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Geometric structure in smooth dual and local Langlands conjecture*

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Abstract. This expository paper first reviews some basic facts about p-adic fields, reductive p-adic groups, and the local Langlands conjecture. If G is a reductive p-adic group, then the smooth dual of G is the set of equivalence classes of smooth irreducible representations of G. The representations are on vector spaces over the complex numbers. In a canonical way, the smooth dual is the disjoint union of subsets known as the Bernstein components. According to a conjecture due to ABPS (Aubert–Baum–Plymen–Solleveld), each Bernstein component has a geometric structure given by an appropriate extended quotient. The paper states this ABPS conjecture and then indicates evidence for the conjecture, and its connection to the local Langlands conjecture.

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