一般的な設定と歴史的な背景：量子化の過程の数学的な定式化，線型 Poisson 構造の量子化と Duflo 同型，Weyl 記号計算法と Moyal star 積
- Kontsevich による滑らかな Poisson 多様体の変形量子化
- 線型な Poisson 構造の量子化と Duflo 同型
- 平坦でない場合の conformal Lie 環の余随伴軌道上の記号計算法
- パラエルミート対称空間の量子化：スペクトルによるアプローチ
- Cayley type の causal 対称空間と Rankin–Cohen ブラケット積の特別な場合

世話人：小林俊行