The 16th Takagi Lectures

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ABSTRACT

M. Kashiwara:

Riemann–Hilbert Correspondence for Holonomic D*-modules*

A classical Riemann–Hilbert problem asks if a liner ordinary differential equation with regular singularities exists for a given monodromy on a curve.

P. Deligne formulated it as a correspondence of the integrable connections with regular singularities on a complex manifold X with a pole on a hypersurface Y on X and the local systems on $X \setminus Y$.

Later the speaker formulated it as an equivalence of the triangulated category $\mathrm{D}^{\mathrm{b}}_{\mathrm{rh}}(\mathcal{D}_X)$ of $\mathcal{D}_X$-modules with regular holonomic $\mathcal{D}_X$-modules as cohomologies and that of $\mathrm{D}^{\mathrm{b}}_{\mathbb{C}\text{-c}}(\mathbb{C}_X)$ of sheaves on X with $\mathbb{C}$-constructible cohomologies. The equivalence is given by the de Rham functor

$$\mathcal{DR}_X \colon \mathrm{D}^{\mathrm{b}}_{\mathrm{rh}}(\mathcal{D}_X) \to \mathrm{D}^{\mathrm{b}}_{\mathbb{C}\text{-c}}(\mathbb{C}_X).$$

Here $\mathcal{DR}_X(\mathscr{M}) = \Omega_X \otimes_{\mathcal{D}_X} \mathscr{M}$ with Ω_X the sheaf of differential forms of top degree.

However, it was a long standing problem to generalize it to the (not necessarily regular) holonomic D-module case. Recently, the speaker succeeded it by using enhanced version of indsheaves (joint work with Andrea D'Agnolo).

There are two ingredients for it. One is the notion of indsheaves. The notion of indsheaves is introduced to treat the "sheaf " of functions with tempered growth.

The other ingredient is adding an extra variable. We consider indsheaves on $M \times \mathbb{R}$, not on the base manifold M. This permits us to capture the growth of solutions at singular points.

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F. Catanese:

Kodaira Fibrations and Beyond: Methods for Moduli Theory

Kodaira fibred surfaces are a remarkable example of projective classifying spaces, and there are still many intriguing open questions concerning them, especially the slope question. The topological characterization of Kodaira fibrations is emblematic of the use of topological methods in the study of moduli spaces of surfaces and higher dimensional complex algebraic varieties, and their compactifications. Our tour through algebraic surfaces and their moduli (with results valid also for higher dimensional varieties) shall deal with fibrations, questions on monodromy and factorizations in the mapping class group, old and new results on Variation of Hodge Structures, Galois coverings, deformations and rigid varieties (there are rigid Kodaira fibrations). These questions lead to interesting algebraic surfaces, for instance surfaces isogenous to a product with automorphisms acting trivially on cohomology, hypersurfaces in Bagnera–de Franchis varieties, Inoue-type surfaces, remarkable surfaces constructed from VHS.

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J.-P. Demailly:

Recent Progress towards the Kobayashi and Green–Griffiths–Lang Conjectures

The study of entire holomorphic curves contained in projective algebraic varieties is intimately related to fascinating questions of geometry and number theory. The aim of the lectures is to present recent progress on the geometric side of the problem.

The Green–Griffiths–Lang conjecture stipulates that for every projective variety X of general type over $\mathbb{C}$, there exists a proper algebraic subvariety of X containing all non constant entire curves $f : \mathbb{C} \to X$. Using the formalism of directed varieties, we will show that this assertion holds true in case X satisfies a strong general type condition that is related to a certain jet-semistability property of the tangent bundle T_X. It is then possible to exploit this result to investigate the long-standing conjecture of Kobayashi (1970), according to which every general algebraic hypersurface of dimension n and degree at least $2n + 1$ in the complex projective space $\mathbb{P}^{n+1}$ is hyperbolic.

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S.-T. Yau:

From Riemann and Kodaira to Modern Developments on Complex Manifolds

In the first talk, I shall report on the theory of complex manifolds that are related to Riemann and Kodaira, and also some modern development related to uniformization theorems.

In the second talk, I shall talk about the theory of Kähler Einstein metric and the theory of Donaldson–Uhlenbeck–Yau theory of Hermitian Yang Mills connections. A brief discussion on mirror symmetry will be touched.