The 9th Takagi Lectures

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Lecture Hall (Room No. 420)
Research Institute for Mathematical Sciences
Kyoto University, Kyoto, Japan

ABSTRACT

S. Brendle: ***Evolution equations in Riemannian geometry***

A fundamental question in Riemannian geometry is to find canonical metrics on a given smooth manifold. In the 1980s, R. Hamilton proposed an approach to this question based on parabolic partial differential equations. The goal is to start from a given initial metric and deform it to a canonical metric by means of an evolution equation. There are various natural evolution equations for Riemannian metrics, including the Ricci flow and the conformal Yamabe flow. We will discuss the global behavior of the solutions to these equations. In particular, we will describe how these techniques can be used to prove the Differentiable Sphere Theorem.

C. E. Kenig: ***Critical nonlinear dispersive equations: global existence, scattering, blow-up and universal profiles***

We will discuss some recent developments in the area of nonlinear dispersive and wave equations, concentrating on the long-time behavior of solutions to critical problems. The issues that arise are global well-posedness, scattering and finite time blow-up. In this direction we will discuss a method to study such problems (which we call the "concentration compactness/rigidity theorem" method) developed by C. Kenig and F. Merle. The ideas used are natural extensions of the ones used earlier, by many authors, to study critical nonlinear elliptic problems, for instance in the context of the Yamabe problem and in the study of harmonic maps. They also build up on earlier works on energy critical defocusing problems. Elements of this program have also proved fundamental in the determination of "universal profiles" at the blow-up time. This has been carried out in recent works of Duyckaerts, C. Kenig and F. Merle. The method will be illustrated with some concrete examples.